Straight Lines

Definition 1 *The* ***general form*** *of an equation representing a straight line is.*

$$Ax + By = C$$

where A is a non-negative real number, and B and C are any real numbers.

Example 2 *The equation*

$$5x + 2y = -10$$

is in general form.

Example 3 *The equation*

$$2y - x = 8$$

is not in general form because after we rearragne the left-hand side,

$$-x + 2y = 8$$

the coefficient of x is negative. We can fix this by multiplying both sides of the equation by -1.

$$\begin{aligned} -1(-x + 2y) &= -1(8) \\ x - 2y &= -8 \end{aligned}$$

and this is now the general form.

Definition 4 *The* ***slope-intercept form*** *of a linear equation is*

$$y = mx + b$$

where m and b are any real numbers. m is called the ***slope*** *of the straight line, and the y-intercept of the line is the point $(0, b)$.*

Theorem 5 *Two straight lines are parallel if and only if they have the same slope. (The only lines that have no slope are the vertical lines, and all vertical lines are parallel to each other.)*

Theorem 6 *Two straight lines are perpendicular if and only if the product of their slopes is -1. (Vertical lines have no slopes, they are only perpendicular to horizontal lines.)*

note: If two numbers multiply to -1, they are called negaitve reciprocals of each other.

Exercise 7 *Find the intercept-slope form of the straight line that has the general form $5x + 2y = -10$.*

Solution 8 *We need to solve the formula for y. First we subtract $2y$,then we divide by 2.*

$$\begin{aligned} 5x + 2y &= -10 \\ 2y &= -10 - 5x \\ 2y &= -5x - 10 \\ y &= \frac{-5x - 10}{2} \\ y &= -\frac{5}{2}x - 5 \end{aligned}$$

Note: the last step can be justified as follows. We start with

$$y = \frac{-5x - 10}{2}$$

To divide is to multiply by the reciprocal:

$$y = (-5x - 10) \cdot \frac{1}{2}$$

Then we apply the law of distributivity and siplify.

Exercise 9 *Find the general form of the straight line that has the intercept-slope form* $y = -\frac{2}{3}x + 5$.

Solution 10 *We first multiply both sides of the equation by* 3*, the denominator. Then we arrange both sides so that* x *and* y *are on the same side. We must be careful to end up with a positive coefficient in front of* x.

$$\begin{aligned} y &= -\frac{2}{3}x + 5 \\ 3y &= -2x + 15 \\ 2x + 3y &= 15 \end{aligned}$$

Exercise 11 *Find the equation of the straight line with slope* $\frac{2}{3}$ *and y-intercept* $(0, -2)$.

Solution 12 *We are looking for the slope-intercept form*

$$y = mx + b$$

We already know that the slope is $\frac{2}{3}$*, and so now our equation is*

$$y = \frac{2}{3}x + b$$

We next use the information that the point $(0, -2)$ *lies on the line. This means that if we plug in* $x = 0$ *and* $y = -2$*, the equal sign must be true. This gives us an equation we can solve for* b.

$$\begin{aligned} y &= \frac{2}{3}x + b \\ -2 &= \frac{2}{3}(0) + b \\ -2 &= b \end{aligned}$$

Thus the answer is

$$y = \frac{2}{3}x - 2.$$

Exercise 13 *Find the equation of the line with slope* $m = -\frac{3}{4}$ *that passes through the point* $(-2, 5)$.

Solution 14 *We are looking for the slope-intercept form*

$$y = mx + b$$

We already know that the slope is $-\frac{3}{4}$*, and so now our equation is*

$$y = -\frac{3}{4}x + b$$

We next use the information that the point $(-2, 5)$ *lies on the line. This means that if we plug in* $x = -2$ *and* $y = 5$*, the equal sign must be true. This gives us an equation we can solve for* b*.*

$$\begin{aligned} y &= -\frac{3}{4}x + b \\ 5 &= -\frac{3}{4}(-2) + b \\ 5 &= \frac{3}{2} + b \\ \frac{7}{2} &= b \end{aligned}$$

Thus the answer is

$$y = -\frac{3}{4}x - \frac{7}{2}.$$

Exercise 15 *Find the equation of the straight line passing through the point* $(-2, 3)$ *and* $(2, -5)$*.*

Solution 16 *We are looking for the spole-intercept form of the line. From the points, we can find the slope using the formula*

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

In our case, let's call $(-2, 3)$ *the first point and* $(2, -5)$ *the second point. So the slope is*

$$m = \frac{-5 - 3}{2 - (-2)} = \frac{-8}{4} = -2$$

note: if we label the points differently, we still get the same slope, from the computiation

$$m = \frac{3 - (-5)}{-2 - 2} = \frac{8}{-4} = -2$$

Now that we have the slope, our equation is

$$y = -2x + b$$

We will obtain an equation for b *by plugging in any of the two given points into our equation. We'll use* $(2, -5)$*.*

$$\begin{aligned} -5 &= -2(2) + b \\ -5 &= -4 + b \\ -1 &= b \end{aligned}$$

And so the equation is $y = -2x - 1$*.*
We can easily check this result by plugging in the two points:

$$\begin{aligned} -5 &= -2(2) - 1 \text{ and} \\ 3 &= -2(-2) - 1 \end{aligned}$$

Because this line contains the two points given, it has to be the line uniquely determined by the points.

Exercise 17 *Find the equation of the straight line that is parallel to* $2x - 3y = -10$ *and passes through the point* $(2, 7)$.

Solution 18 *Since parallel lines have the same slopes, we will have the slope of our line once we find the slope of* $2x - 3y = -10$. *We will do this by solving the formula for* y.

$$\begin{aligned} 2x - 3y &= -10 \\ 2x &= 3y - 10 \\ 2x + 10 &= 3y \\ 3y &= 2x + 10 \\ y &= \frac{2x+10}{3} = \frac{2}{3}x + \frac{10}{3} \end{aligned}$$

Now we know that the slope of our line is $m = \frac{2}{3}$. *To find the slope-intercept form of our line, we plug in the coordinates of the point given.*

$$\begin{aligned} y &= \frac{2}{3}x + b \\ 7 &= \frac{2}{3}(2) + b \\ 7 &= \frac{4}{3} + b \\ \frac{17}{3} &= b \end{aligned}$$

And so the solution is $y = \frac{2}{3}x + \frac{17}{3}$.

Exercise 19 *Find the equation of the straight line that is perpendicular to* $2x + 5y = -1$ *and passes through the point* $(2, -1)$.

Solution 20 *Let us first find the slope of the other line given. We do this by solving the formula for* y.

$$\begin{aligned} 2x + 5y &= -1 \\ 5y &= -2x - 1 \\ y &= \frac{-2x-1}{5} = -\frac{2}{5}x - \frac{1}{5} \end{aligned}$$

Our line is perpendicular to a line with slope $-\frac{2}{5}$. *So our slope must be the negative reciprocal of it,* $\frac{5}{2}$. *(We can check by multiplying the two slopes:* $-\frac{2}{5}\left(\frac{5}{2}\right) = -1$.*)*

Solution 21 *Now we obtain* b *as before.*

$$\begin{aligned} y &= \frac{5}{2}x + b \\ -1 &= \frac{5}{2}(2) + b \\ -1 &= 5 + b \\ -6 &= b \end{aligned}$$

So our equation is $y = \frac{5}{2}x - 6$.