

## Word Problems - SOLUTIONS

1. Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

Solution: Let us denote the number of big ones by  $x$ . Then the number of small ones is  $2x + 3$ . We obtain the equation expressing the total weight:

$$\begin{aligned} 3(2x + 3) + 4x &= 79 && \text{distribute} \\ 6x + 9 + 4x &= 79 && \text{combine like terms} \\ 10x + 9 &= 79 && \text{subtract 9} \\ 10x &= 70 && \text{divide by 10} \\ x &= 7 \end{aligned}$$

The number of big ones is then 7, and so the number of small ones is  $2(7) + 3 = 17$ .

We check: the number of small ones, 17 is indeed 3 more than twice the number of big ones, 7. The total weight is  $7(4) + 17(3) = 28 + 51 = 79$ . Thus the solution is 7 big, 17 small.

2. One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.

Solution: Let us denote the shorter side by  $x$ . Then the longer side is  $5x - 7$ . We obtain the equation for the perimeter:

$$\begin{aligned} 2x + 2(5x - 7) &= 118 && \text{distribute} \\ 2x + 10x - 14 &= 118 && \text{combine like terms} \\ 12x - 14 &= 118 && \text{add 14} \\ 12x &= 132 && \text{divide by 12} \\ x &= 11 \end{aligned}$$

Thus the shorter side is 11 cm, the longer side is  $5(11) - 7 = 48$  cm. We check: the perimeter is  $2(11) + 2(48) = 118$  cm and 48 is indeed 7 shorter than five times 11.

Thus the solution is: 11 cm by 48 cm

3. One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the area of the rectangle is  $430 \text{ cm}^2$ .

Solution: Let us denote the shorter side by  $x$ . Then the longer side is  $5x - 7$ . We obtain the equation for the area:

$$x(5x - 7) = 430$$

Since this equation is quadratic, we will reduce one side to zero, and factor the other side to solve the equation.

$$\begin{aligned} x(5x - 7) &= 430 \\ 5x^2 - 7x &= 430 \\ 5x^2 - 7x - 430 &= 0 \end{aligned}$$

We factor the left hand side by grouping. Since  $5(-430) = -2150$ , we are looking for  $p$  and  $q$  such that

$$\begin{aligned} pq &= -2150 \\ p + q &= -7 \end{aligned}$$

Since  $pq$  is negative, one number must be negative, and the other positive. Since  $p + q$  is negative, the one with the larger absolute value is the negative. Since they are relatively close to each other, the absolute values must be close to  $\sqrt{2150} \simeq 46.3681$ . We find 43 and  $-50$ .

$$\begin{aligned} 5x^2 - 50x + 43x - 430 &= 0 \\ 5x(x - 10) + 43(x - 10) &= 0 \\ (5x + 43)(x - 10) &= 0 \\ 5x + 43 &= 0 \quad \text{or} \quad x - 10 = 0 \\ 5x &= -43 \quad \text{or} \quad x = 10 \\ x_1 &= -\frac{43}{5} \quad \text{and} \quad x_2 = 10 \end{aligned}$$

Since distances can not be negative,  $x = -\frac{43}{5}$  is ruled out. If  $x = 10$ , then the other side is  $5(10) - 7 = 43$ .

We check:  $43 = 5(10) - 7$  and  $10(43) = 430$ .

Thus the rectangle's dimensions are 10 cm by 43 cm.

4. The sum of two consecutive even integers is  $-170$ . Find these numbers.

Solution: Let us denote the shorter number by  $x$ . Then the larger number is  $x + 2$ . The equation expresses the sum of the numbers.

$$\begin{aligned} x + x + 2 &= -170 && \text{combine like terms} \\ 2x + 2 &= -170 && \text{subtract 2} \\ 2x &= -172 && \text{divide by 2} \\ x &= -86 \end{aligned}$$

Then the larger number must be  $-86 + 2 = -84$ . Thus the numbers are  $-86$  and  $-84$ .

5. We have a jar of coins, all quarters and dimes. All together, they are worth \$17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?

Solution: Let us denote the number of dimes by  $x$ . Then the number of quarters must be  $x + 13$ . We obtain the equation by expressing the total value, in pennies:

$$\begin{aligned} 10x + 25(x + 13) &= 1760 && \text{distribute} \\ 10x + 25x + 325 &= 1760 && \text{combine like terms} \\ 35x + 325 &= 1760 && \text{subtract 325} \\ 35x &= 1435 && \text{divide by 35} \\ x &= 41 \end{aligned}$$

Thus we have 41 dimes and  $41 + 13 = 54$  quarters. We check:  $41(0.10) + 54(0.25) = 4.10 + 13.50 = 17.60$

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Thus the solution is 41 dimes and 54 quarters.

6. The sum of three consecutive odd integers is 57. Find these numbers.

Let us denote the smallest number by  $x$ . Then the other two numbers must be  $x + 2$  and  $x + 4$ . The equation expresses the sum of the three numbers.

$$\begin{aligned}x + x + 2 + x + 4 &= 57 && \text{combine like terms} \\3x + 6 &= 57 && \text{subtract 6} \\3x &= 51 && \text{divide by 3} \\x &= 17\end{aligned}$$

Thus the three numbers are 17, and  $17 + 2 = 19$ , and  $17 + 4 = 21$ . We check: indeed,  $17 + 19 + 21 = 57$ .

Thus the solution is 17, 19, and 21.

7. One side of a rectangle is 3 cm shorter than four times the other side. Find the sides if the perimeter of the rectangle is 204 cm.

Solution: Let us denote the shorter side by  $x$ . Then the longer side is  $4x - 3$ . We obtain the equation for the perimeter:

$$\begin{aligned}2x + 2(4x - 3) &= 204 && \text{distribute} \\2x + 8x - 6 &= 204 && \text{combine like terms} \\10x - 6 &= 204 && \text{add 6} \\10x &= 210 && \text{divide by 10} \\x &= 21\end{aligned}$$

Thus the shorter side is 21 cm, the longer side is  $4(21) - 3 = 81$  cm. We check: the perimeter is  $2(21) + 2(81) = 42 + 162 = 204$  cm and 81 is indeed 3 shorter than four times 21.

Thus the solution is: 21 cm by 81 cm

8. The opposite of a number is 18 more than twice the number. Find this number.

Solution: Let us denote the number by  $x$ . The the two things that we are comparing are:

$$\begin{array}{ll} \text{the opposite of the number is} & -x \\ \text{twice the number is} & 2x \end{array}$$

Now we make these two equal by adding the difference to the SMALLER number. Since twice the number is 18 less than the opposite of the number, they will be equal once we add 18 to the smaller one.

$$\begin{aligned}-x &= 2x + 18 && \text{add } x \\0 &= 3x + 18 && \text{subtract 18} \\-18 &= 3x && \text{divide by 3} \\-6 &= x\end{aligned}$$

Thus the number is  $-6$ . Indeed, twice  $-6$  is  $-12$  which is 18 less than 6, the opposite of  $-6$ .

Thus the number is  $-6$ .

9. Two times a number is 5 less than the sum of 80 and the opposite of the number. Find this number.

Solution: let us denote the number by  $x$ . The two things we compare:

$$\begin{array}{ll} \text{two times a number} & : \quad 2x \\ \text{the sum of 80 and the opposite of the number} & : \quad 80 + (-x) = 80 - x \end{array}$$

We make these two equal by adding the difference to the smaller one:

$$\begin{array}{ll} 2x + 5 = 80 - x & \text{add } x \\ 3x + 5 = 80 & \text{subtract 5} \\ 3x = 75 & \text{divide by 3} \\ x = 25 & \end{array}$$

Thus the number is 25. We check: twice 25 is 50 and the sum of 80 and the opposite of 25 is 55. 55 is indeed 5 more than 50. Thus the solution is: the number is 25.

10. Red pens cost \$1 each, blue ones cost \$1.50 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$58?

Solution: Let us denote the number of blue pens by  $x$ . Then the number of red pens is  $5x - 7$ . The equation will express the total cost of the pens:

$$\begin{array}{ll} 1(5x - 7) + 1.50(x) = 58 & \text{distribute} \\ 5x - 7 + 1.5x = 58 & \text{combine like terms} \\ 6.5x - 7 = 58 & \text{add 7} \\ 6.5x = 65 & \text{divide by 6.5} \\ x = 10 & \end{array}$$

Thus we bought 10 blue and  $5(10) - 7 = 43$  red pens. We check:

$$\begin{array}{l} 43 = 5(10) - 7 \\ 1(43) + 1.50(10) = 43 + 15 = 58 \end{array}$$

Thus our solution is correct; we bought 10 blue and 43 red pens.

11. 55 people showed up on the party. There were 3 less women than men. How many men were there?

Solution: Let us denote the number of women by  $x$ . Then  $x + 3$  men showed up. The equation expresses the number of people:

$$\begin{array}{ll} x + x + 3 = 55 & \text{combine like terms} \\ 2x + 3 = 55 & \text{subtract 3} \\ 2x = 52 & \text{divide by 2} \\ x = 26 & \end{array}$$

Thus there were 26 women and 29 men on the party.

12. Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?

Solution: Let us denote by  $x$  the amount that Betty paid. Then Ann paid  $x + 2$ . The equation expresses the total amount paid:

$$\begin{aligned}x + x + 2 &= 38 && \text{combine like terms} \\2x + 2 &= 38 && \text{subtract 2} \\2x &= 36 && \text{divide by 2} \\x &= 18\end{aligned}$$

Thus Betty paid \$18 and Ann paid \$20.