**Pre-Algebra**   Perimeter and Circumference

Definition 1 *The **perimeter** of a geometric object is the length of its boundary.*

One easy way to think about perimeter is like this: Suppose the geometric object is a piece of property. How much fence do we need to buy if we wanted to build a fence around it?

It follows from the definition that perimeter is a measurement of **length**, and thus it is measured in inches, feet, miles, meters, etc.

Based on the definition, we derive the following formulas.

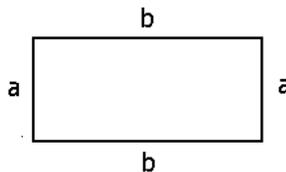The perimeter of a triangle with sides a, b, and c, shown on the picture below is $P = a + b + c$,
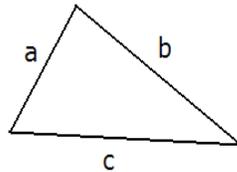
since we simply need to add these three sides.

Example 1 *Find the perimeter of a triangle with sides 9 in, 10 in, and 11 in long.*

Solution 1 *The perimeter is simply the sum of the length of the three sides.*

$$P = 9 \ in \ + 10 \ in \ + 11 \ in \ = 30 \ in$$

The perimeter of a rectangle with sides a and b, shown on the picture below is $P = 2a + 2b$.

This formula also reflects that we simply added all four sides. The opposite sides of every rectangle are of equal length, and so both a and b appears twice.

Example 2 *Find the perimeter of a rectangle with sides 3 m and 11 m.*

Solution 2 *We add all four sides:*

$$P = 2a + 2b = 2 \left(3 \ m \right) + 2 \left(11 \ m \right) = 28 \ m$$

Definition 2 *The **circumference** of a circle is the length of its boundary.*

Circumference is exactly the same concept as perimeter, only mathematicians use this word in case of a circle. There is a formula for the circumference. We will state this formula, but to really prove it, one has to use tools from calculus. The circumference of a circle, denoted by C is

$$C = 2\pi r$$

where r is the radius of the circle
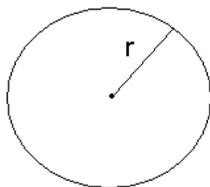
and π is a fixed number, its value is approximately $\pi = 3.141\,592\,654$ (and that's just an approximation!) π is a famous and fascinating number, with many interesting properties. π is an irrational number, and so its decimal presentation is very difficult: it never terminates and it does not follow a repetative pattern as $\dfrac{1}{3} = 0.33333333...$ does. This is the reason that we often use the letter π and not a decimal approximation of it.

Note: Your computations are always more accurate if you use the π button on your calculator instead of typing in 3.14. While the calculator shows $8 - 9$ digits, its memory stores the number with an even higher precision.

Example 3 *Find the circumference of a circle whose radius is 2 feet.*

Solution 3 *We simply apply the formula. Since π is irrational, the numbers rarely will be integers. We will show the result up to four decimal places (that is, four digits after the decimal point), using the correct rules for rounding. We will also carry the units in the computations.*

$$C = 2\pi r = 2\pi\,(2 \text{ ft}) = 12.566\,4 \text{ ft}$$

Sometimes people don't approximate π and show a result we call exact value, in this case 4π ft. The next example shows why it is sometimes more convenient to work with the exact value, so numbers will be 'nice'.

Example 4 *Find the radius of a circle, given that its circumference is 26π mi.*

Solution 4 *We again use the circumference formula. This time it will give us an equation for r.*

$$
\begin{aligned}
C &= 2\pi r & \text{circumference is given} \\
26\pi \text{ mi} &= 2\pi r & \text{divide by } 2\pi \\
\frac{26\pi \text{ mi}}{2\pi} &= r & \text{simplify} \\
r &= 13 \text{ mi}
\end{aligned}
$$

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Example 5 *Find the perimeter of the object shown on the picture below. Angles that look like right angles are right angles.*
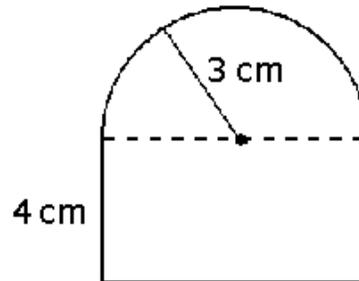
Solution 5 *The bottom side is* 6 *cm long since we can fit the radius twice on it. Thus the three straight sides add up to*

$$Length_1 = 4 \ cm + 6 \ cm + 4 \ cm = 14 \ cm$$

To this we have to add the length of the curved part, which is the circumference of a semi-circle. We compute the circumference of the circle, and then divide it by 2*.*

$$Length_2 = \frac{C}{2} = \frac{2\pi r}{2} = \pi r = \pi \left(3 \ cm\right) = 9.\,424\,8 \ cm$$

Now we just add these two to obtain the perimeter.

$$P = Length_1 + Length_2 = 14 \ cm + 9.\,424\,8 \ cm = 23.\,424\,8 \ cm$$