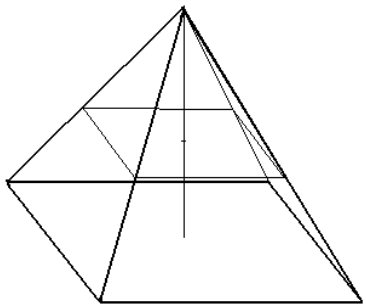
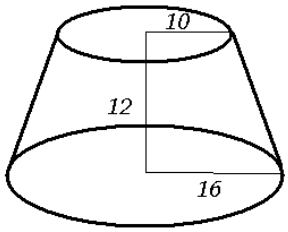
1. Compute the volume of the pyramid with a square base if all of its edges are 10 feet long.

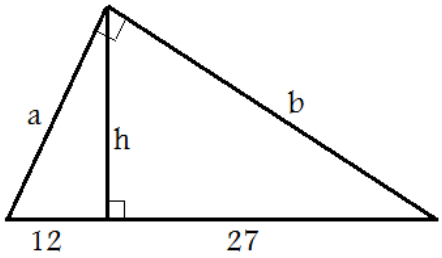
2. Consider the pyramid from problem #1. Suppose we cut it into two parts by a plane parallel to its base so that both parts have the same height. What is the ratio of the volmes of the two objects?



3. Compute the volume of the conical frustum shown on the picture below. Dimensions are in centimeters.



4. Find a, b, and h based on the picture below.



5. Find the value of n if we know that the $n-$sided regular polygon has inner angles measuring 168.75° each.

6. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,2,6,8,10\}$, and $B = \{2,5,8,10\}$, and $C = \{2,3,5,7\}$. Find each of the following.

a) $(A \cup B) \cap C$ c) $A \cap \overline{B}$ e) $\overline{A} \cap \overline{B}$ g) $(A \cup B) \cap (A \cup C)$

b) $A \cup (B \cap C)$ d) $\overline{A \cap B}$ f) $\overline{A \cup C}$ h) $(A \cap B) \cup (A \cap C)$

i) We randomly pull an element of A. If it is an element of C, we get \$5. If not, we pay \$2. What is the expected value for this game?

7. We roll two dice. What is the probability that the sum of the two numbers rolled is 7, given that the sum of the two numbers rolled is larger than 4?

8. We have 20 marbles in a bag: 17 red and 3 blue. We randomly pull three marbles, without replacement. If we pull three red marbles, we pay \$10. In any other case, we receive \$10 for every blue marbles pulled. (i.e. one blue marble means we recieve \$10, two blue marbles means we receive \$20, three of them means we receive \$30.) Find the expected value for this game.

9. We have 10 marbles in a bag: 7 red, 2 blue, and 1 yellow. We randomly pull two marbles, with replacement. Find the expected value of the number of blue marbles pulled.

10. We have 10 marbles in a bag: 7 red, 2 blue, and 1 yellow. We randomly pull two marbles, without replacement. Find the expected value of the number of blue marbles pulled.

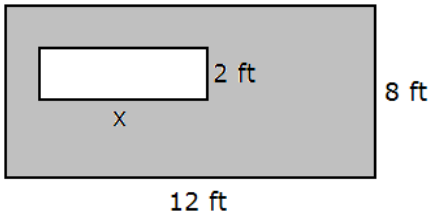
11. Find the present value of \$100000, twenty years from now. Assume a compound annual interest rate of 5%, compounded

    a) annually b) monthly c) daily d) continuously

12. We pull two cards from $\{1, 2, ..., 10\}$, without replacement. If the product of the two number pulled is even, we pay \$3. If it is odd, we receive \$7. Find the expected value of this game.

13. We pull two cards from $\{1, 2, ..., 10\}$, with replacement. If the product of the two number pulled is even, we pay \$3. If it is odd, we receive \$7. Find the expected value of this game.

14. We drop small objects on the rectangle shown on the picture below. Find the value of x if the following is given: the probability of an object landing in the shaded area is $\frac{43}{48}$.



15. A collection of 20 calculators are being tested. Turns out that 5 out of the 20 are defective. We randomly draw two claculators. What is the probability that both calculators drawn are defective? Assume

    a) replacement b) no replacement

16. A collection of 20 calculators are being tested. Turns out that 5 out of the 20 are defective. We randomly draw three claculators. What is the probability that exactly two calculators drawn are defective? Assume

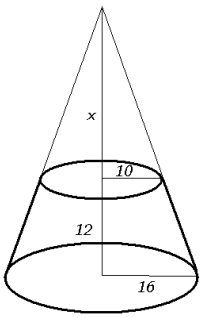
    a) replacement b) no replacement

Answers

1. $\frac{1}{3}(10\,\text{ft})^2\sqrt{50}\,\text{ft} = \frac{500}{3}\sqrt{2}\,\text{ft}^3 \approx 235.702\,260\,4\,\text{ft}^3$

2. bottom part to top part: 7 to 1

3. First find height of the original cone using similar triangles:



$$\begin{aligned} \frac{16}{10} &= \frac{12+x}{x} \\ 16x &= 10(12+x) \\ 16x &= 120+10x \\ 6x &= 120 \\ x &= 20 \end{aligned}$$

So the entire cone had height 32 centimeters long. Now the volume can be computed as a difference of two volumes (both cones):

$$V = V_{\text{big}} - V_{\text{small}} = \frac{1}{3}\pi(16\,\text{cm})^2(32\,\text{cm}) - \frac{1}{3}\pi(10\,\text{cm})^2(20\,\text{cm}) = 2064\pi\,\text{cm}^3 \approx 6484.247\,237\,\text{cm}^3$$

4. $h = 18 \quad a = \sqrt{468} = 6\sqrt{13} \quad b = \sqrt{1053} = 9\sqrt{13}$

5. $n = 32$

6. a) $\{2,5\}$ b) $\{1,2,5,6,8,10\}$ c) $\{1,6\}$ d) $\{1,3,4,5,6,7,9\}$ e) $\{3,4,7,9\}$ f) $\{2\}$
   g) $\{1,2,5,6,8,10\}$ h) $\{2,8,10\}$ i) $-\$0.6$

7. $\frac{1}{5}$

8. $P(3\text{ red}) = \frac{17}{20}\left(\frac{16}{19}\right)\left(\frac{15}{18}\right) = \frac{34}{57}$
   $P(2\text{ red}) = \frac{17}{20}\left(\frac{16}{19}\right)\left(\frac{3}{18}\right)(3) = \frac{34}{95}$
   $P(1\text{ red}) = \frac{17}{20}\left(\frac{3}{19}\right)\left(\frac{2}{18}\right)(3) = \frac{17}{380}$
   $P(0\text{ red}) = \frac{3}{20}\left(\frac{2}{19}\right)\left(\frac{1}{18}\right) = \frac{1}{1140}$

   $\frac{34}{57}(-10) + \frac{34}{95}(10) + \frac{17}{380}(20) + \frac{1}{1140}(30) = -\frac{167}{114} = -1.464\,912\,280\,701\,8$

9. $P(2\text{ blue}) = \frac{2}{10}\left(\frac{2}{10}\right) = \frac{1}{25}$
   $P(0\text{ blue}) = \frac{8}{10}\left(\frac{8}{10}\right) = \frac{16}{25}$
   $P(1\text{ blue}) = 1 - \left(\frac{1}{25} + \frac{16}{25}\right) = \frac{8}{25}$

   $E = \frac{1}{25}(2) + \frac{8}{25}(1) + \frac{16}{25}(0) = \frac{2}{5}$

10. $P(2\text{ blue}) = \frac{2}{10}\left(\frac{1}{9}\right) = \frac{1}{45}$
    $P(0\text{ blue}) = \frac{8}{10}\left(\frac{7}{9}\right) = \frac{28}{45}$
    $P(1\text{ blue}) = 1 - \left(\frac{1}{45} + \frac{28}{45}\right) = \frac{16}{45}$

$$E = \frac{1}{45}(2) + \frac{16}{45}(1) + \frac{28}{45}(0) = \frac{2}{5}$$

11. a) \$37 688.95 b) \$36 864.45 c) \$36 790.46 d) \$36 787.94

12. $-\frac{7}{9}$

13. $-\frac{1}{2}$

14. **5** ft equation: $\frac{96-2x}{96} = \frac{43}{48}$

15. a) $\left(\frac{5}{20}\right)^2 = \frac{1}{16}$ b) $\left(\frac{5}{20}\right)\left(\frac{4}{19}\right) = \frac{1}{19}$

16. a) $\binom{3}{1}\left(\frac{5}{20}\right)^2\left(\frac{15}{20}\right) = \frac{9}{64}$ b) $\binom{3}{1}\left(\frac{5}{20}\right)\left(\frac{4}{19}\right)\left(\frac{15}{18}\right) = \frac{5}{38}$