Math 118 Extra Credit Problems

1. Partitions.

   (a) Find two sets A and B that satisfy all of the following conditions:

      i. Both A and B have infinitely many elements.

      ii. $A \cap B = \varnothing$

      iii. $A \cup B = \mathbb{N}$

   (b) Express $\mathbb{N}$ as the union of three infinite, pairwise disjoint sets. In other words, find sets A, B, and C such that

      i. A, B, and C have infinitely many elements.

      ii. (pairwise disjoint) $A \cap B = \varnothing$ and $A \cap C = \varnothing$ and $B \cap C = \varnothing$.

      iii. $A \cup B \cup C = \mathbb{N}$

   (c) Express $\mathbb{N}$ as the union of infinitely many, infinite, and pairwise disjoint sets.

2. Find three infinite sets A, B, and C such that $A \cap B$ is an infinite set, $A \cap C$ is an infinite set, $B \cap C$ is an infinite set, and $A \cap B \cap C = \varnothing$.

3. S is a subset of $\mathbb{N}$, satisfying the following two properties.

   $1 \in S$

   For every natural number k, if $k \in S$, then $k + 1 \in S$.

   Describe the set S.

4. (Hilbert) You are the manager of an infinitely long motel shown on the picture below. Your office is the room labeled M. Every room can accomodate one person, and so happens, every room is occupied. Every room is equipped with a speaker, so you can talk to every guest from your office. One rainy night a person arrives and needs a room. Make room for the new guest! (Nobody can board in the mamager's office, and no room can be occupied by more than one person. Yet, it is possible to free up a room. How?)