We will first establish a language. Consider the statement

$$\frac{2}{5} \text{ of } 70 \text{ is } 28$$

In this statement, we will call $\frac{2}{5}$ the *fraction*, 70 the *of-number*, and 28 the *is-number*. While this language is not at all elegant, it will prove to be useful.

$$\underset{\text{Fraction}}{\frac{2}{5}} \text{ of } \underset{\text{of}}{70} \text{ is } \underset{\text{is}}{28}$$

Every basic fraction or percent problem is based on this connection. In such a problem, any two of the three quantities can be given, and we would have to solve for the third one. In either case, we can use the formula

$$(\text{is}) = \text{F} \cdot (\text{of})$$

Example 1 *(Type 1) Find* 15% *of* 400.

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = \quad x$
$F = \quad \frac{15}{100}$
$(\text{of}) = \quad 400$

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) \\ x &= \frac{15}{100} \cdot 40 \\ x &= 60 \end{aligned}$$

Thus 15% of 400 is 60.

Example 2 *(Type 2)* 21 *is what percent of* 350*?*

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = \quad 21$
$F = \quad x$
$(\text{of}) = \quad 350$

We will substitute these into the formula and solve for x.

$$\begin{aligned} (\text{is}) &= \text{F} \cdot (\text{of}) & \\ 21 &= x \cdot 350 & \text{divide by } 350 \\ \frac{21}{350} &= x & \end{aligned}$$

We obtained the value of x, but not as a percent. We will convert x into a percent. First we simplify

$$x = \frac{21}{350} = \frac{\cancel{7} \cdot 3}{\cancel{7} \cdot 50} = \frac{3}{50} = \frac{3 \cdot 2}{50 \cdot 2} = \frac{6}{100} = 6\%$$

Thus 21 is 6% of 350.
We can check by computing 6% of 350:

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ (\text{is}) &= \frac{6}{100} \cdot 350 = 21\end{aligned}$$

Thus our solution, 6% is correct.

Example 3 *(Type 3)* 24% *of what number is* 72*?*

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = \quad 72$
$F = \quad \frac{24}{100}$
$(\text{of}) = \quad x$

We will substitute these into the formula and solve for x.

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ 72 &= \frac{24}{100} \cdot x \qquad \text{divide by } \frac{24}{100} \\ \frac{72}{\frac{24}{100}} &= x \\ 300 &= x\end{aligned}$$

The computation is $72 \div \frac{24}{100} = \frac{72}{1} \cdot \frac{100}{24} = \frac{3 \cdot 24}{1} \cdot \frac{100}{24} = \frac{300}{1} = 300$. Thus 72 is 24% of 300.
We can check by computing 24% of 300:

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ (\text{is}) &= \frac{24}{100} \cdot 300 = 72\end{aligned}$$

Thus our solution is correct.
We have focused on the steps of solving such equations. Computations can be simplified by reducing fractions or working with decimals instead of fractions. We will work out three more basic problems to demonstrate these.

Example 4 *(Type 1) Find* 16% *of* 3600.

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

$(\text{is}) = \quad x$
$F = \quad 0.16$
$(\text{of}) = \quad 3600$

We will substitute these into the formula and solve for x. $0.16 \cdot 360 = 57.6$

$$\begin{aligned}(\text{is}) &= \text{F} \cdot (\text{of}) \\ x &= 0.16 \cdot 3600 \\ x &= 576\end{aligned}$$

Thus 16% of 3600 is 576.

Example 5 *(Type 2)* 45 *is what percent of* 18*?*

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. Because the is-number is smaller than the of-number, we should expect a percentage larger than 100%.

(is) = 27
F = x
(of) = 18

We will substitute these into the formula and solve for x.

$$\begin{array}{rcll} \text{(is)} & = & \text{F} \cdot \text{(of)} & \\ 27 & = & x \cdot 18 & \text{divide by 18} \\ \dfrac{27}{18} & = & x & \\ x & = & 1.5 = 150\% & \end{array}$$

Thus 27 is 150% of 18.
We can check by computing 150% of 18:

$$\begin{array}{rcl} \text{(is)} & = & \text{F} \cdot \text{(of)} \\ \text{(is)} & = & 1.5 \cdot 18 = 27 \end{array}$$

Thus our solution, 6% is correct.

Example 6 *(Type 3)* 120% *of what number is* 150*?*

We first write a table, listing the three quantities, is-number, fraction, and of-number. We need to identify the two quantities given, and call the third one x. In this case,

(is) = 150
F = 1.2
(of) = x

We will substitute these into the formula and solve for x.

$$\begin{array}{rcll} \text{(is)} & = & \text{F} \cdot \text{(of)} & \\ 150 & = & 1.2 \cdot x & \text{divide by 1.2} \\ \dfrac{150}{1.2} & = & x & \\ 125 & = & x & \end{array}$$

We can check by computing 120% of 125:

$$\begin{array}{rcl} \text{(is)} & = & \text{F} \cdot \text{(of)} \\ \text{(is)} & = & 1.2 \cdot 125 = 150 \end{array}$$

Thus our solution, 125 is correct.
The following examples all boil down to one of the basic problems shown above. One advice: before starting computations, re-write the problem to a basic question. Once we have this question, the problem is easy to solve.

Example 7 *We placed* \$ 8000 *into a bank account with an annual* 7% *of interest rate. How much money do we have in the account a year later?*

Solution 1: We compute 7% of 8000 and add the result to 8000. The basic question is: What is 7% of 8000? (Type 1)
(is) = x
F = 0.07
(of) = 8000
We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 0.07 \cdot 8000 \\ x &= 560 \end{aligned}$$

Thus we earned \$ 560 in interest, and now we have \$ 8000 + \$ 560 = \$ 8560.
Solution 1: This is a neat shortcut that will become very important in other problems. If a quantity is increased by 7%, then it "grew up" from 100% of itself to 107% of itself. We can find the amount in the bank, if we simply compute 107% of 8000. The basic question is: What is 107% of 8000?
(is) = x
F = 1.07
(of) = 8000
We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ x &= 1.07 \cdot 8000 \\ x &= 8560 \end{aligned}$$

Thus we now have \$ 8560.

Example 8 *The population of a town has decreased from* 80 000 *to* 68 000. *What percent of a decrease does this represent?*

Solution 1: We subtract 68 000 from 80 000 to determine the change. $80\,000 - 68\,000 = 12\,000$. Now the question is: 12 000 is what percent of 80 000? (Type 2)
(is) = 12 000
F = x
(of) = 80 000
We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 12\,000 &= x \cdot 80\,000 \\ \frac{12\,000}{80\,000} &= x \\ 0.15 &= x \end{aligned}$$

Thus

$$x = 0.15 = \frac{0.15}{1} = \frac{0.15\,(100)}{1\,(100)} = \frac{15}{100} = 15\%$$

This is a 15% decrease.

Solution 2: Instead of comparing the chage to the original condition, we can compare the new condition to the original condition and interpret the result. The question may be re-phrased as: 68 000 is what percent of 80 000? Then

(is) = 68 000
$F =$ x
(of) = 80 000

We substitute the data into the formula:

$$\begin{aligned} \text{(is)} &= \text{F} \cdot \text{(of)} \\ 68\,000 &= x \cdot 80\,000 \\ \frac{68\,000}{80\,000} &= x \\ 0.85 &= x \end{aligned}$$

Thus

$$x = 0.85 = \frac{0.85}{1} = \frac{0.85\,(100)}{1\,(100)} = \frac{85}{100} = 85\%$$

Since the population has decreased from 100% of itself to 85% of itself, our result reflects a 15% decrease.

Example 9 *Tom got a* 4% *raise in his job. Now he makes* 2496 *per month. How much was he making before the raise?*

It would be a mistake to simply subtract 4% of 2496 from 2496, because the raise was 4% of a smaller, unknown number! The trick is to realize that Tom is now making exactly 104% of his original pay. The question is thus (Type 3) 104% of what number is 2496?

(is) = 2496
F = 1.04
(of) = x

We substitute the data into the formula:

$$\begin{array}{rcll} \text{(is)} & = & \text{F} \cdot \text{(of)} & \\ 2496 & = & 1.04 \cdot x & \text{divide by } 1.04 \\ \dfrac{2496}{1.04} & = & x & \\ 2400 & = & x & \end{array}$$

Thus his original pay was \$ 2400 per month. We can check by taking 104% of 2400 and find that it is indeed 2496.

Example 10 *A TV set went on a* 35% *sale. The sale price is* \$ 312. *Find the original price.*

This problem is similar to the previous one in the sense that it is type 3, and the method used previously is essential. It would be a mistake to simply add 35% of 312 from 312, because the deduction was 35% of a larger, unknown number! The trick is to realize that the current price is 65% of the original price. The difference is that we subtract 35% this time. The question is thus (Type 3) 65% of what number is 312?

(is) = 312
F = 0.65
(of) = x

We substitute the data into the formula:

$$\begin{array}{rcll} \text{(is)} & = & \text{F} \cdot \text{(of)} & \\ 312 & = & 0.65 \cdot x & \text{divide by } 0.65 \\ \dfrac{312}{0.65} & = & x & \\ 480 & = & x & \end{array}$$

Thus the original price was \$ 480. We can check by taking 65% of 480 and find that it is indeed 312.

Exercises

1. What fraction of $2\frac{1}{8}$ is $\frac{1}{4}$?
2. 3.45 times what number is 4. 14?
3. Eighty-seven percent of 300 is what number?
4. One hundred thirty percent of what number is 78?
5. Fifteen percent of what number is 10.5?
6. What percent of 450 is 288?
7. What percent of 460 is 1150?
8. Three hundred forty-seven percent of what number is 2429?
9. Fifteen percent of the town's population are students. If there are 1800 students living in the town, how many people live there?
10. Paul earned $ 128 this week in his part time job. If this was a sixty percent increase from last week, how much money did he make last week?
11. A TV went on a 14% sale. The sale price is $ 412. 8. Find the original price of the TV.
12. Overnight, the number of bacteria increased by one hundred sixty percent. There are now 650000 bacteria. How many was there yesterday?
13. Susan got a 10% raise. A week later, she got another 10% raise. If we express the two raises as one, what percentage do we obtain?
14. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value? (Hint: if no value for the stock is given, make up a few different numbers.)

Answers for Exercises

1. What fraction of $2\frac{1}{8}$ is $\frac{1}{4}$? $\frac{2}{17}$
2. 3.45 times what number is 4.14? 1.2
3. Eighty-seven percent of 300 is what number? 261
4. One hundred thirty percent of what number is 78? 60
5. Fifteen percent of what number is 10.5? 70
6. What percent of 450 is 288? 64%
7. What percent of 460 is 1150? 250%
8. Three hundred forty-seven percent of what number is 2429? 700
9. Fifteen percent of the town's population are students. If there are 1800 students living in the town, how many people live there? 12000
10. Paul earned $ 128 this week in his part time job. If this was a sixty percent increase from last week, how much money did he make last week? $ 80
11. A TV went on a 14% sale. The sale price is $ 412.8. Find the original price of the TV. $ 480
12. Overnight, the number of bacteria increased by one hundred sixty percent. There are now 650000 bacteria. How many was there yesterday? 250000
13. Susan got a 10% raise. A week later, she got another 10% raise. If we express the two raises as one, what percentage do we obtain? 21%
14. A stock loses 60% of its value. What must the percent of increase be to recover all of its lost value? (Hint: if no value for the stock is given, make up a few different numbers.) 150%