1. The Fibonacci Sequence is a fascinating sequence. It starts with 1 and 1, and for every new element, we add the last two elements. The first few elements are $1, 1, 2, 3, 5, 8, 13, \ldots$ We denote the nth element of the Fibonacci Sequence by F_n. Find the 10th element of the sequence, i.e. find the value of F_{10}.

2. A trick: We start with any three-digit number. We first create a six-digit number by writing down the three-digit number twice. For example, if we start with 140, our six-digit number is 140140. No matter what number we started with, this number will be divisible by 7. Divide the six-digit number by 7. This number is divisible by 11. Divide. The result is divisible by 13. Divide. What do you notice? Can you explain it?

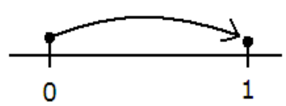
3. What is the remainder if we divide 3^{140} by 7?

4. The set of all natural numbers is arranged in an infinite triangular shape as indicated below.

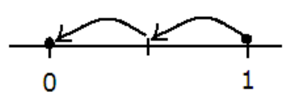
				1				
			2	3	4			
		5	6	7	8	9		
	10	11	12	13	14	15	16	
17	18	19	20	21	22	23	24	25

   Find the number in the middle of the 40th row.

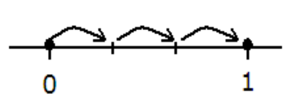
5. A cricket is sitting on the number line, on zero. It decides to jump to one.



   Then it changes its mind and wants to get back to zero. Since it is a bit tired, it uses two equal jumps to get back to zero.



   Then it changes its mind again, and wants to get back to one. Since it is more tired, it uses three equal jumps to get back to one.



   Then it jumps back to zero, using four equal jumps. Then it jumps back to one, using five equal jumps. And so on, in the nth trip, using n equal jumps, never stopping. Is this cricket going to land on every point between zero and one? Explain your answer.

6. Partitions.

   (a) Find two sets A and B that satisfy all of the following conditions:

      i. Both A and B have infinitely many elements.

      ii. $A \cap B = \varnothing$

      iii. $A \cup B = \mathbb{N}$

   (b) Express $\mathbb{N}$ as the union of three infinite, pairwise disjoint sets. In other words, find sets A, B, and C such that

      i. A, B, and C have infinitely many elements.

      ii. (pairwise disjoint) $A \cap B = \varnothing$ and $A \cap C = \varnothing$ and $B \cap C = \varnothing$.

      iii. $A \cup B \cup C = \mathbb{N}$

   (c) Express $\mathbb{N}$ as the union of infinitely many, infinite, and pairwise disjoint sets.

7. Find three infinite sets A, B, and C such that $A \cap B$ is an infinite set, $A \cap C$ is an infinite set, $B \cap C$ is an infinite set, and $A \cap B \cap C = \varnothing$.

8. S is a subset of $\mathbb{N}$, satisfying the following two properties.

   1. $1 \in S$
   2. For every natural number k, if $k \in S$, then $k + 1 \in S$

   Describe the set S.

9. S is the smallest subset of $\mathbb{N}$ with the following properties.

   1. $2 \in S$
   2. If $x \in S$, then $3x \in S$
   3. If $x \in S$, then $x + 5 \in S$

   (a) List the 10 smallest elements of S.

   (b) Find the smallest number that is larger than 118 and is not in S.

10. We increased a quantity Q by $g\%$. Then we decreased it by $d\%$, and found that we now again have the original quantity Q. Find all values of g and d so that they are both integers.

11. (Hilbert) You are the manager of an infinitely long motel shown on the picture below. Your office is the room labeled M. Every room can accomodate one person, and so happens, every room is occupied. Every room is equipped with a speaker, so you can talk to every guest from your office. One rainy night a person arrives and needs a room. Make room for the new guest! (Nobody can board in the mamager's office, and no room can be occupied by more than one person. Yet, it is possible to free up a room. How?)