1. Convert each of the following decimals to fractions.

   (a) $1.18 =$
   (b) $1.1\overline{8} =$
   (c) $1.\overline{18} =$
   (d) $0.\overline{118} =$

2. We toss a coin seven times.

   (a) How many outcomes are possible?
   (b) How many different ways can we get two times heads and five times tails?

3. We throw a die twice.

   (a) How many outcomes are possible where the sum of the two numbers rolled is less than five?
   (b) How many outcomes are possible where the two numbers rolled are different?

4. Ten points are given on a circle. We connect every point with all other points on the circle.

   (a) How many line segments are drawn?
   (b) How many different triangles are there on the figure?
   (c) How many different four-sided polygons are there on the figure?

5. The population of a town has decreased from 450 000 to 418 500 What percent of a decrease does this represent?

6. Ten years ago, the population of a town was 60 000. After five years, the population has increased by 20%. After another five years, the population again has increased by 20%.

   (a) How many people live in the town today?
   (b) By what percent did the population grow in the last ten years?

7. We borrowed \$5000 for three years, with a simple annual interest rate of 8%. How much do we need to pay back at the end of the three years?

8. We borrowed \$5000 for three years, with a compound annual interest rate of 8%. How much do we need to pay back at the end of the three years?

9. We asked 120 people if they listen to radio, watch TV or read books. 85 watch TV, 69 listen to radio, and 74 read books. 51 watch TV and listen to radio, 57 watch TV and read books, and 43 listen to radio and read books. 29 do all three.

   (a) Draw a Venn diagram depicting the information given.
   (b) How many people do exactly two of these activities?
   (c) How many people read books or watch TV?
   (d) How many people do neither of these?

10. Consider a rectangle with sides 5 ft and 12 ft.

    (a) Find the perimeter of the rectangle. Include units in your computation and answer.

    (b) Find the area of the rectangle. Include units in your computation and answer.

11. (Round your final answer to the nearest penny.) We placed \$ 500 into a bank account. How much money do we have in the account after 20 years if the account comes with an annual

    (a) simple interest rate of 7%?

    (b) compound interest rate of 7%?

12. Identities in set theory. Among numbers, an **identity** is a statement of equality that is true for every number. For example, the equation $x + 3x = 4x$ is an identity. Consider statements such as $(A')' = A$, $A \cup U = U$, or $A \cup \varnothing = A$. These statements are true for every set A, and so we may call them identities. Find the other side of the following identities.

    (a) $A \setminus A' =$

    (b) $A \cap \varnothing =$

    (c) $A \cup A' =$

    (d) $A \cap A' =$

    (e) $A \cap U =$

13. If X and Y are sets, how do we prove that X is a subset of Y? The procedure is always the same: we prove that the definition of $X \subseteq Y$ is true. Recall the definition: X is a **subset** of Y, (denoted by $X \subseteq Y$) if every element of X also belongs to Y. In short:

$$X \subseteq Y \quad \text{if} \quad \text{for all } x \in X, \text{ we also have } x \in Y$$

    Claim: For every sets A and B, $(A \cup B)' \subseteq A' \cap B'$

    Proof. Let x be any element of $(A \cup B)'$ We will show that then x is also an element of $A' \cap B'$.

    1. If x is an element of $(A \cup B)'$, then x is not an element of $A \cup B$. (by the definition of complement).

    2. Since x is not an element of $A \cup B$, it cannot belong to neither A, nor B. (by the definition of union).

    3. Since x is not in A, then it is an element of A'. Similarly, x is an element of B' (by the definition of complement).

    4. Since x is in both A' and B', it is an element of their intersection, $A' \cap B'$ (by the definition of intersection). ♠

    The same argument with set theory and logic notation: Let $x \in (A \cup B)'$ be given.

$$\begin{array}{lcl} x \in (A \cup B)' & \Longrightarrow & x \notin A \cup B \\ x \notin A \cup B & \Longrightarrow & x \notin A \quad \text{and} \quad x \notin B \\ & & x \in A' \quad \text{and} \quad x \in B' \quad \Longrightarrow \quad x \in A' \cap B' \spadesuit \end{array}$$

    Use the method shown above to prove the following statement:

$$A' \cap B' \subseteq (A \cup B)'$$

14. Proofs of identities in set-theory. We prove the general statement $X = Y$ among sets, we usually prove two separate statements: that $X \subseteq Y$ and $Y \subseteq X$. In the previous problem, we actually proved that

$$(A \cup B)' = A' \cap B'$$

This is one of the two statements we call the **DeMorgan's Laws.** Prove the other DeMorgan's Law:

$$(A \cap B)' = A' \cup B'$$