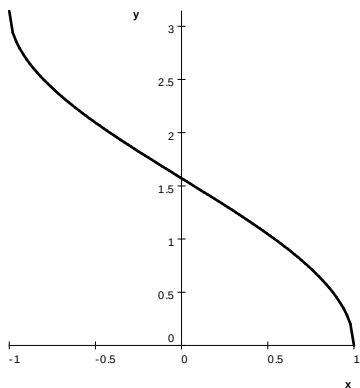


Sample Exam for the Final Exam - Version 2

SOLUTIONS

1. .



domain: $-1 \leq x \leq 1$

range: $0 \leq f(x) \leq \pi$

2. .

(a) $\sqrt{3}$

(b) $\frac{-\sqrt{6} + \sqrt{2}}{4}$

3. Prove each of the following identities.

(a) $\frac{\cos x - \cos y}{\cos x + \cos y} = -\tan \frac{x+y}{2} \tan \frac{x-y}{2}$

Solution:

$$\begin{aligned} LHS &= \frac{\cos \left(\frac{x+y}{2} + \frac{x-y}{2} \right) - \cos \left(\frac{x+y}{2} - \frac{x-y}{2} \right)}{\cos \left(\frac{x+y}{2} + \frac{x-y}{2} \right) + \cos \left(\frac{x+y}{2} - \frac{x-y}{2} \right)} = \\ &= \frac{\cos \frac{x+y}{2} \cos \frac{x-y}{2} - \sin \frac{x+y}{2} \sin \frac{x-y}{2} - \left(\cos \frac{x+y}{2} \cos \frac{x-y}{2} + \sin \frac{x+y}{2} \sin \frac{x-y}{2} \right)}{\cos \frac{x+y}{2} \cos \frac{x-y}{2} - \sin \frac{x+y}{2} \sin \frac{x-y}{2} + \left(\cos \frac{x+y}{2} \cos \frac{x-y}{2} + \sin \frac{x+y}{2} \sin \frac{x-y}{2} \right)} = \\ &= \frac{-2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}}{2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}} = RHS \end{aligned}$$

(b) $\cot 3x = \frac{3 \tan^2 x - 1}{\tan^3 x - 3 \tan x}$

$$\begin{aligned} \text{Solution: } LHS &= \frac{1}{\tan 3x} = \frac{1}{\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}} = \frac{1 - \tan x \tan 2x}{\tan x + \tan 2x} = \frac{1 - \tan x \frac{2 \tan x}{1 - \tan^2 x}}{\tan x + \frac{2 \tan x}{1 - \tan^2 x}} \\ &= \frac{1 - \tan x \frac{2 \tan x}{1 - \tan^2 x}}{\tan x + \frac{2 \tan x}{1 - \tan^2 x}} \cdot \frac{1 - \tan^2 x}{1 - \tan^2 x} = \frac{1 - \tan^2 x - 2 \tan^2 x}{\tan x (1 - \tan^2 x) + 2 \tan x} = \frac{1 - 3 \tan^2 x}{3 \tan x - \tan^3 x} = RHS \end{aligned}$$

4. .

(a) $\pm \frac{9}{\sqrt{106}}$

(b) $-\frac{96}{265}$

(c) $-\frac{23}{264}$

(d) $-\frac{44}{125}$

5. $-5^\circ \quad 55^\circ \quad 115^\circ \quad 175^\circ \quad 235^\circ \quad 295^\circ$
 $-25^\circ \quad 35^\circ \quad 95^\circ \quad 155^\circ \quad 215^\circ \quad 275^\circ$

6. $\beta_1 = 27.622^\circ \quad \gamma_1 = 112.378^\circ \quad c = 5.754$

7. .

(a) $50.2655 \frac{\text{m}}{\text{s}}$

(b) $0.2513 \frac{1}{\text{s}}$

(c) $(-161.8034, 117.557)$