Extra Credit Assignment 5

1. The **Fibonacci sequence**, (F_n) is $1, 1, 2, 3, 5, 8, 13, 21, ...$ Every element is obtained by adding the previous two elements. The recursive definition is

$$\begin{aligned} F_1 &= 1,\ F_2 = 1, \text{ and} \\ F_{n+2} &= F_n + F_{n+1} \quad \text{for all natural number } n, \text{ with } n \geq 1 \end{aligned}$$

   Consider now another sequence, (q_n) that is formed by taking the ratios of consecutive elements of the Fibonacci sequence. That is, $\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, ...$

$$q_n = \frac{F_n}{F_{n+1}} \quad \text{for all natural number } n.$$

   Compute the exact value of the number that these ratios approach.

2. Is there a Fibonacci-type of a sequence that does not grow to be indefinitely large (positive or negative)?

3. (Viete's picture) Consider the picture shown below. Line AB is tangent to the unit circle, where D is the point of tangency. Let α denote angle DCE . Match each of the six trigonometric functions with the length of each of the line segments given.

   $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, $\sec\alpha$, $\cot\alpha$ and AC, AD, BC, BD, CE, DE