

Review Problems

1. Convert to radians.

- a) 45° b) -120° c) 10° d) 135° e) 270°

2. Convert to degrees.

- a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

3. Find the exact value of each of the following expressions.

- a) $\sin 120^\circ$ c) $\tan \frac{7\pi}{4}$ e) $\cos(-90^\circ)$ g) $\sec\left(-\frac{5\pi}{3}\right)$ i) $\tan(-270^\circ)$
 b) $\cos 120^\circ$ d) $\sin \frac{5\pi}{6}$ f) $\cos \frac{5\pi}{6}$ h) $\sin\left(-\frac{\pi}{2}\right)$

4. Find all angles coterminal with 120° . Express your answer

- a) in degrees b) in radians
 c) Find all coterminal angles α such that $-500^\circ < \alpha < 500^\circ$.

5. True or false?

- a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .
 b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .

6. Simplify each of the following. Assume that x is positive.

- a) $(-16)^{-3/4}$ e) $\frac{\sin 10^\circ}{\cos 80^\circ}$ i) $(\sqrt{3 + \sqrt{5}} - \sqrt{3 - \sqrt{5}})^2$
 b) $-16^{-3/4}$ f) $\sin\left(\frac{\pi}{3}\right)$ j) $\cos \frac{\pi}{6}$
 c) $(x^{2/3})^{3/4} \left(\frac{1}{\sqrt{x}}\right)$ g) $\tan\left(\frac{\pi}{6}\right)$ k) $\csc \frac{\pi}{4}$
 d) $\frac{2}{(\sqrt{5} - 1)^2}$ h) $(1 + \sqrt{3})^3 (1 - \sqrt{3})^3$ l) $\frac{3 + \sqrt{3}}{3 - \sqrt{3}}$

7. Simplify each of the following.

- a) -3^{-2} e) $\log_{81} 3$ i) $\ln(e^3)$ m) $9^{\log_3 7}$ q) $e^{\ln 2}$
 b) $(-3)^{-2}$ f) $\log_5 1$ j) $\log_{49} 7$ n) $5^{\log_{25} 36}$ r) $\log_{100} 1000$
 c) $81^{-3/4}$ g) $\log_5 0$ k) $\log_4 \left(\frac{1}{32}\right)$ o) $\log_8 \left(\frac{1}{16}\right)$ s) $3^{\log_9 2}$
 d) $16^{7/4}$ h) $\log_m(m^4)$ l) $\log_8 16$ p) $\log_{27} \left(\frac{1}{9}\right)$ t) $\ln\left(\frac{1}{e^2}\right)$

8. Simplify each of the following expressions.

- a) $\log_5(5^b)$ c) $\log_2 \left(\left(\frac{1}{8}\right)^p\right)$ e) $3^{\log_3 4}$ g) $3^{\log_9 2}$
 b) $\log_5(125^m)$ d) $\log_{16}(2^x)$ f) $9^{\log_3 5}$ h) $2^{\log_8 10}$

9. Find the exact value of each of the following expressions.

a) $\sin 30^\circ \cos 30^\circ \tan 30^\circ - \sin 60^\circ \cos 60^\circ \tan 60^\circ$

f) $\sin^4\left(\frac{\pi}{4}\right) + \cos^4\left(\frac{\pi}{4}\right)$

b) $\sin 60^\circ \sec 30^\circ - 3 \tan 30^\circ - \cot 45^\circ$

g) $\sin\left(\frac{5\pi}{2}\right) - \cos\left(\frac{7\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right) - \cos(7\pi)$

c) $\cos\frac{\pi}{6} + \sin\frac{\pi}{3} - \tan\frac{\pi}{3}$

h) $\log_{10}(0.01) + \log_8\left(\frac{1}{2}\right) - \ln\left(\frac{1}{e^5}\right)$

d) $\cos\frac{\pi}{3} + 2 \sin\frac{\pi}{4} - \tan\frac{\pi}{4} + \cos\frac{\pi}{6}$

i) $\cos 420^\circ - \tan 210^\circ + \sec 240^\circ + \cot 135^\circ$

e) $\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ$

10. Place each of the following logarithms between two consecutive integers.

a) $\log_2 20$

c) $\log_3 10$

e) $\log_{10} 314$

g) $\log_2 1000$

b) $\log_5 100$

d) $\log_{10} 2011$

f) $\log_2 100$

h) $\log_3 100$

11. Re-write each of the following logarithmic statements as an exponential statement.

a) $\ln x = 10$

c) $\log_7 21 = y$

e) $\log_A B = C$

g) $\log_x 5 = 2$

b) $\log_3 P = -2$

d) $\log_x y = 4$

f) $\log_2(x+1) = -2$

h) $\ln(x-5) = 1$

12. Re-write each of the following exponential statements as a logarithmic statement.

a) $A^B = C$

b) $4^x = 7$

c) $3^{A-1} = 12$

d) $e^x = 24$

13. Solve each of the following equations for x . Present the exact value of each solution. (Hint: re-write the logarithmic statements as exponential statements!)

a) $\log_5 x = 3$

d) $\ln(x-1) = -2$

g) $\frac{2}{3} \ln(x-1) - 1 = 5$

b) $\log_x 16 = 2$

e) $\log_3(2x^2 - 5) = 3$

h) $\frac{\log_2(5x+1) - 1}{3} = -1$

c) $\frac{1}{2} \log_2(3x+1) = 3$

f) $\log_{10}(x-2) = -1$

14. Solve each of the following equations. Make sure to check your solutions.

a) $\sqrt{x+6} = 1 + \sqrt{x-1}$

b) $\sqrt{x} + \sqrt{x-5} = 5$

c) $x^2 + x = 1$

d) $\sqrt{3x+1} = \sqrt{x+4} + 3$

15. Find the domain of each of the following functions.

a) $f(x) = \frac{1}{5x+3}$

d) $f(x) = \sqrt{x^2 - 10x + 24}$

g) $f(x) = \log_3(x+7)$

b) $f(x) = \sqrt{5x+3}$

e) $f(x) = \sqrt{x-3} - 2$

h) $f(x) = \log_5(x^2 + 6x)$

c) $f(x) = \frac{1}{x^2 - 10x + 24}$

f) $f(x) = \frac{x+5}{\sqrt{x-3} - 2}$

i) $f(x) = \tan x$

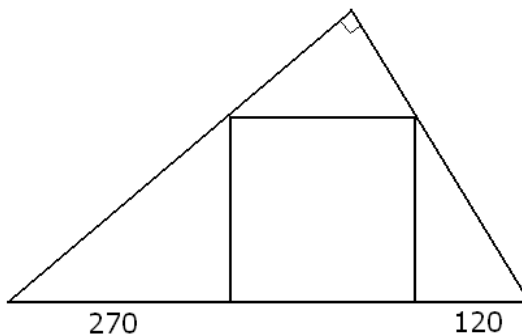
16. Find the exact values for $\sin \alpha$ and $\cos \alpha$ if we know that $0 < \alpha < 90^\circ$ and $\tan \alpha = \frac{3}{10}$.

17. The first element in an arithmetic sequence is 80. Find the common difference if we also know that s_9 is eighteen times a_{11} .

18. Given the arithmetic sequence by $a_1 = -16$ and $d = \frac{1}{3}$, find all values of n so that $s_n = 50$.

19. Given the arithmetic sequence by $a_1 = -10$ and $d = \frac{1}{2}$, find all values of n so that $s_n = -100$.

20. Let (a_n) be an arithmetic sequence. Find the first element and common difference if the fifth element is twice the opposite of the first element and the product of the first and fourth elements is the opposite of the ninth element.
21. Find the height of a tree if the angle of elevation of its top changes from 15° to 25° as the observer advances 120 ft toward its base.
22. Find an equation of the straight line passing through the intersections of $(x + 4)^2 + (y - 4)^2 = 25$ and $(x - 10)^2 + (y - 2)^2 = 125$.
23. Consider the circle $(x - 10)^2 + (y - 2)^2 = 125$. Find the equation of all tangent lines drawn to the circle that have slope $m = 2$.
24. A satellite is 1000 miles above the equator of the Earth. The radius of Earth is 3960 miles. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?
25. A circle has a radius of 10 units. A point P is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from P to the circle.
26. The sum of a and three times b is 100. What is the smallest possible value of $a^2 + b^2$?
27. Find the area of a regular polygon of 12 sides is written into a circle of radius 4 cm.
28. Prove each of the following identities.
- a) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$ d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$
- b) $\tan x - \csc x \sec x (1 - 2 \cos^2 x) = \cot x$ e) $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$
- c) $\frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$
29. Graph each of the following functions.
- a) $f(x) = \frac{1}{x}$ b) $f(x) = \frac{1}{x^2}$ c) $f(x) = |x|$ d) $f(x) = x^3$
30. Graph each of the following pairs of functions together, in the same coordinate system.
- a) $f(x) = x^2$ and $g(x) = (x + 3)^2$ d) $f(x) = 2^x$ and $g(x) = \log_2 x$
- b) $f(x) = x^3$ and $g(x) = (x + 3)^3$ e) $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{1/2} x$
- c) $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ f) $f(x) = \log_2 x$ and $g(x) = \log_{1/2} x$
31. The picture below shows a square within a right triangle. Find the length of the sides in the square.



Answers

- 1.) a) $\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{\pi}{18}$ d) $\frac{3\pi}{4}$ e) $\frac{3\pi}{2}$
- 2.) a) 30° b) -20° c) 75° d) -540° e) 315°
- 3.) a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) -1 d) $\frac{1}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$ g) 2 h) -1 i) undefined
- 4.) a) $120^\circ + k \cdot 360^\circ$ where $k = 0, 1, -1, 2, -2, 3, -3, \dots$
 b) $\frac{2\pi}{3} + 2k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, \dots$ c) $-240^\circ, 120^\circ, 480^\circ$
- 5.) a) True: $\alpha = 40^\circ + k \cdot 360^\circ \implies 2\alpha = 80^\circ + k \cdot 720^\circ$
 b) False: Let $\beta = 190^\circ$. Clearly 10° and 190° are not co-terminal, but $2\beta = 380^\circ$ which is co-terminal with $2\alpha = 20^\circ$.
- 6.) a) undefined b) $-\frac{1}{8}$ c) 1 d) $\frac{\sqrt{5}+3}{4}$ e) 1 f) $\frac{\sqrt{3}}{2}$ g) $\frac{\sqrt{3}}{3}$ h) -8 i) 2 j) $\frac{\sqrt{3}}{2}$
 k) $\sqrt{2}$ l) $2 + \sqrt{3}$
- 7.) a) $-\frac{1}{9}$ b) $\frac{1}{9}$ c) $\frac{1}{27}$ d) 128 e) $\frac{1}{4}$ f) 0 g) undefined h) 4 i) 3 j) $\frac{1}{2}$ k) $-\frac{5}{2}$
 l) $\frac{4}{3}$ m) 49 n) 6 o) $-\frac{4}{3}$ p) $-\frac{2}{3}$ q) 2 r) $\frac{3}{2}$ s) $\sqrt{2}$ t) -2
- 8.) a) b b) $3m$ c) $-3p$ d) $\frac{1}{4}x$ e) 4 f) 25 g) $\sqrt{2}$ h) $\sqrt[3]{10}$
- 9.) a) $-\frac{1}{2}$ b) $-\sqrt{3}$ c) 0 d) $\sqrt{2} + \frac{\sqrt{3}}{2} - \frac{1}{2}$ e) 1 f) $\frac{1}{2}$ g) $\frac{1}{2}$ h) $\frac{8}{3}$ i) $-\frac{\sqrt{3}}{3} - \frac{5}{2}$
- 10.) a) $4 < \log_2 20 < 5$ b) $2 < \log_5 100 < 3$ c) $2 < \log_3 10 < 3$ d) $3 < \log_{10} 2011 < 4$
 e) $2 < \log_{10} 314 < 3$ f) $6 < \log_2 100 < 7$ g) $9 < \log_2 1000 < 10$ h) $4 < \log_3 100 < 5$
- 11.) a) $e^{10} = x$ b) $3^{-2} = P$ c) $7^y = 21$ d) $x^4 = y$ e) $A^C = B$ f) $\frac{1}{4} = x + 1$ g) $x^2 = 5$
 h) $e = x - 5$ 12.) a) $\log_A C = B$ b) $x = \log_4 7$ c) $A - 1 = \log_3 12$ d) $x = \ln 24$
- 13.) a) 125 b) 4 c) 21 d) $\frac{1}{e^2} + 1$ e) $4, -4$ f) 2.1 g) $e^9 + 1$ h) $-\frac{3}{20}$
- 14.) a) 10 b) 9 c) $\frac{-1 + \sqrt{5}}{2}, \frac{-1 - \sqrt{5}}{2}$ d) 21

15.) a) $x \neq -\frac{3}{5}$ b) $x \geq -\frac{3}{5}$ c) $x \neq 4, 6$ d) $x \leq 4$ or $x \geq 6$ e) $x \geq 3$ f) $x \geq 3$ but $x \neq 7$

g) $x > -7$ h) $x < -6$ or $x > 0$ i) $x \neq \frac{\pi}{2} + k\pi$ $k \in \mathbb{Z}$

16.) $\sin \alpha = \frac{3}{\sqrt{109}}$, $\cos \alpha = \frac{10}{\sqrt{109}}$ 17.) -5 18.) 100 19.) $16, 25$ 20.) $a = -4, d = 3$ or $a = d = 0$

21.) 75.588461 ft 22.) $y = 7x + 7$ 23.) $y = 2x + 7$ and $y = 2x - 43$ 24.) $1299 \frac{\text{mi}}{\text{s}}$

25.) 72.063758° 26.) 1000 27.) 48 cm^2

28.) a) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

$$\text{LHS} = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\left(\frac{1}{\sin x}\right)} + \frac{\cos x}{\left(\frac{1}{\cos x}\right)} = \sin^2 x + \cos^2 x = 1 = \text{RHS}$$

b) $\tan x - \csc x \sec x (1 - 2 \cos^2 x) = \cot x$

$$\begin{aligned} \text{LHS} &= \tan x - \csc x \sec x (1 - 2 \cos^2 x) = \frac{\sin x}{\cos x} - \frac{1}{\sin x} \left(\frac{1}{\cos x}\right) (1 - 2 \cos^2 x) \\ &= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} + \frac{2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x - 1 + 2 \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - 1 + \cos^2 x}{\sin x \cos x} \\ &= \frac{1 - 1 + \cos^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x = \text{RHS} \end{aligned}$$

c) $\frac{1 - \cos x}{1 + \cos x} = \frac{\sec x - 1}{\sec x + 1}$

$$\text{RHS} = \frac{\sec x - 1}{\sec x + 1} = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \frac{\frac{1}{\cos x} - \frac{\cos x}{\cos x}}{\frac{1}{\cos x} + \frac{\cos x}{\cos x}} = \frac{\frac{1 - \cos x}{\cos x}}{\frac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} = \text{LHS}$$

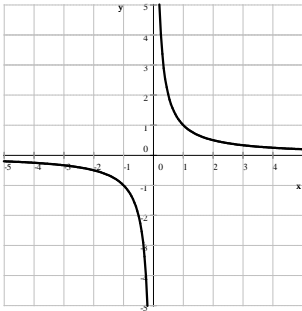
d) $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

$$\begin{aligned} \text{LHS} &= \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x}}{\frac{\cos x \sin x}{\cos x \sin x} + \frac{\sin x \cos x}{\sin x \cos x}} = \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{\cos x \sin x + \sin x \cos x}{\cos x \sin x}} = \frac{\frac{\sin x + \cos x}{\cos x \sin x}}{\frac{1}{\cos x \sin x}} \\ &= \frac{\sin x + \cos x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{1} = \sin x + \cos x = \text{RHS} \end{aligned}$$

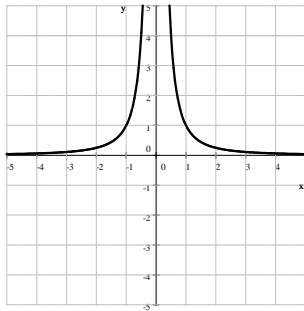
e) $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{(1 + \sin^2 x + 2 \sin x) - (1 + \sin^2 x - 2 \sin x)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2 \sin x - 1 - \sin^2 x + 2 \sin x}{\cos^2 x} = \frac{4 \sin x}{\cos^2 x} = 4 \frac{\sin x}{\cos x} \frac{1}{\cos x} = 4 \tan x \sec x = \text{RHS} \end{aligned}$$

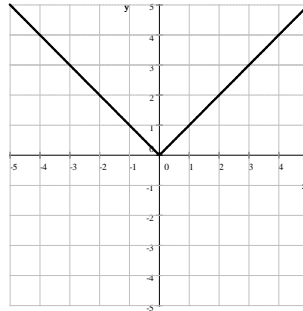
29.) a) $f(x) = \frac{1}{x}$



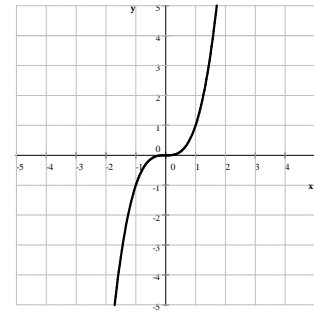
b) $f(x) = \frac{1}{x^2}$



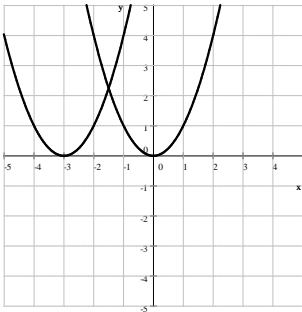
c) $f(x) = |x|$



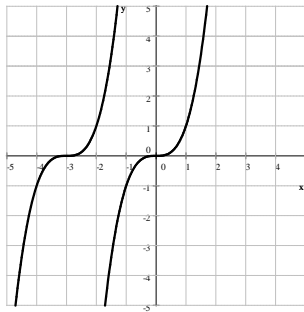
d) $f(x) = x^3$



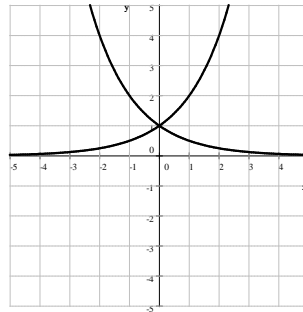
30.) a)



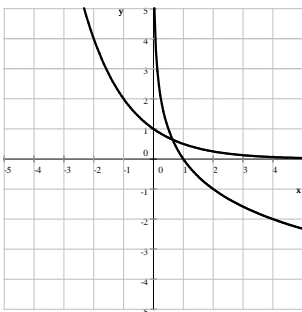
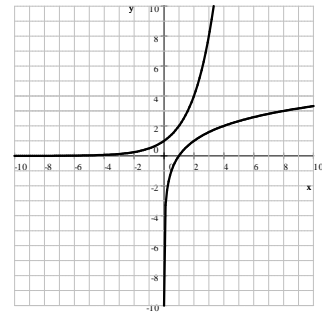
b)



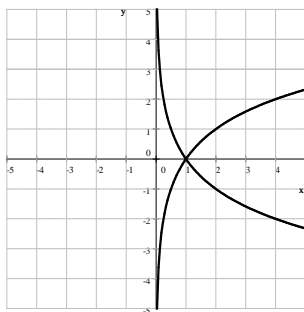
c)



d)



e)



f)

31.) 180