

## Review Problems

- Write each of the following as a single logarithm.
  - $\ln(x+1) + \ln(x-1)$
  - $\log_{10}(24x^3) - \log_{10}(3x)$
  - $\frac{1}{2} \ln(100x^{100})$
  - $\frac{1}{2} \log_2(36x^{16}) - 2 \log_2(3x) + \log_2 6 - 6 \log_2 x$
  - $(\log_3 4)(\log_4 7)$
- Let  $a = \log_5 3$ . Write each of the following in terms of  $a$ .
  - $\log_5 45$
  - $\log_5 75$
  - $\log_3 5$
  - $\log_5 675$
  - $\log_{45} 675$
- Graph each of the following.
  - $f(x) = \frac{2x-3}{x-1}$
  - $f(x) = 2\sqrt[3]{x+3} - 1$
  - $f(x) = \log_2(8x) - 1$
- Find the domain for each of the following functions.
  - $f(x) = \frac{\sqrt{5x-x^2}}{x-5}$
  - $f(x) = \log_2(x^2 - 6x + 5) + \sqrt{x+3}$
  - $h(x) = \frac{1}{-1 + \ln x}$
  - $h(t) = \sqrt{3 - \sin t}$
- We place \$1000 into a bank account with an annual compound interest rate of 8%, compounded annually. How long does it take for the money in the account to reach \$1000 000?
- Suppose that  $\alpha$  is an angle in the first quadrant with  $\sin \alpha = \frac{3}{5}$  and  $\beta$  is an angle in the second quadrant with  $\cos \beta = -\frac{5}{13}$ . Compute the exact value of each of the following.
  - $\cos \alpha$
  - $\tan \alpha$
  - $\sin 2\alpha$
  - $\cos 2\alpha$
  - $\tan 2\alpha$
  - $\sin \beta$
  - $\tan \beta$
  - $\sin 2\beta$
  - $\cos 2\beta$
  - $\tan 2\beta$
  - $\sin(\alpha + \beta)$
  - $\cos(\alpha + \beta)$
  - $\tan(\alpha + \beta)$
- Compute the exact value of each of the following.
  - $\sin 75^\circ$
  - $\tan 255^\circ$
  - $\frac{\tan 25^\circ + \tan 20^\circ}{1 - (\tan 25^\circ)(\tan 20^\circ)}$
  - $\cos 105^\circ$
- Prove, using the sum formula that for all  $x$ ,
  - $\sin(x + 180^\circ) = -\sin x$
  - $\cos(x + 180^\circ) = -\cos x$
  - $\tan(x + 180^\circ) = \tan x$
- Suppose that  $\alpha$  is an angle with  $\cos \alpha = -\frac{2}{3}$ . Compute the exact value of  $\cos 2\alpha$ .
- Suppose that  $\alpha$  and  $\beta$  are acute angles with  $\cot \alpha = 7$  and  $\cos \beta = \frac{3}{\sqrt{10}}$ . Compute the exact value of each of the following.
  - $\tan \beta$
  - $\tan 2\beta$
  - $\tan(\alpha + 2\beta)$
- Find the exact value of  $\tan \alpha$  if  $\tan \beta = -\frac{1}{5}$  and  $\tan(\alpha + \beta) = \frac{7}{17}$ .

12. Find the exact value of  $\tan \alpha$  if  $\tan 2\alpha = \frac{4}{3}$ .

13. Find a formula for  $\cot(x + y)$ .

14. Solve each of the following equations.

a)  $\cos 2x = \sin x$

f)  $\sin x + 1 = \cos^2 x + 2$

j)  $\log_2(1 - 3x) + \log_2(3 - x) = 7$

b)  $\sin 2x = \cos x$

g)  $\sqrt{x - 2} + \sqrt{x + 3} = 5$

k)  $\log_3(3x - 5) - \log_3(x - 6) = -2$

c)  $\cos 2x = \cos x$

h)  $\log_2(2x - 7) - \log_2(5x - 1) = -3$

d)  $\sin x + \sin 2x = \tan x$

i)  $\frac{3}{8} \ln(2x - 5) = 21$

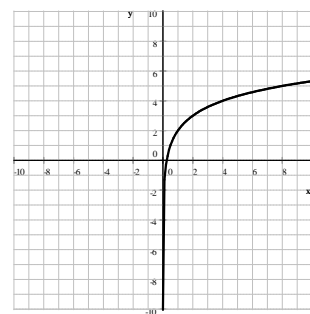
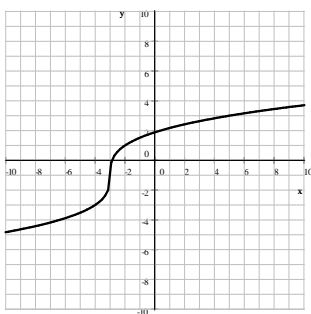
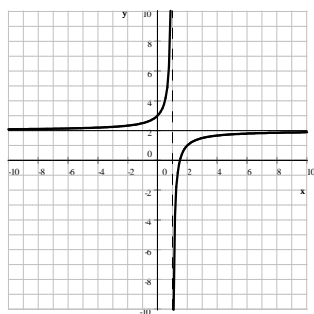
e)  $\cos x + \frac{\sin^2 x}{\cos x} + \sin x + \sin 2x = \frac{1}{\cos x}$

## Answers

1.) a)  $\ln(x^2 - 1)$     b)  $\log_{10}(8x^2)$     c)  $\ln(10x^{50})$     d) 2    e)  $\log_3 7$

2.) a)  $2a + 1$     b)  $a + 2$     c)  $\frac{1}{a}$     d)  $3a + 2$     e)  $\frac{3a + 2}{2a + 1}$

3.) a)  $f(x) = \frac{2x - 3}{x - 1} = -\frac{1}{x - 1} + 2$     b)  $f(x) = 2\sqrt[3]{x + 3} - 1$     c)  $f(x) = \log_2(8x) - 1 = \log_2 x + 2$



4.) a)  $[0, 5)$     b)  $[-3, 1) \cup (5, \infty)$     c)  $(0, \infty) \setminus \{e\}$     d)  $\mathbb{R}$     5.)  $t = \frac{\ln 1000}{\ln 1.08} \approx 89.756521$  years

6.) a)  $\frac{4}{5}$     b)  $\frac{3}{4}$     c)  $\frac{24}{25}$     d)  $\frac{7}{25}$     e)  $\frac{24}{7}$     f)  $\frac{12}{13}$     g)  $-\frac{12}{5}$     h)  $-\frac{120}{169}$

i)  $-\frac{119}{169}$     j)  $\frac{120}{119}$     k)  $\frac{33}{65}$     l)  $-\frac{56}{65}$     m)  $-\frac{33}{56}$

7.) a)  $\frac{\sqrt{6} + \sqrt{2}}{4}$     b)  $\sqrt{3} + 2$     c) 1    d)  $\frac{\sqrt{2} - \sqrt{6}}{4}$

8.) a)  $\sin(x + 180^\circ) = \sin x \cos 180^\circ + \cos x \sin 180^\circ = \sin x(-1) + \cos x(0) = -\sin x$

b)  $\cos(x + 180^\circ) = \cos x \cos 180^\circ - \sin x \sin 180^\circ = \cos x(-1) - \sin x(0) = -\cos x$

c)  $\tan(x + 180^\circ) = \frac{\tan x + \tan 180^\circ}{1 - \tan x \tan 180^\circ} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \frac{\tan x}{1} = \tan x$

9.)  $-\frac{1}{9}$     10.) a)  $\frac{1}{3}$     b)  $\frac{3}{4}$     c) 1    11.)  $\frac{2}{3}$     12.) -2 or  $\frac{1}{2}$

13.)  $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

14.) a)  $-\frac{\pi}{2} + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$  where  $k \in \mathbb{Z}$     b)  $\frac{\pi}{2} + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$  where  $k \in \mathbb{Z}$

c)  $2k\pi, \pm \frac{2\pi}{3} + 2k\pi$  where  $k \in \mathbb{Z}$     d)  $k\pi, \pi + 2k\pi, \pm \frac{1}{3}\pi + 2k\pi$  where  $k \in \mathbb{Z}$

e)  $k\pi, \pm \frac{2\pi}{3} + 2k\pi$  where  $k \in \mathbb{Z}$     f)  $\frac{\pi}{2} + 2k\pi$  where  $k \in \mathbb{Z}$     g) 6    h) 5

i)  $\frac{1}{2}e^{56} + \frac{5}{2}$     j) -5    k) no solution