Review Problems

1. A manufacturer estimates that when q thousand units of a particular commodity are produced each month, the total cost will be $C(q) = 0.4q^2 + 3q + 40$ thousand dollars, and all q units can be sold at a price of $p(q) = 22.2 - 1.2q$ thousand dollars per unit. Find the maximal profit.

2. Graph each of the following equations. In case of each of these equations, determine whether y is a function of x or not.

   a) $y = x^2 - 6x + 8$ c) $x = y^2$ e) $xy = 1$ g) $x^2y = 1$

   b) $x^2 = y^2$ d) $x^2 + y = 4$ f) $2x + 3y = 4$ h) $x^2 + y^2 = 9$

3. Simplify each of the following.

   a) $\log_{10}(8x^7) + \frac{1}{2}\log_{10}(25x^{16}) - 2\log_{10}(2x^3)$ c) $\ln(a^2 - 4) - \ln(a - 2)$

   b) $5^{-\log_{25} 11}$ d) $8^{\log_8 m - \log_2(3m)}$

4. Solve each of the following equations.

   a) $\sqrt{2x-1} + \sqrt{x-1} = 5$ d) $\frac{2}{3}\ln(2x-5) + 3 = 7$ g) $\log_3(x-4) + \log_3(2x-5) = 2$

   b) $3^{2x-1} = 2^{2-x}$ e) $\log_{3x+1}\left(\frac{1}{4}\right) = -1$ h) $\frac{1}{x} + \frac{1}{x-2} = \frac{3}{4}$

   c) $4^x - 5 \cdot 2^x = -4$ f) $\log_3(x-3) - \log_3(4x-2) = -2$

5. Solve each of the following inequalities.

   a) $\frac{5x-2}{2x+1} \geq 2$ b) $\frac{x}{2-x} > 4$

6. Perform each of the following divisions.

   a) $4x^5 - x^3 + 8$ by $x^2 + x - 2$ b) $x^5 - 1$ by $x - 1$

7. Suppose that $x = \log_5 10$ and $y = \log_9 5$. Express $\log_{24} 60$ in terms of x and y.

8. Find the points where the circles $(x-1)^2 + (y+7)^2 = 50$ and $(x-5)^2 + (y+4)^2 = 25$ intersect each other.

9. Find an equation of the tangent line drawn to the graph of $4x - 2y + x^2 + y^2 = 20$ at the point $(-6, 4)$.

10. Solve each of the following triangles.

    a) $a = 4$ $b = 17$ $c = 7$ d) $a = 4$ $c = 3$ $\gamma = 43°$

    b) $a = 4$ $c = 10$ $\alpha = 43°$ e) $a = 4$ $c = 3$ $\beta = 43°$

    c) $a = 4$ $c = 3$ $\alpha = 43°$ f) $a = 4$ $b = 7$ $c = 9$

11. Graph each of the following functions.

    a) $f(x) = \frac{2}{x+1} + 3$ b) $f(x) = 2^{x-3} - 3$ c) $f(x) = |x+3| - 2$ d) $f(x) = (x-1)^3 - 1$

12. Sketch the graph of each of the following functions.

a) $f(x) = (x+2)(x-1)(x-4)$ b) $f(x) = -x^3 + 4x$ c) $f(x) = -\frac{1}{2}x(x-3)^2$

13. Sketch the graph of each of the following functions.

a) $f(x) = x - 2$ d) $f(x) = \frac{1}{x-2}$ g) $f(x) = \frac{(x-2)^5}{(x-2)^7}$ j) $f(x) = \frac{(x-2)^5}{(x-2)^4}$

b) $f(x) = (x-2)^2$ e) $f(x) = \frac{1}{(x-2)^2}$ h) $f(x) = \frac{(x-2)^2}{(x-2)^5}$ k) $f(x) = \frac{(x-2)^7}{(x-2)^5}$

c) $f(x) = (x-2)^3$ f) $f(x) = \frac{1}{(x-2)^3}$ i) $f(x) = \frac{(x-2)^6}{(x-2)^6}$ l) $f(x) = \frac{(x-2)^5}{(x-2)^2}$

14. Let (a_n) be a geometric sequence with $a_1 = 50$ and $r = \frac{2}{3}$.

a) Find an approximate value (up to 4 decimal places) of a_7.

b) Find an approximate value (up to 4 decimal places) of s_{10}.

15. Find the exact value of the infinite sum of the geometric sequence $180, 60, 20, \ldots$.

16. Compute the exact value of $\sin\frac{x}{2}$ if $\cos x = \frac{1}{3}$.

17. Compute the exact value of $\tan 22.5°$.

18. a) Write $\sin 5x \cos 17x$ as a sum or difference.

b) Write $\cos 8x - \cos 20x$ as a product.

19. Prove that if α and β are angles in a triangle such that

$$(\sin\alpha + \sin\beta)^2 + (\cos\alpha - \cos\beta)^2 = 2$$

then the triangle has a right angle.

20. Let l be the line $y = \frac{5}{12}x$. Let k be the line that bisects the angle formed by l and the positive part of the $x-$axis. Find the equation of k.

21. Graph each of the following functions on the interval $[-2\pi, 2\pi]$

a) $f(x) = \sin x$ b) $f(x) = \cos x$

22. Compute each of the following limits.

a) $\lim_{x\to-\infty}(-2x^5 + 8x^2)$ e) $\lim_{x\to-\infty}\frac{3x^2-1}{5x^2-3x+2}$ i) $\lim_{x\to\infty}\frac{-x^3+2x+1}{x-3}$

b) $\lim_{x\to\infty}(-2x^5 + 8x^2)$ f) $\lim_{x\to-\infty}\frac{100x-1}{5x^2-3x+2}$ j) $\lim_{x\to-\infty}(2^x)$

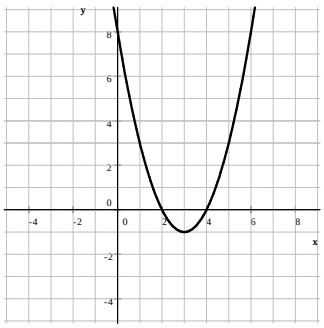
c) $\lim_{x\to-\infty}(-2x^5 + 8x^6)$ g) $\lim_{x\to-\infty}\log_2 x$ k) $\lim_{x\to\infty}\frac{2^{x+5}}{4^{x-1}}$

d) $\lim_{x\to\infty}(-2x^5 + 8x^6)$ h) $\lim_{x\to\infty}\frac{2x^2+3x+1}{3x^2-5x+2}$ l) $\lim_{x\to\infty}\frac{3^{x+1}\cdot\left(\frac{1}{3}\right)^{-x+2}}{9^{x-1}}$

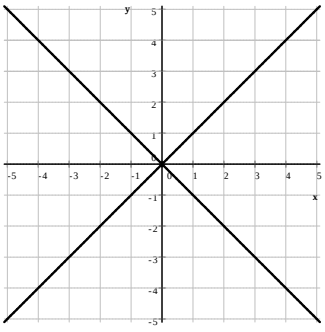
Review Problems - Answers

1. $17 600

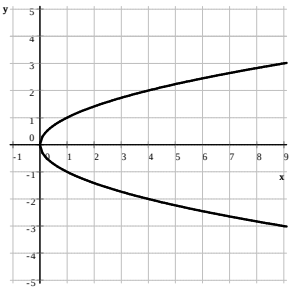
2. a) $y = x^2 - 6x + 8$
Solution: This is a function.
Algebraically: There is a unique formula to compute y in terms of x.
Geometrically: its graph passes the vertical line test.



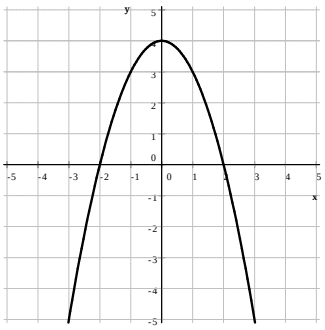
b) $x^2 = y^2$
Solution: Not a function.
Algebraically: If we solve for y, we get $y = x$ or $y = -x$.
$x^2 = y^2$
$0 = y^2 - x^2$
$0 = (y + x)(y - x)$
$y_1 = -x \qquad y_2 = x$
Geometrically: its graph fails the vertical line test,



c) $x = y^2$
Solution: Not a function
Algebraically: when we solve for y, we get that $y = \pm\sqrt{x}$.
$x = y^2$
$0 = y^2 - x$
$0 = y^2 - (\sqrt{x})^2$
$0 = (y + \sqrt{x})(y - \sqrt{x})$
$y_1 = -\sqrt{x} \qquad y_2 = \sqrt{x}$
Geometrically: the graph fails the vertical line test.



d) $x^2 + y = 4$
Solution: Function. We solve for y and see that it is a unique formula. $y = -x^2 + 4$ Its graph passes the vertical line test.

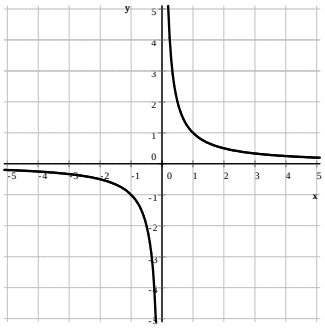


e) $xy = 1$

Solution: Function.

Algebraically: We solve for y and see that it is a unique formula. $y = \dfrac{1}{x}$

Geometrically: Its graph passes the vertical line test.

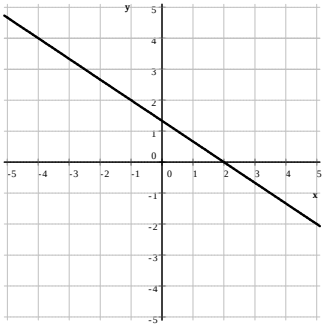


f) $2x + 3y = 4$

Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} 2x + 3y &= 4 && \text{subtract } 2x \\ 3y &= -2x + 4 && \text{divide by } 3 \\ y &= \frac{-2x+4}{3} \\ y &= -\frac{2}{3}x + \frac{4}{3} \end{aligned}$$

Its graph passes the vertical line test.

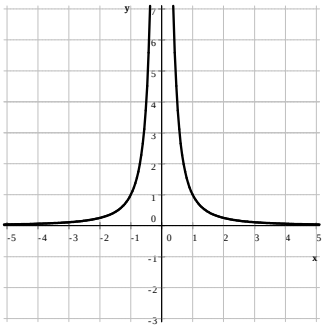


g) $x^2y = 1$

Solution: Function. We solve for y and see that it is a unique formula.

$$\begin{aligned} x^2y &= 1 && \text{divide by } x^2 \\ y &= \frac{1}{x^2} \end{aligned}$$

Its graph passes the vertical line test.

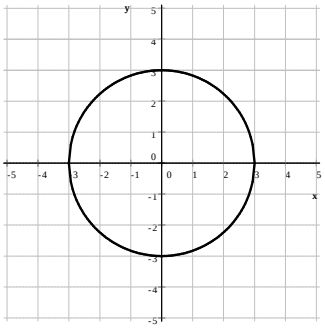


h) $x^2 + y^2 = 9$

Solution: Not a function. We solve for y and see that it is not a unique formula.

$$\begin{aligned} x^2 + y^2 &= 9 && \text{subtract } x^2 \\ y^2 &= 9 - x^2 \\ y &= \pm\sqrt{9 - x^2} \end{aligned}$$

Its graph fails the vertical line test.



3. a) $1 + 9\log_{10} x$ b) $\dfrac{1}{\sqrt{11}}$ c) $\ln(a+2)$ d) $\dfrac{1}{27m^2}$

4. a) 5 (145 doesn't work) b) $\log_{18} 12$ c) $0, 2$ d) $\dfrac{1}{2}(e^6 + 5)$ e) 1 f) 5 g) 7 h) $\dfrac{2}{3}, 4$

5. a) $\left(-\infty, -\dfrac{1}{2}\right) \cup [4, \infty)$ b) $\left(\dfrac{8}{5}, 2\right)$

6. a) $4x^3 - 4x^2 + 11x - 19$ R $41x - 30$ b) $x^4 + x^3 + x^2 + x + 1$

7. $x-1=\log_5 2$ and $\dfrac{1}{2y}=\log_5 3$

$$\log_{24} 60=\frac{\log_5 60}{\log_5 24}=\frac{2\log_5 2+\log_5 3+1}{3\log_5 2+\log_5 3}=\frac{2(x-1)+\dfrac{1}{2y}+1}{3(x-1)+\dfrac{1}{2y}}=\frac{4xy-2y+1}{6xy-6y+1}$$

8. $(8,-8)$ and $(2,0)$

9. $\dfrac{4}{3}(x+6)=y-4$ or $y=\dfrac{4}{3}x+12$

10. a) no such triangle b) no such triangle c) $\gamma\approx 30.763\,7^\circ$ $\beta\approx 106.236\,3^\circ$ $b\approx 5.631\,197$

d) $\alpha_1\approx 65.413\,084^\circ$ $\beta_1\approx 71.586\,916^\circ$ $b_1\approx 4.173\,634$

$\alpha_2\approx 114.586\,916^\circ$ $\beta_2\approx 22.413\,084^\circ$ $b_2\approx 1.677\,195\,36$

e) $b\approx 2.729\,013$ $\gamma\approx 48.566\,112^\circ$ $\alpha\approx 88.433\,888^\circ$

f) $\alpha\approx 17.975\,28^\circ$ $\beta\approx 35.264\,39^\circ$ $\gamma\approx 126.760\,33^\circ$

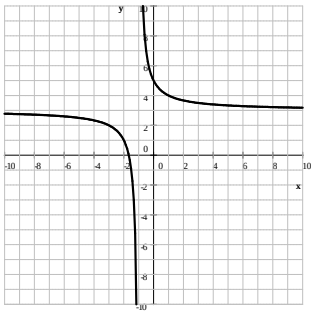
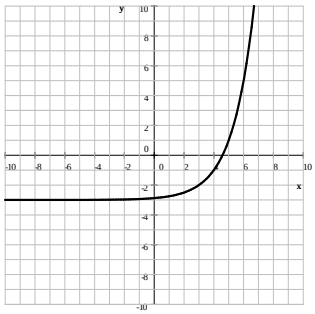
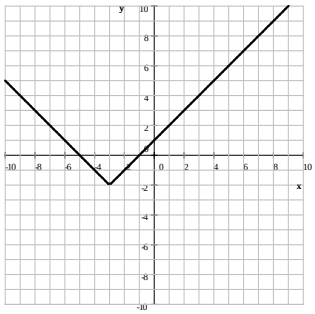
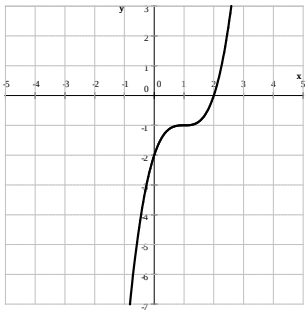
11. a) $f(x)=\dfrac{2}{x+1}+3$ b) $f(x)=2^{x-3}-3$ c) $f(x)=|x+3|-2$ d) $f(x)=(x-1)^3-1$

start with $y=\dfrac{1}{x}$ start with $y=2^x$ start $y=|x|$ start with $y=x^3$

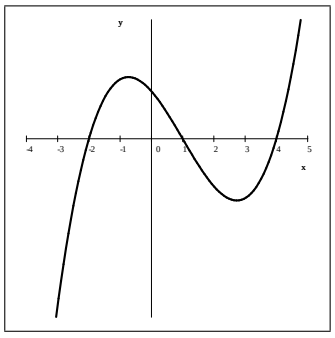
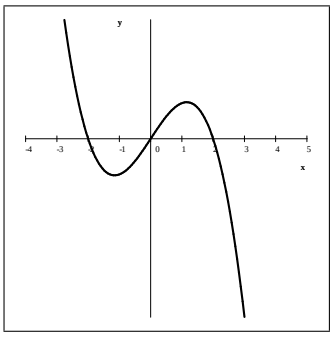
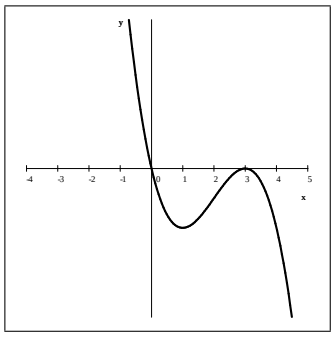
shift by 1 to the left, shift by 3 to the right shift by 3 to the left shift by 1 to the right

stretch by 2 along $y-$axis shift down by 3 units shift down by 2 units shift down by 1 unit

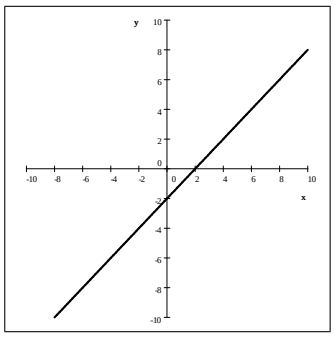
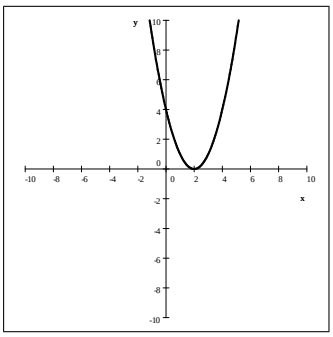
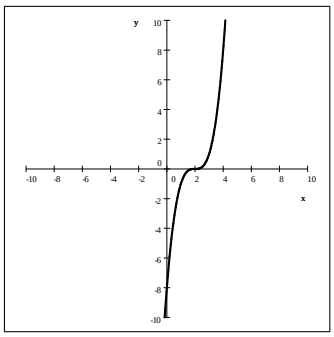
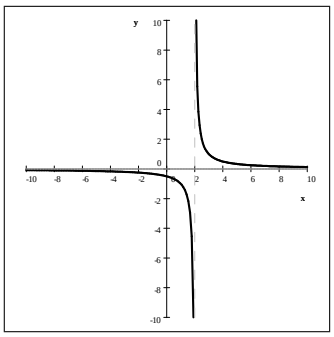
shift up by 3 units

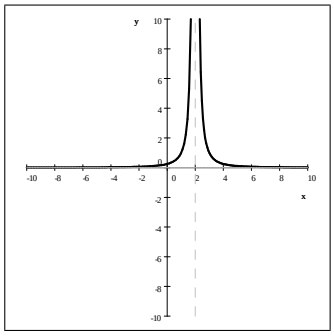
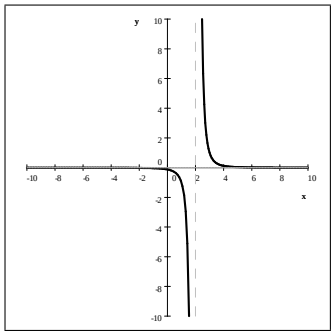
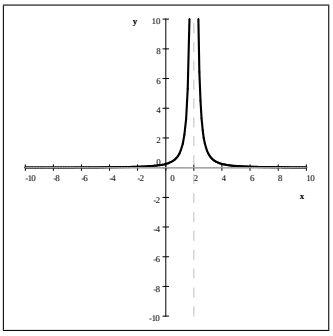
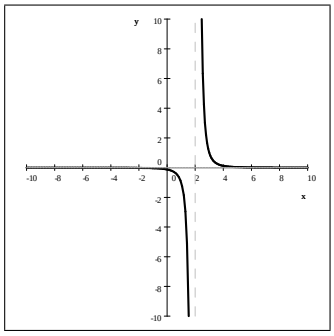
12. a) $f(x)=(x+2)(x-1)(x-4)$ b) $f(x)=-x^3+4x$ c) $f(x)=-\dfrac{1}{2}x(x-3)^2$

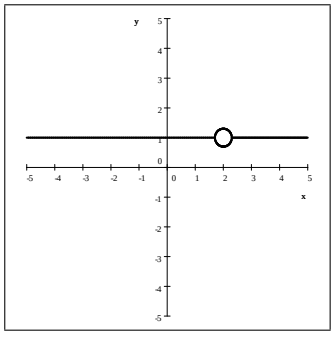
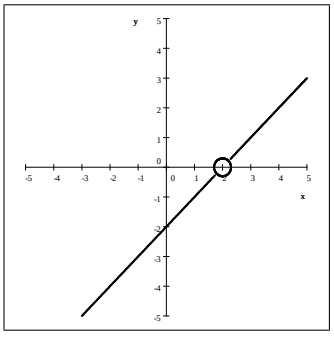
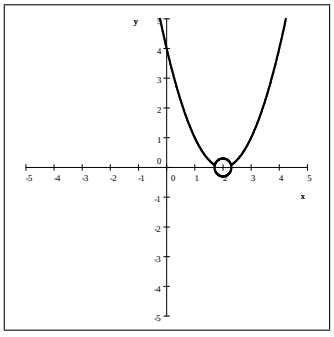
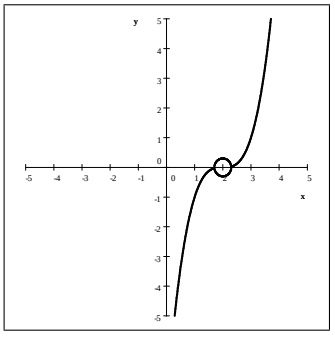
13. a) $f(x) = x - 2$ b) $f(x) = (x-2)^2$ c) $f(x) = (x-2)^3$ d) $f(x) = \dfrac{1}{x-2}$

e) $f(x) = \dfrac{1}{(x-2)^2}$ f) $f(x) = \dfrac{1}{(x-2)^3}$ g) $f(x) = \dfrac{(x-2)^5}{(x-2)^7}$ h) $f(x) = \dfrac{(x-2)^2}{(x-2)^5}$

i) $f(x) = \dfrac{(x-2)^6}{(x-2)^6}$ j) $f(x) = \dfrac{(x-2)^5}{(x-2)^4}$ k) $f(x) = \dfrac{(x-2)^7}{(x-2)^5}$ l) $f(x) = \dfrac{(x-2)^5}{(x-2)^2}$

14. a) $50\left(\dfrac{2}{3}\right)^6 \approx 4.389\,574\,76$ b) $50\dfrac{1-\left(\dfrac{2}{3}\right)^{10}}{1-\left(\dfrac{2}{3}\right)} \approx 147.398\,77$

15. 270

16. $\pm\dfrac{\sqrt{3}}{3}$

17. $\sqrt{2}-1$

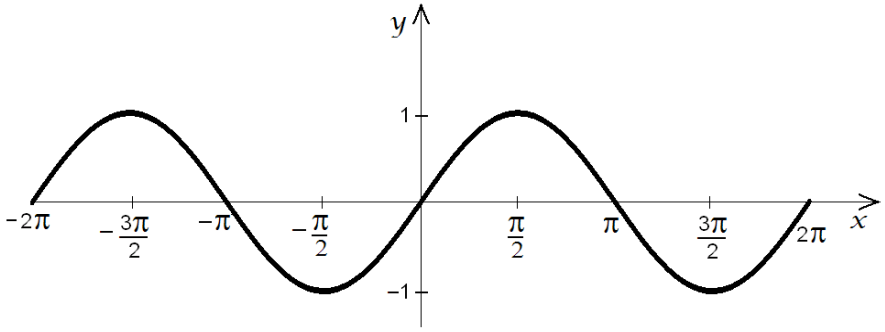
18. a) $\dfrac{1}{2}(\sin 22x - \sin 12x)$ b) $2\sin 14x \sin 6x$

19.

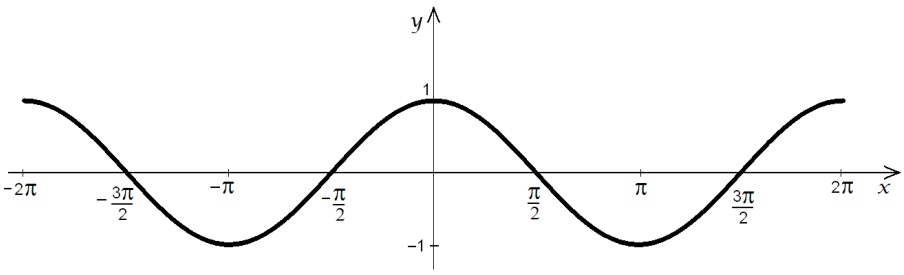
$$\begin{aligned}
(\sin\alpha+\sin\beta)^2+(\cos\alpha-\cos\beta)^2 &= 2\\
\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta+\cos^2\alpha+\cos^2\beta-2\cos\alpha\cos\beta &= 2\\
2+2\sin\alpha\sin\beta-2\cos\alpha\cos\beta &= 2\\
2\sin\alpha\sin\beta-2\cos\alpha\cos\beta &= 0\\
-2\left(\cos\alpha\cos\beta-\sin\alpha\sin\beta\right) &= 0\\
-2\cos\left(\alpha+\beta\right) &= 0\\
\alpha+\beta &= 90^\circ \quad \Longrightarrow \gamma = 90^\circ
\end{aligned}$$

20. $y=\frac{1}{5}x$ or $y=-5x$

21. a) $f(x)=\sin x$



b) $f(x)=\cos x$



22. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $\frac{3}{5}$ f) 0 g) undefined h) $\frac{2}{3}$ i) $-\infty$ j) 0 k) 0 l) 3