Review Problems

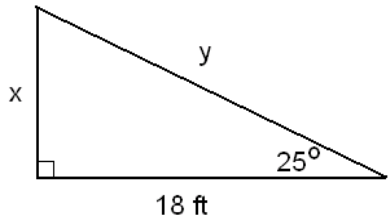
Please note that Quiz 7 will also cover topics covered on Quizzes 1-6 and Exam 1. Please review those topics as well, even if they do not appear on this document.

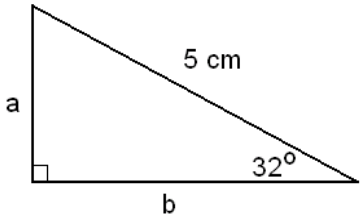
1. Solve each of the following equations.
   a) $\sqrt{x+6}+\sqrt{11-x}=5$
   b) $\sqrt{x+5}-\sqrt{x-2}=\sqrt{x-10}$
   c) $\sqrt{x-3}+1=\sqrt{x+2}$
   d) $\sqrt{x-1}-1=\sqrt{2x-9}$

2. Consider the arithmetic sequence with first element 30 and common difference -2.
   a) Compute a_{100} and s_{100}.
   b) Find n so that $a_n=-254$
   c) Find n so that $s_n=0$
   d) Find n so that $s_n=234$

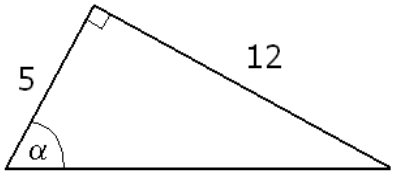
3. Find the first element and common difference in the arithmetic sequence if
   a) $a_{17}=-76$ and $a_{20}=-94$
   b) $a_{19}=-19$ and $s_{19}=-418$
   c) $a_{20}=86$ and $s_{45}=4680$
   d) $s_{33}=1320$ and $s_{53}=-530$

4. Find the exact value and an approximate value for the legth of x, y, a and b, based on the picture shown below.





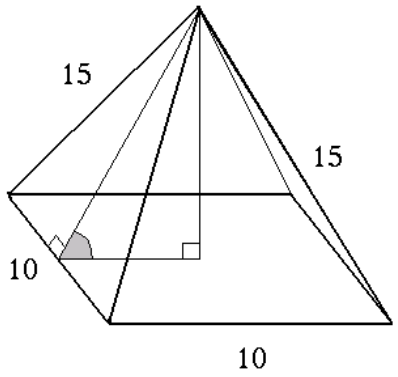
5. Find the exact value for all six trigonometric function of α, based on the following picture.



6. Find the angle that the straight line $y=\frac{2}{3}x+5$ forms with the positive half of the x-axis. Present your answer as
   a) an exact value
   b) an approximation, accurate up to four decimal places.

7. We are driving toward a tower. The angle of elevation is 28°. Then we drive 550 ft toward the tower. Now the angle of elevation is 40°. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.

8. The picture below shows a straight pyramid with a square base. The sides of the base are 10 in long. The other sides are 15 in long.



   a) Find the height of a triangluar edge.

   b) Use part a) to find the height of the pyramid.

   c) Find the measure of the angle shown on the picture.

9. Consider a regular polygon with 10 sides that is written in a circle of radius 28.

   a) Find the side of the polygon. b) Find the area of the polygon.

10. The sum of a number a and three times b is 120.

    a) Find the maximal value of their product
    b) Find the minimal value of the sum of their squares

11. Find an equation of the perpendicular bisector for the line segment AB where $A(-5, 11)$ and $B(1, -1)$.

12. Find the coordinates of all points where the given circles intersect each other.

    a) $(x-3)^2 + (y-2)^2 = 25$ and $(x+3)^2 + (y+1)^2 = 10$

    b) $(x+6)^2 + (y+2)^2 = 125$ and $x^2 + (y-6)^2 = 25$

13. What is the radius of a circle if a sector of 48° has an area of 50 m^2.

14. Prove each of the following the identities.

    a) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

    b) $\cos x (\sec x - \cos x) = \sin^2 x$

    c) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

Answers

1. a) $-5, 10$ b) 11 ($-\frac{19}{3}$ doesn't work) c) 7 d) 5 (17 doesn't work)

2. a) $a_{100} = -168$ and $s_{100} = -6900$ b) 143 c) 31 d) $13, 18$

3. a) $a = 20,\ d = -6$ b) $a = -25,\ d = \frac{1}{3}$ c) $a = -28,\ d = 6$ d) $a = 120,\ d = -5$

4. $x = 18 \tan 25^\circ \approx 8.393\,538$ $\quad y = \dfrac{18}{\cos 25^\circ} \approx 19.860\,8$

   $a = 5 \sin 32^\circ \approx 2.649\,596$ cm $\quad b = 5 \cos 32^\circ \approx 4.240\,240$ cm

5. $\sin\alpha = \dfrac{12}{13},\ \cos\alpha = \dfrac{5}{13},\ \tan\alpha = \dfrac{12}{5},\ \csc\alpha = \dfrac{13}{12},\ \sec\alpha = \dfrac{13}{5},\ \cot\alpha = \dfrac{5}{12}$

6. a) $\tan^{-1}\left(\dfrac{2}{3}\right)$ same as $\arctan\left(\dfrac{2}{3}\right)$ b) $33.690\,07^\circ$

7. 798.290 ft

8. a) $10\sqrt{2} \simeq 14.142$ in b) $5\sqrt{7} \simeq 13.229$ in c) $69.295\,19^\circ$

9. a) 17.3049517 b) $2304.118\,189$

10. a) 1200 b) 1440

11. $\dfrac{1}{2}(x+2) = y - 5$ or $y = \dfrac{1}{2}x + 6$

12. a) $(0, -2)$ and $(-2, 2)$ b) $(-4, 9)$ and $(4, 3)$

13. $\dfrac{5\sqrt{15}}{\sqrt{\pi}}$ m

14. a) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

    $\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + 2\sin x \cos x = \text{RHS}$

    b) $\cos x\,(\sec x - \cos x) = \sin^2 x$

    $\text{LHS} = \cos x\,(\sec x - \cos x) = \cos x \left(\dfrac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$

    c) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

    $\text{LHS} = \dfrac{\sin x}{1 - \cos x} = \dfrac{\sin x}{1 - \cos x} \cdot \dfrac{1 + \cos x}{1 + \cos x} = \dfrac{\sin x\,(1 + \cos x)}{1 - \cos^2 x} = \dfrac{\sin x\,(1 + \cos x)}{\sin^2 x} = \dfrac{1 + \cos x}{\sin x} = \text{RHS}$