

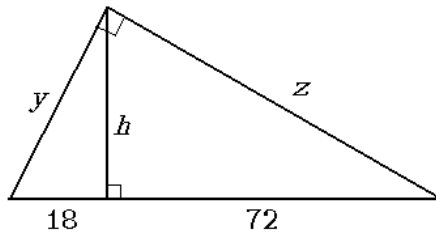
- Solve the equation $\frac{x}{2} - \frac{x+1}{5} = \frac{x}{3} + 1$. Make sure to check your solution.
- Solve the equation $9x^2 - 12x = 1$.
 - Check your solution using exact values.
- Simplify each of the following. Write all answers with positive exponents.

a) -2^{-2}	d) $\frac{1}{\left(\frac{1}{3}\right)^{-2}}$	f) $\frac{a}{b^{-2}}$	h) $\left(\frac{x^{-5}}{y^4}\right)^{-3}$
b) $(-2)^{-2}$			i) $2^{-1} - 3^{-1}$
c) $\left(\frac{1}{2}\right)^{-3}$	e) $\left(\frac{a}{b}\right)^{-2}$	g) $\left(\frac{p}{q^{-3}}\right)^{-2}$	
- Rationalize the denominator in $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$.
- An arch is in the shape of a semicircle. At a point along the base 2 foot from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.
- Let x denote 3^{143} . Simplify and write in terms of x the following expression: $3^{143} - 2 \cdot 3^{144} + 3^{145}$
- Completely factor each of the following over the real numbers.

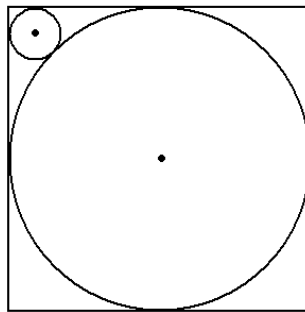
a) $80x^3 + 30x^4 - 5x^5$	d) $2x^7 - 54x$	h) $4x^2 - 24x + 40$
b) $x^3 + 8$	e) $-48x - 3x^3$	i) $6ax - 2x - 10bx + 30abx$
c) $\frac{1}{2}x^2 - 4x + 8$	f) $6x^2 - 13x - 5$	j) $12x - x^2 - 29$
	g) $12x^4 - 4x^3$	k) $b^2c - a^2c - c + a^2b^2c$
- Solve each of the following inequalities.

a) $x^2 + 4 > 6x$	b) $x^2 \leq 6x$	c) $x^2 - 6x \leq -11$	d) $4x - 1 \geq 4x^2$
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- Consider the circle $(x-1)^2 + (y+2)^2 = 25$
 - Find all points on the circle with x -coordinate -9
 - Find all points on the circle with x -coordinate -4
 - Find all points on the circle with x -coordinate -2
- Find both coordinates of all intersection points for each of the following pairs of equations given.
 - $y = x^2 - 6x + 2$ and $y = 2x - 5$
 - $y = x^2 - 4x + 24$ and $y = 6x - 1$
 - $y = x + x^2 + 15$ and $y = -x + 4$
 - $(x+2)^2 + (y-1)^2 = 5$ and $(x-2)^2 + (y-3)^2 = 45$
 - $(x+3)^2 + (y-1)^2 = 25$ and $(x-3)^2 + (y-4)^2 = 10$
 - $(x-1)^2 + (y+4)^2 = 40$ and $2y = x + 6$
- Consider the circle $2y - 3x + x^2 + y^2 = x + 45$. Find an equation drawn to this circle at the point $P(-5, -2)$.

12. A person is standing 12 feet away from a street light that is 30 feet tall. If she is 5 feet tall, how long is her shadow?
13. The population of a city has increased by 4% during the last year. If the population now is 13 977 600, how big was it a year ago, before the raise?
14. Suppose that m and n are real number such that m is 5 greater than twice n . Find each of the following.
- the smallest value of $n^2 + m^2$
 - the smallest value of nm
 - the greatest value of $n^2 - m^2$
15. Find the exact value of the longest line segment that can be drawn inside the rectangular prism with sides 5 cm, 8 cm, and 10 cm long.
16. Compute the exact values of h , y , and z based on the picture below.



17. A large and a smaller sphere are placed into a crete as shown on the picture below. If the radius of the smaller sphere is 1 unit, find the exact value for the radius of the larger sphere.



Answers

1. -36

2. a) $\frac{2 \pm \sqrt{5}}{3}$

b) If $x = \frac{2 + \sqrt{5}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9 \left(\frac{2 + \sqrt{5}}{3} \right)^2 - 12 \left(\frac{2 + \sqrt{5}}{3} \right) = 9 \frac{(2 + \sqrt{5})^2}{3^2} - 4(2 + \sqrt{5}) \\ &= (2 + \sqrt{5})^2 - 4(2 + \sqrt{5}) = 4 + 5 + 4\sqrt{5} - 8 - 4\sqrt{5} = 1 = \text{RHS} \end{aligned}$$

and if $x = \frac{2 - \sqrt{5}}{3}$, then

$$\begin{aligned} \text{LHS} &= 9 \left(\frac{2 - \sqrt{5}}{3} \right)^2 - 12 \left(\frac{2 - \sqrt{5}}{3} \right) = 9 \frac{(2 - \sqrt{5})^2}{3^2} - 4(2 - \sqrt{5}) \\ &= (2 - \sqrt{5})^2 - 4(2 - \sqrt{5}) = 4 + 5 - 4\sqrt{5} - 8 + 4\sqrt{5} = 1 = \text{RHS} \end{aligned}$$

3. a) $-\frac{1}{4}$ b) $\frac{1}{4}$ c) 8 d) $\frac{1}{9}$ e) $\frac{b^2}{a^2}$ f) ab^2 g) $\frac{1}{p^2q^6}$ h) $x^{15}y^{12}$ i) $\frac{1}{6}$

4. $\frac{7 + 2\sqrt{10}}{3}$

5. 17 feet

6. $x - 6x + 9x = 4x$

7. a) $-5x^3(x+2)(x-8)$ b) $(x+2)(x^2-2x+4)$ c) $\frac{1}{2}(x-4)^2$ d) $2x(x^2-3)(x^4+3x^2+9)$

e) $-3x(x^2+16)$ f) $6\left(x-\frac{5}{2}\right)\left(x+\frac{1}{3}\right)$ g) $4x^3(3x-1)$ h) $4(x^2-6x+10)$

i) $2x(3a-1)(5b+1)$ j) $-(x-6+\sqrt{7})(x-6-\sqrt{7})$ k) $c(a^2+1)(b-1)(b+1)$

8. a) $x < 3 - \sqrt{5}$ or $x > 3 + \sqrt{5}$ b) $0 \leq x \leq 6$ c) no solution d) $x = \frac{1}{2}$

9. a) there is no such point b) $(-4, -2)$ c) $(-2, 2)$ and $(-2, -6)$

10. a) $(7, 9)$ and $(1, -3)$ b) $(5, 29)$ c) no intersection points d) $(-4, 0)$ e) $(0, 5)$ and $(2, 1)$
f) no intersection point

11. $y + 2 = -7(x + 5)$

12. 2.4 feet

13. 13440000

14. a) 5, when $n = -2$ and $m = 1$ b) $-\frac{25}{8}$, when $n = -\frac{5}{4}$ and $m = \frac{5}{2}$

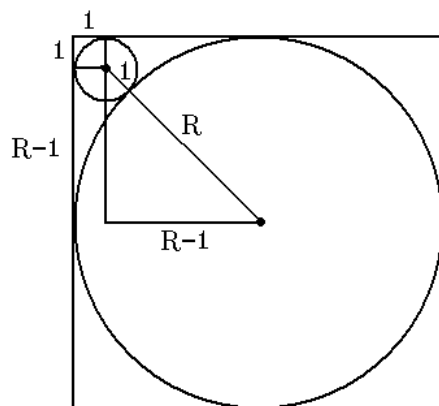
c) $\frac{25}{3}$, when $n = -\frac{10}{3}$ and $m = -\frac{5}{3}$

15. $\sqrt{189} \text{ cm} = 3\sqrt{21} \text{ cm}$

16. $h = 36$ $y = 18\sqrt{5}$ $z = 36\sqrt{5}$

17. $3 + 2\sqrt{2}$

Solution: Consider the right triangle shown on the picture below.



We write the Pythagorean theorem for this triangle.

$$\begin{aligned} (R-1)^2 + (R-1)^2 &= (R+1)^2 \\ 2(R-1)^2 &= (R+1)^2 \\ 2(R^2 - 2R + 1) &= R^2 + 2R + 1 \\ 2R^2 - 4R + 2 &= R^2 + 2R + 1 \\ R^2 - 6R + 1 &= 0 \\ R^2 - 6R + 9 - 9 + 1 &= 0 \\ (R-3)^2 - 8 &= 0 \\ (R-3)^2 - (\sqrt{8})^2 &= 0 \\ (R-3-\sqrt{8})(R-3+\sqrt{8}) &= 0 \\ R_1 = 3 + \sqrt{8} \quad R_2 = 3 - \sqrt{8} \end{aligned}$$

Since $R_2 = 3 - \sqrt{8} \approx 0.171573$ is less than 1, it is ruled out as solution. So the answer is $R = 3 + \sqrt{8}$ or $3 + 2\sqrt{2}$