1. Compute each of the following sums.

   a) $2 + 4 + 6 + ... + 2016$
   b) $11 + 22 + 33 + ... + 143$
   c) $63 + 71 + 79 + ... + 2015$
   d) $100 + 101 + 102 + ... + 300$

2. Simplify each of the following.

   a) $\sqrt{x^2}$ b) $(\sqrt{x})^2$ c) $\sqrt[3]{x^3}$ d) $(\sqrt[3]{x})^3$ e) $\sqrt[4]{x^4}$ f) $(\sqrt[4]{x})^4$

3. Simplify each of the following and re-write using only integer exponents. Assume all variables represent positive numbers. a) $\sqrt[5]{x^{10}}$ b) $\sqrt[28]{y^{24}}$ c) $\sqrt{\sqrt{\sqrt{z}}}$

4. Convert to radians.

   a) $45°$ b) $-120°$ c) $10°$ d) $135°$ e) $270°$

5. Convert to degrees.

   a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

6. Use the unit circle to find the exact value of each of the following expressions.

   a) $\sin 120°$
   b) $\cos 120°$
   c) $\tan \frac{7\pi}{4}$
   d) $\sin \frac{5\pi}{6}$
   e) $\cos(-90°)$
   f) $\cos \frac{5\pi}{6}$
   g) $\sec\left(-\frac{5\pi}{3}\right)$
   h) $\sin\left(-\frac{\pi}{2}\right)$
   i) $\tan(-270°)$
   j) $\sec\left(\frac{5\pi}{3}\right)$

7. The following are all equal to each other, except for one. Which one?

   A) $\cos 150°$ B) $\sin(-60°)$ C) $\cos 210°$ D) $\cos(-120°)$ E) $\sin 300°$

8. Consider the expression $\cos(180° - \alpha)$. All of the following expressions are equal to $\cos(180° - \alpha)$, except for one. Which one?

   A) $-\cos\alpha$ B) $-\sin(90° - \alpha)$ C) $\sin(\alpha - 90°)$ D) $\cos\alpha$ E) $-\cos(-\alpha)$

9. Simplify each of the following expressions. Use exact values and rationalize denominators.

   a) $\dfrac{\tan 120° - \tan 45°}{1 + \tan 120° \tan 45°}$

   b) $\dfrac{\sin(-240°)\cos 135° - \sin 30° \sin 315° \tan 300°}{\sin 150° \sec 225° \tan(-30°)}$

   c) $\cot 300° \sec 210° \tan^2 240°$

   d) $\sin\left(\frac{5\pi}{3}\right) - 2\tan\left(-\frac{\pi}{6}\right) + \sin\left(\frac{7\pi}{6}\right)$

   e) $\cos 1° + \cos 2° + \cos 3° + ... + \cos 179° + \cos 180°$

10. Simplify each of the following, i.e. re-write in terms of a trigonometric function of θ.

    a) $\sin(\theta + 360°)$
    b) $\cos(\pi - \theta)$
    c) $\tan(\pi - \theta)$
    d) $\sin(180° - \theta)$
    e) $\sin\left(\frac{\pi}{2} - \theta\right)$
    f) $\cos(\theta + \pi)$
    g) $\sin(\theta - \pi)$
    h) $\tan(\theta + \pi)$
    i) $\tan(-\theta)$
    j) $\cos(360° - \theta)$

11. Solve each of the following equations. You may present your answer in degrees.

    a) $\sin x = -\frac{1}{\sqrt{2}}$
    b) $\cos x = -\frac{\sqrt{3}}{2}$
    c) $\tan x = -\sqrt{3}$
    d) $\sin x = -\frac{3}{2}$
    e) $\cos x = -1$
    f) $\tan x = 0$
    g) $\sin\alpha = -\frac{\sqrt{3}}{2}$
    h) $\cos\beta = -1$
    i) $\tan\gamma = \frac{2}{3}$

12. a) Suppose that α is not an acute angle. Compute the exact value of $\sin\alpha$ if we know that $\cos\alpha = \frac{4}{7}$.

    b) Suppose that β is an angle, not in the second quadrant. Compute the exact value of $\cos\beta$ if we know that $\tan\beta = -3$.

    c*) Suppose that γ is an angle in the second quadrant. Compute the exact value of $\sin\gamma$ and $\cos\gamma$ if we know that $\tan\gamma = M$.

13. Suppose that $\tan x = -\frac{3}{2}$. Compute the exact value of $\sin x \cos x$.

14. Find all angles coterminal with $120°$. Express your answer

    a) in degrees b) in radians

    c) Find all coterminal angles α such that $-500° < \alpha < 500°$.

15. True or false?

    a) If an angle α is co-terminal with $60°$, then 2α is co-terminal with $120°$.

    b) If twice an angle 2β is co-terminal with $120°$, then β is co-terminal with $60°$.

16. Find an equation for the tangent line drawn to the graph of $6y - 14x + x^2 + y^2 + 18 = 0$ to the point $P(1, -5)$.

17. Find both coordinates of the point where the parabola $y = -\frac{1}{2}x^2 + 4x - 7$ and the line $y = 6x - 5$ intersect each other.

18. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

    a) $(x+3)^2 + (y+2)^2 = 10$ and $(x-1)^2 + (y-2)^2 = 10$

    b) $(x+3)^2 + (y+2)^2 = 8$ and $x^2 + (y-1)^2 = 2$

19. Solve each of the following inequalities.

    a) $x^2 < 4x$ b) $x^2 \geq 4$ c) $x^2 + 8 \geq 6x + 1$ d) $\frac{3x-8}{x-2} \leq -1$ e) $\frac{2x-5}{x+4} \leq 2$

20. Simplify each of the following.

    a) 2^{2+3} c) $\ln\left(e^{-3A}\right)$ e) $\log_2\left(\left(\sqrt{8}\right)^{10}\right)$ g) $\log_4\left(\frac{1}{2}\right)$ i) $2^{\log_8 5}$

    b) $2^2 + 2^3$ d) $e^{-3\ln A}$ f) $\log_{\sqrt{2}}\left(8^{20}\right)$ h) $8^{\log_2 5}$ j) $\log_{3/4}\left(\frac{16}{9}\right)$

    k) $3^{\log_3 9} + 3^{\log_3 27}$ o) $\log_{10}(0.001)$ s) $2^{\log_2 3} + 2^{\log_2 7}$

    l) $3^{\log_3 9 + \log_3 27}$ p) $e^{-2\ln 5}$ t) $2^{\log_2 3 + \log_2 7}$

    m) $e^{\ln 5} + e^{\ln 8}$ q) $2^{\log_2 8} + 2^{\log_2 16}$

    n) $e^{\ln 5 + \ln 8}$ r) $2^{\log_2 8 + \log_2 16}$

21. Simplify $\log_2\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}\right)$

22. Simplify each of the following.

    a) $\log_{10} 4 + \log_{10} 25$ c) $\log_2 12 - \log_2 3$ e) $\log_2 6 + \log_2 12 - 2\log_2 3$

    b) $\log_6 2 - \log_6 72$ d) $\log_3 2 + \log_3 18 - 2\log_3 2$ f) $\log_7 0.8 + \log_7 1.25$

23. Write each of the following as a single logarithm. Assume that A and B represent positive numbers.

a) $\log_2 A + \log_2 B$

b) $\log_2 A + 2\log_2 B$

c) $\log_2 A - \log_2 B$

d) $\frac{1}{2}\log_2 A + 2\log_2 B$

e) $1 + \log_3 A$

f) $2 - \log_{10} 5$

g) $3 - \ln 8$

h) $2 - \log_3 A$

i) $2 - \frac{1}{4}\log_3 A$

24. Solve the equation $\sqrt{x+10} - \sqrt{x+3} = \sqrt{2x-11}$

25. Solve each of the following equations.

a) $4 + \log_2(5x) = 7$

b) $2 + \log_3\left((x-1)^2\right) = 6$

c) $\ln\left((x+1)^2\right) = 4$

d) $\frac{2}{3}\log_2\left(\frac{1}{2}x - 2\right) - 1 = 3$

e) $\dfrac{2\log_2\left(\frac{1}{2}x - 2\right) - 1}{3} = 3$

f) $2\log_3(8x+1) - 2 = 6$

g) $3\ln(2x-7) = 8$

h) $\log_2\left(x^2-1\right) = 5$

i) $\ln\left(x^2-1\right) = -5$

26. Solve each of the following equations.

a) $5^x = \frac{1}{25}$

b) $10^{1-5x} = 1000$

c) $3^{2x-1} = 243$

d) $3^{x^2-3x} = 81$

e) $2 \cdot 5^{3x-1} = 50$

f) $2 \cdot 5^{3x-1} = 20$

g) $2 \cdot 3^x + 5 \cdot 3^x = 63$

h) $2 \cdot 3^x + 5 \cdot 3^x = 56$

27. Solve each of the following equations. Make sure to check your solutions.

a) $\log_2(x-1) + \log_2(x-5) = 5$

b) $\log_6(3-x) + \log_6(-x-2) = 2$

c) $\log_2(x+1) + \log_2(x-1) = 3$

d) $\log_5(x-16) - \log_5(4x+20) = -2$

e) $\log_2(3x+6) - \log_2(x+7) = 3$

28. Sketch the graph of each of the following.

a) $f(x) = -(x+3)^3 x^2 (x-2)$

b) $f(x) = x^5 - 4x^3$

c) $f(x) = \log_2 x$

29. Find the inverse for each of the following functions.

a) $f(x) = \frac{1}{3}x - 2$

b) $f(x) = \sqrt[5]{2x-7}$

c) $f(x) = \log_3(2x-1)$

d) $f(x) = \frac{2x-5}{3x+1}$

30. Find the range of the function $f(x) = \dfrac{2x-5}{3x+1}$. (Hint: it is the domain of its inverse.)

31. Prove each of the following identities.

a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

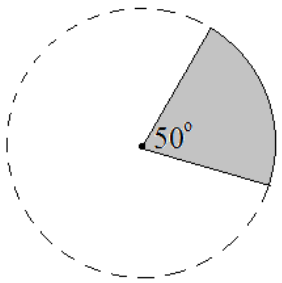
b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

32. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

33. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are 47.5° N and 37.4° N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

34. Prove that the area of any triangle can be computed as $A = \frac{1}{2}ab\sin\gamma$. (Recall that γ is the angle between sides a and b.)

35. Find the perimeter and area of the sector shown on the picture. We know that the circle has a radius of 12 cm.



36. One number a is 10 greater than twice another number b. Find each of the following.

a) The minimal value of $a^2 + b^2$.

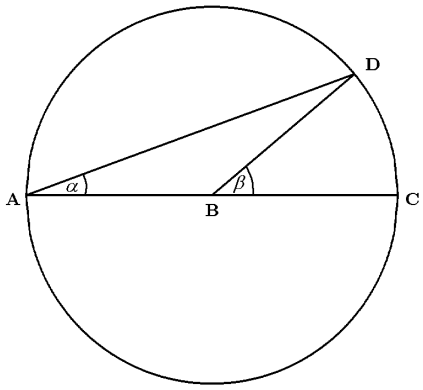
b) The minimal value of ab

c) The maximal value of $b^2 - a^2$

37. a) Compute the area of the triangle determined by the points $A\,(5, 2)$, $B\,(10, 2)$, and $C\,(10, 6)$.

b) Compute the area of the triangle determined by the points $A\,(\log_2 x, \log_2 y)$, $B\,(\log_2 8x, \log_2 y)$, and $C\,(\log_2 x, \log_2 4y)$.

38. Consider the picture shown. Given that B is the center of the circle, prove that $\beta = 2\alpha$.



39. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

a) the perimeter of the polygon

b) the area of the polygon.

40. * The latitude of Seattle, WA is 47.5° N. A satellite above Seattle appears stationary in the sky. Find the speed of the satellite if it is at a height of 1200 miles above the surface of Earth. Assume that the Earth is a sphere with radius 3960 miles.

41. * Solve each of the following equations.

a) $\sqrt{4 - x^2} + \sqrt{x^2 - 4} = 3x + 6$

b) $x^2 - 8x + 17 = \sin y$

Answers

1. a) 1017 072 b) 1001 c) 254 555 d) 40 200

2. a) $|x|$ b) x c) x d) x e) $|x|$ f) x

3. a) x^2 b) $\sqrt[7]{y^6}$ c) $\sqrt[8]{z}$

4. a) $\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{\pi}{18}$ d) $\frac{3\pi}{4}$ e) $\frac{3\pi}{2}$

5. a) 30° b) -20° c) 75° d) -540° e) 315°

6. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) -1 d) $\frac{1}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$ g) 2 h) -1 i) undefined j) 2

7. D

8. D

9. a) $\sqrt{3}+2$ b) -3 c) 2 d) $\frac{\sqrt{3}-3}{6}$ e) -1

10. a) $\sin\theta$ b) $-\cos\theta$ c) $-\tan\theta$ d) $\sin\theta$ e) $\cos\theta$ f) $-\cos\theta$ g) $-\sin\theta$ h) $\tan\theta$
    i) $-\tan\theta$ j) $\cos\theta$

11. a) $x = -45^\circ + k\cdot 360^\circ$ or $x = -135^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$
    $x = -\frac{\pi}{4} + 2k\pi$ or $x = -\frac{3\pi}{4} + 2k\pi$ where $k\in\mathbb{Z}$
    b) $x = \pm 150^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$ $x = \pm\frac{5\pi}{6} + 2k\pi$ where $k\in\mathbb{Z}$
    c) $x = -60^\circ + k\cdot 180^\circ$ where $k\in\mathbb{Z}$ $x = -\frac{\pi}{3} + k\pi$ where $k\in\mathbb{Z}$
    d) no solution e) $x = 180^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$ $x = \pi + 2k\pi$ where $k\in\mathbb{Z}$
    f) $x = k\cdot 180^\circ$ where $k\in\mathbb{Z}$ $x = k\pi$ where $k\in\mathbb{Z}$
    g) $\alpha = -60^\circ + k\cdot 360^\circ$ or $\alpha = -120^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$
    h) $\beta = 180^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$ i) $\gamma = 33.690\,07^\circ + k\cdot 180^\circ$ where $k\in\mathbb{Z}$

12. a) $-\frac{\sqrt{33}}{7}$ b) $\frac{\sqrt{10}}{10}$ c) $\cos\gamma = \frac{-1}{\sqrt{M^2+1}}$ $\sin\gamma = \frac{-M}{\sqrt{M^2+1}}$

13. $-\frac{6}{13}$

14. a) $120^\circ + k\cdot 360^\circ$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$
    b) $\frac{2\pi}{3} + 2k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$ c) $-240^\circ, 120^\circ, 480^\circ$

15. a) If an angle α is co-terminal with 60°, then 2α is co-terminal with 120°.
    True: $\alpha = 60^\circ + k\cdot 360^\circ$ where $k\in\mathbb{Z}$ multiply by 2 and get that
    $2\alpha = 120^\circ + k\cdot 720^\circ$ where $k\in\mathbb{Z}$
    b) If twice an angle 2β is co-terminal with 120°, then β is co-terminal with 60°.
    False: Let $\beta = 240^\circ$. Clearly 240° is not co-terminal to 60°, but $2\beta = 480^\circ$ is co-terminal with 120°.

16. $y = -3x - 2$

17. $(-2,-17)$

18. a) $(-2,1)$ and $(0,-1)$ b) $(-1,0)$

19. a) $(0,4)$ b) $(-\infty,-2]\cup[2,\infty)$ c) $\left(-\infty,3-\sqrt{2}\right]\cup\left[3+\sqrt{2},\infty\right)$ d) $\left(2,\frac{5}{2}\right]$ e) $(-4,\infty)$

20. a) 32 b) 12 c) $-3A$ d) $\frac{1}{A^3}$ e) 15 f) 120 g) $-\frac{1}{2}$ h) 125 i) $\sqrt[3]{5}$ j) -2
k) 36 l) 243 m) 13 n) 40 o) -3 p) $\frac{1}{25}$ q) 24 r) 128 s) 10 t) 21

21. $\frac{1}{32}$

22. a) 2 b) -2 c) 3 d) 2 e) 3 f) 0

23. a) $\log_2 AB$ b) $\log_2\left(AB^2\right)$ c) $\log_2\left(\frac{A}{B}\right)$ d) $\log_2\left(B^2\sqrt{A}\right)$ e) $\log_3(3A)$ f) $\log_{10}20$
g) $\ln\left(\frac{e^3}{8}\right)$ h) $\log_3\left(\frac{9}{A}\right)$ i) $\log_3\left(\frac{9}{\sqrt[3]{A}}\right)$

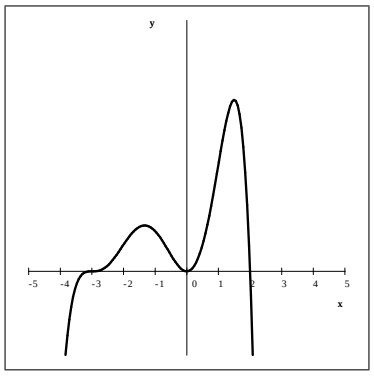
24. 6 (-19 does not work)

25. a) $\frac{8}{5}$ b) $10,-8$ c) $-e^2-1, e^2-1$ d) 132 e) 68 f) 10 g) $\frac{1}{2}e^{8/3}+\frac{7}{2}$ h) $\pm\sqrt{33}$
i) $\pm\sqrt{\frac{1}{e^5}+1}$

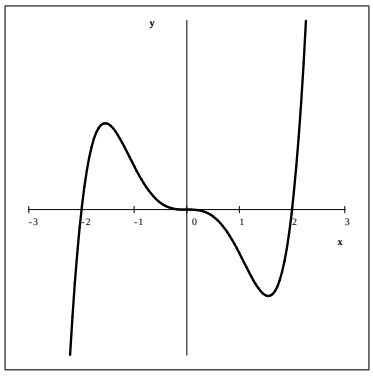
26. a) -2 b) $-\frac{2}{5}$ c) 3 d) $-1,4$ e) 1 f) $\frac{1}{3}\left(1+\log_5 10\right)$ g) 2 h) $\log_3 8$

27. a) 9 (-3 does not work) b) -6 (7 does not work) c) 3 (-3 does not work)
d) 20 e) no solution

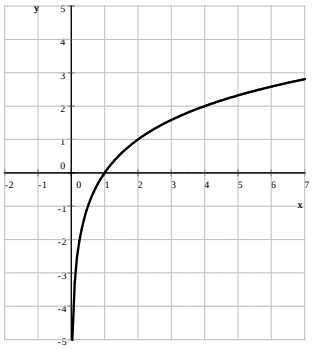
28. a) $f(x)=-(x+3)^3x^2(x-2)$



b) $f(x)=x^5-4x^3$



c) $f(x)=\log_2 x$



29. a) $f^{-1}(x)=3x+6$ b) $f^{-1}(x)=\frac{x^5+7}{2}$ c) $f^{-1}(x)=\frac{1+3^x}{2}$ d) $f^{-1}(x)=\frac{x+5}{-3x+2}$

30. $\left\{x : x\neq\frac{2}{3}\right\}$

31. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

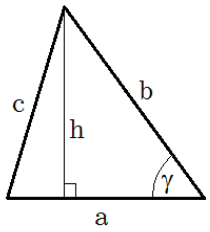
b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

32. $71.\,670\,246\,2°$

33. 698 miles

34. The area is $A = \dfrac{1}{2}ah$ and h, the height belonging to side a is $h = b\sin\gamma$.



35. $P = \left(24 + \dfrac{10}{3}\pi\right)$ cm $\approx 34.\,471\,975\,512$ cm
$A = 20\pi$ cm^2 $\approx 62.\,831\,853\,0718$ cm^2

36. a) 20 b) $-\dfrac{25}{2}$ c) $\dfrac{100}{3}$

37. a) 10 unit^2 b) 3 unit^2

38. a) $2nR\sin\left(\dfrac{180°}{n}\right)$

b) $nR\sin\left(\dfrac{180°}{n}\right) R\cos\left(\dfrac{180°}{n}\right)$
$= nR^2\sin\left(\dfrac{180°}{n}\right)\cos\left(\dfrac{180°}{n}\right)$

39. Line segments AB and BD are both radii in the circle, and so they are equal. So ABD triangle is isosceles and so the angles opposite AB and BD are also equal to each other. Thus $\measuredangle ADB = \alpha$. The third angle in triangle ADB is $180° - 2\alpha$. Angles ABD and DBC are supplementary beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ABD + \measuredangle DBC &= 180° \\ 180° - 2\alpha + \beta &= 180° && \text{subtract } 180° \\ -2\alpha + \beta &= 0 && \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$

40. $912.\,644\,57\dfrac{\text{mi}}{\text{h}}$

41. a) -2 b) $x = 4$, $y = \dfrac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

Last revised: November 2, 2016