Review Problems

Please note that Quiz 6 will also cover topics cevered on Quizzes 1-5 and exam 1. Please review those topics as well, even if they do not appear on this document.

1. Factor by completing the square $12x - 18x^2 + 12$
2. Simplify each of the following compound inequalities. If it is not possible, state so.
   a) $x > -3$ or $x \geq 5$ c) $x \geq 4$ or $x < 10$ e) $x \leq 0$ or $x > 9$
   b) $x > -3$ and $x \geq 5$ d) $x \geq 4$ and $x < 10$ f) $x \leq 0$ and $x > 9$
3. Solve each of the following inequalities.
   a) $x^2 > 9x$ c) $24x - 3x^2 - 48 \geq 0$ e) $x^2 - 2x + 3 < 0$
   b) $8x - 2x^2 + 2 \leq 0$ d) $x^2 < 9$ f) $10x - 5x^2 - 20 \leq 0$
4. Suppose that (a_n) is an arithmetic sequence with $a = 80$ and $d = -7$. Compute each of the following.
   a) a_{10} and s_{10} c) $n =?$ so that $a_n = -39$
   b) a_{30} and s_{30} d) $n =?$ so that $s_n = 494$
5. Find the first element and common difference of the arithmetic sequence with $a_7 = 10$ and $s_7 = -35$.
6. Find the first element and common difference of the arithmetic sequence with $a_{21} = -12$ and $s_{21} = -882$.
7. The first element in an arithmetic sequence is 10. Find the common difference in the sequence such that a_5, a_{51}, and a_{55}, in this order, are sides of a right triangle.
8. For each of the following pairs of graphs, find the coordinates of all points where they intersect.
   a) $(x-2)^2 + (y+1)^2 = 20$ and $x - 2y = 19$
   b) $(x+1)^2 + (y-3)^2 = 50$ and $x - 7y + 72 = 0$
   c) $(x+1)^2 + (y-4)^2 = 17$ and $x + 4y = 15$
   d) $(x-2)^2 + (y-1)^2 = 25$ and $3y = 4x - 30$
   e) $(x-1)^2 + (y+7)^2 = 25$ and $y = -\frac{1}{2}x - \frac{3}{2}$
   f) $(x+2)^2 + (y-1)^2 = 10$ and $(x-1)^2 + (y+2)^2 = 4$
   g) $(x+2)^2 + (y-1)^2 = 10$ and $(x-4)^2 + (y+2)^2 = 25$
9. Find an equation of the tangent line drawn to $8x + x^2 + y^2 = 6y$ to the point $(-7, -1)$.
10. Two numbers a and b are such that $3a + b = 20$. Find the exact vaue of each of the following.
    a) smallest value of $a^2 + b^2$. b) greatest value of ab. c) greatest value of $a^2 - b^2$.

Review Problems - Answers

Please note that Quiz 6 will also cover topics cevered on Quizzes 1-5 and exam 1. Please review those topics as well, even if they do not appear on this document.

1.) $-18\left(x-\frac{1-\sqrt{7}}{3}\right)\left(x-\frac{1+\sqrt{7}}{3}\right)$

2.) a) $x>-3$ b) $x\geq 5$ c) $\mathbb{R}$ (all real numbers)

d) the expressions can't be simplified, but the notation can be modified: $4\leq x<10$

e) can't be simplified f) $\varnothing$ (no real number)

The same answers in interval notation:
a) $(-3,\infty)$ b) $[5,\infty)$ c) $(-\infty,\infty)$ d) $[4,10)$ e) $(-\infty,0]\cup(9,\infty)$ f) $\varnothing$

3.) a) $(-\infty,0)\cup(9,\infty)$ b) $\left(-\infty,2-\sqrt{5}\right]\cup\left[2+\sqrt{5},\infty\right)$ c) 4 d) $(-3,3)$

e) no solution f) $\mathbb{R}$

4.) a) $a_{10}=17$ $s_{10}=485$ b) $a_{30}=-123$ $s_{30}=-645$ c) 18 d) 13

5.) $a=-20, d=5$ 6.) $a=-72, d=3$ 7.) $\frac{1}{2}$

8.) a) no intersection point b) $(-2,10)$ c) $(3,3)$ and $(-5,5)$ d) $(6,-2)$

e) $(1,-2)$ and $(5,-4)$ f) $(1,0)$ and $(-1,-2)$ g) $(-1,-2)$ and $(1,2)$

9.) $-\frac{3}{4}(x+7)=y+1$ or $y=-\frac{3}{4}x-\frac{25}{4}$ 10.) a) 40 b) $\frac{100}{3}$ c) 50

Solution for 7: We express a_5, a_{51}, and a_{55} in terms of a and d. It may be useful to note it now that if these elements are sides of a rigth triangle in this order, then d must be positive.

$$\begin{aligned} a_5 &= a + 4d = 10 + 4d \\ a_{51} &= a + 50d = 10 + 50d \\ a_{55} &= a + 54d = 10 + 54d \end{aligned}$$

We write the Pythagorean theorem for these three quantities

$$\begin{aligned} (a_5)^2 + (a_{51})^2 &= (a_{55})^2 \\ (10 + 4d)^2 + (10 + 50d)^2 &= (10 + 54d)^2 \end{aligned}$$

We solve this quadratic equation for d.

$$\begin{aligned} (10 + 4d)^2 + (10 + 50d)^2 &= (10 + 54d)^2 \\ 100 + 80d + 16d^2 + 100 + 1000d + 2500d^2 &= 100 + 1080d + 2916d^2 \\ 2516d^2 + 1080d + 200 &= 2916d^2 + 1080d + 100 \\ 0 &= 400d^2 - 100 \\ 0 &= 100\left(4d^2 - 1\right) \\ 0 &= 100\,(2d + 1)\,(2d - 1) \\ d &= \pm\frac{1}{2} \end{aligned}$$

Since d must be positive, $d = \dfrac{1}{2}$.