

1. Let $f(x) = \frac{x-2}{x+5}$ and $g(x) = \frac{5x+2}{-x+1}$. Compute each of the following.

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|-----------|-------------------|--------------|-------------------|
| a) $f(0)$ | d) $g(\sqrt{2})$ | f) $f(g(0))$ | i) $f(1) + f(3)$ |
| b) $g(0)$ | e) $f(\sqrt{20})$ | g) $g(f(3))$ | j) $2f(5)$ |
| c) $g(1)$ | | h) $f(1+3)$ | k) $f(2 \cdot 5)$ |

2. Simplify each of the following. Assume that x is positive.

- | | | |
|--|------------------------------------|--|
| a) $(-16)^{-3/4}$ | d) $\frac{2}{(\sqrt{5}-1)^2}$ | f) $(\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}})^2$ |
| b) $-16^{-3/4}$ | | |
| c) $(x^{2/3})^{3/4} \left(\frac{1}{\sqrt{x}}\right)$ | e) $(1+\sqrt{3})^3 (1-\sqrt{3})^3$ | g) $\frac{3+\sqrt{3}}{3-\sqrt{3}}$ |

3. Simplify each of the following. Assume that a is positive.

- | | | | | |
|--------------------------------------|--------------------------------------|--------------------------------------|----------------------|--|
| a) $\log_4 \left(\frac{1}{2}\right)$ | e) $\log_2 \left(\frac{1}{8}\right)$ | i) $\log_4(-8)$ | m) $\log_5 1$ | q) $\log_9 \left(\frac{1}{\sqrt{27}}\right)$ |
| b) $\log_4 8$ | f) $\log_8 2$ | j) $\log_8 \left(\frac{1}{4}\right)$ | n) $\log_1 5$ | r) $\log_a \left(\sqrt[7]{a^2}\right)$ |
| c) $\log_{10} 1000$ | g) $\log_{\sqrt{2}} 8$ | k) $\log_{(1/5)} 125$ | o) $\log_a (a^{17})$ | s) $2^{\log_2 8}$ |
| d) $\log_{100} 1000$ | h) $\log_4 8$ | l) $\log_{0.1} 100\,000$ | p) $\log_{(-5)} 25$ | t) $3^{\log_3 8}$ |

4. Simplify each of the following.

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|---------------------|-----------------------|-----------------------|----------------------|-----------------------|
| a) $\cos 30^\circ$ | e) $\tan 270^\circ$ | i) $\sin 315^\circ$ | m) $\cos 330^\circ$ | q) $\sec(-600^\circ)$ |
| b) $\sin 0^\circ$ | f) $\cos 120^\circ$ | j) $\cot(-90^\circ)$ | n) $\tan 240^\circ$ | r) $\csc(-180^\circ)$ |
| c) $\sin 90^\circ$ | g) $\tan 210^\circ$ | k) $\sin 180^\circ$ | o) $\csc(-45^\circ)$ | |
| d) $\sin 120^\circ$ | h) $\sec(-120^\circ)$ | l) $\cos(-180^\circ)$ | p) $\sin 495^\circ$ | s) $\cot(-120^\circ)$ |

5. Find the exact value of each of the following expressions and simplify.

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|---|--|
| a) $\sin 210^\circ - \cos 240^\circ \tan 30^\circ$ | e) $\sin 20^\circ \cos 70^\circ + \cos 20^\circ \sin 70^\circ$ |
| b) $\sin(-210^\circ) - \cos 240^\circ \tan 330^\circ$ | f) $\sin 30^\circ \cos 150^\circ \tan 120^\circ - \sin(-60^\circ) \cos 120^\circ \tan 150^\circ$ |
| c) $\frac{\sin 10^\circ}{\cos 80^\circ}$ | g) $\sin 120^\circ \sec -30^\circ - 3 \tan 30^\circ - \cot 135^\circ$ |
| d) $\sin^4(135^\circ) + \cos^4(135^\circ)$ | h) $\cos 60^\circ + 2 \sin 45^\circ - \cot 450^\circ + \cos 210^\circ$ |

6. Approximate each of the following by placing them between two consecutive integers.

- | | | | | |
|-----------------|-----------------|---------------------|------------------|--------------------------------------|
| a) $\log_2 100$ | b) $\log_2 200$ | c) $\log_{10} 2014$ | d) $\log_5 2014$ | e) $\log_2 \left(\frac{1}{3}\right)$ |
|-----------------|-----------------|---------------------|------------------|--------------------------------------|

7. Re-write each of the following as an exponential statement.

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|-----------------------|--------------------------|--------------------|
| a) $x = \log_7(2y-1)$ | c) $p = \log_q T$ | e) $2 = \log_x 20$ |
| b) $3 = \log_B(A-1)$ | d) $x+2 = \log_a(x^2+1)$ | |

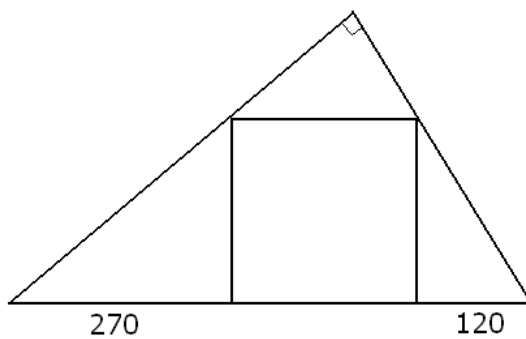
8. Re-write each of the following as an exponential statement. Assume that all variables represent positive numbers.
- a) $3^{x-2} = 60$ b) $10^{2x-5} = 2012$ c) $\left(\frac{1}{3}\right)^{a+1} = b - 2$ d) $A^B = C$
9. Solve each of the following equations. Make sure to check your solutions.
- a) $\sqrt{x+6} = 1 + \sqrt{x-1}$ c) $x^2 + x = 1$
b) $\sqrt{x} + \sqrt{x-5} = 5$ d) $\sqrt{3x+1} = \sqrt{x+4} + 3$
10. Solve each of the following inequalities.
- a) $3x^2 \geq 15x$ b) $28x + 88 \leq -2x^2$ c) $\frac{1}{3}x^2 < 4x - 12$ d) $\frac{1}{3}x^2 \leq 4x - 12$
11. Find the domain of each of the following functions.
- a) $f(x) = \frac{1}{5x+3}$ c) $f(x) = \frac{1}{x^2 - 10x + 24}$ e) $f(x) = \sqrt{x-3} - 2$
b) $f(x) = \sqrt{5x+3}$ d) $f(x) = \sqrt{x^2 - 10x + 24}$ f) $f(x) = \frac{1}{\sqrt{x-3} - 2}$
12. Find the exact values for $\sin \alpha$ and $\cos \alpha$ if we know that $0 < \alpha < 90^\circ$ and $\tan \alpha = \frac{3}{10}$.
13. Find the height of a tree if the angle of elevation of its top changes from 15° to 25° as the observer advances 120 ft toward its base.
14. Find the radius of the circle if
- a) an arc subtended by a central angle of 30° has a length of 20 cm.
b) a sector of 30° has an area of 20 cm^2 .
15. Compute the area and perimeter of a regular 12-sided polygon written into a circle with radius 9 cm. Present the exact value and approximation of your answer.
16. Find all points of intersections of $(x+4)^2 + (y-4)^2 = 25$ and $(x-10)^2 + (y-2)^2 = 125$.
17. Consider the circle $(x-10)^2 + (y-2)^2 = 125$. Find the equation of the tangent line drawn to the circle at $(0, 7)$.
18. A circle has a radius of 10 units. A point P is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from P to the circle.
19. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
- a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
b) Compute the distance between the two points of tangency on a common tangent line.
20. A satellite is 1000 miles above the surface of the equator. The radius of Earth is 3960 miles. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?
21. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.
22. The sum of a and three times b is 100. What is the smallest possible value of $a^2 - (2b)^2$?

23. Prove the identity $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

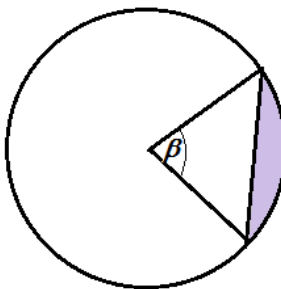
24. Graph each of the following.

a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x - 2)^2 + (y - 1)^2 = 25$ d) $f(x) = \frac{1}{x}$

25. The picture below shows a square within a right triangle. Find the length of the sides in the square.



26. Compute the area of the shaded region shown on the picture below. The radius of the circle is 3 meters and $\beta = 80^\circ$. Present the exact value and approximate value of the answer.



Answers

- 1.) a) $-\frac{2}{5}$ b) 2 c) undefined d) $-7\sqrt{2} - 12$ e) $\frac{14}{5}\sqrt{5} - 6$ f) 0 g) 3 h) $\frac{2}{9}$ i) $-\frac{1}{24}$
- 2.) a) undefined b) $-\frac{1}{8}$ c) 1 d) $\frac{\sqrt{5} + 3}{4}$ e) -8 f) 2 g) $\sqrt{3} + 2$
- 3.) a) $-\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) $\frac{3}{2}$ e) -3 f) $\frac{1}{3}$ g) 6 h) $\frac{3}{2}$ i) undefined j) $-\frac{2}{3}$ k) -3
- 4.) a) -5 m) 0 n) undefined o) 17 p) undefined q) $-\frac{3}{4}$ r) $\frac{2}{7}$ s) 8 t) 8
- 1.) $-\frac{1}{2}$ m) 0 n) $\frac{3}{2}$ o) undefined f) $-\frac{1}{2}$ g) $\frac{\sqrt{3}}{3}$ h) -2 i) $-\frac{\sqrt{2}}{2}$ j) 0 k) 0
- 1) -1 m) $\frac{\sqrt{3}}{2}$ n) $\sqrt{3}$ o) $-\sqrt{2}$ p) $\frac{\sqrt{2}}{2}$ q) -2 r) undefined s) $\frac{\sqrt{3}}{3}$

5.) a) $\frac{\sqrt{3}}{6} - \frac{1}{2}$ b) $-\frac{\sqrt{3}}{6} + \frac{1}{2}$ c) 1 d) $\frac{1}{2}$ e) 1 f) 1 g) $2 - \sqrt{3}$ h) $\sqrt{2} - \frac{\sqrt{3}}{2} + \frac{1}{2}$

6.) a) $6 < \log_2 100 < 7$ b) $7 < \log_2 200 < 8$ c) $3 < \log_{10} 2014 < 4$ d) $4 < \log_5 2014 < 5$

e) $-2 < \log_2 \left(\frac{1}{3}\right) < -1$

7.) a) $7^x = 2y - 1$ b) $B^3 = A - 1$ c) $q^p = T$ d) $a^{x+2} = x^2 + 1$ e) $x^2 = 20$

8.) a) $x - 2 = \log_3 60$ b) $2x - 5 = \log_{10} 2012$ c) $a + 1 = \log_{1/3}(b - 2)$ d) $B = \log_A C$

9.) a) 10 b) 9 c) $\frac{-1 \pm \sqrt{5}}{2}$ d) 21

10.) a) $(-\infty, 0] \cup [5, \infty)$ b) $[-7 - \sqrt{5}, -7 + \sqrt{5}]$ c) no solution d) 6

11.) a) $x \neq -\frac{3}{5}$ b) $x \geq -\frac{3}{5}$ c) $x \neq 4, 6$ d) $x \leq 4$ or $x \geq 6$ e) $x \geq 3$ f) $x \geq 3$ but $x \neq 7$

12.) $\sin \alpha = \frac{3}{\sqrt{109}}$, $\cos \alpha = \frac{10}{\sqrt{109}}$ 13.) 75.588461 ft

14.) a) $r = \frac{120}{\pi}$ cm ≈ 38.197186342 cm b) $r = \sqrt{\frac{240}{\pi}}$ cm ≈ 8.740387 cm

15.) $P = 216 \sin 15^\circ$ cm ≈ 55.90491 cm $A = 972 \sin 15^\circ \cos 15^\circ$ cm² ≈ 243 cm² 16.) (0, 7) and (-1, 0)

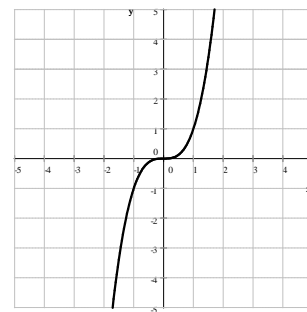
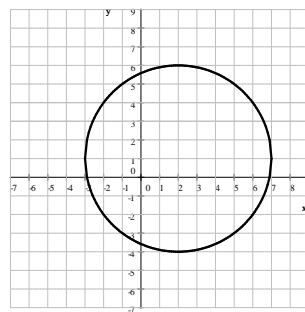
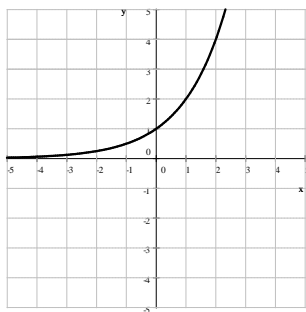
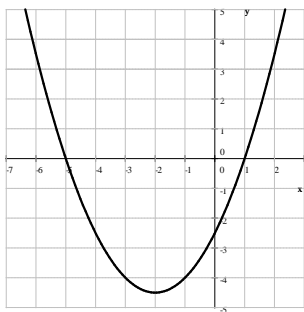
17.) $y = 2x + 7$ 18.) $2 \sin^{-1} \left(\frac{10}{17}\right) \approx 72.063758^\circ$ 19.) a) $2 \sin^{-1} \left(\frac{3}{20}\right) \approx 17.253853^\circ$ b) $\sqrt{391}$ unit

20.) $1299 \frac{\text{mi}}{\text{h}}$ 21.) 17 feet 21.) -8000

23.) $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{(1 + \sin^2 x + 2 \sin x) - (1 + \sin^2 x - 2 \sin x)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2 \sin x - 1 - \sin^2 x + 2 \sin x}{\cos^2 x} = \frac{4 \sin x}{\cos^2 x} = 4 \frac{\sin x}{\cos x} \frac{1}{\cos x} = 4 \tan x \sec x = \text{RHS} \end{aligned}$$

24.) a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x - 2)^2 + (y - 1)^2 = 25$ d) $f(x) = \frac{1}{x}$



25.) 180 26.) $(2\pi - 9 \sin 40^\circ \cos 40^\circ)$ m² ≈ 1.85155 m²

Last revised: February 25, 2014