Please note that Quiz 12 will also cover topics covered on Quizzes 1-11 and Exams 1 and 2. Please review those topics as well, even if they do not appear in this document.

1. Simplify each of the following.

   a) $\log_3\left(\frac{1}{\sqrt{27}}\right)$ b) $2^{\log_2 8}$ c) $2^{\log_2 24}$ d) $\log_2(\cos 45°)$ e) $\log_{\sin 45°} 8$

2. a) Suppose that α is an acute angle, i.e. $0 < \alpha < 90°$. Find the exact value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \frac{1}{3}$. Rationalize the denominator in your answers.

   b) Suppose that β is an acute angle, i.e. $0 < \beta < 90°$. Find the value of $\sin\beta$ and $\cos\beta$ if $\tan\beta = 4$. Rationalize the denominator in your answer.

   c) Suppose that γ is an acute angle, i.e. $0 < \gamma < 90°$. Find the value of $\sin\gamma$, $\cos\gamma$, and $\tan\gamma$ if $\sec\gamma = \frac{5}{2}$. Rationalize the denominator in your answer.

3. Simplify each of the following. (Write it in terms of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$.

   a) $\sin(\alpha + 360°)$ c) $\tan(\alpha - 180°)$ e) $\cos(-\alpha)$ g) $\sin(\alpha - 180°)$

   b) $\cos(\alpha + 180°)$ d) $\sin(180° - \alpha)$ f) $\tan(-\alpha)$ h) $\tan(180° - \alpha)$

4. Consider the expression $\cos(180° - \alpha)$. All of the following expressions are equal to $\cos(180° - \alpha)$, except for one. Which one?

   A) $-\cos\alpha$ B) $-\sin(90° - \alpha)$ C) $\sin(\alpha - 90°)$ D) $\cos\alpha$ E) $-\cos(-\alpha)$

5. Simplify each of the following. Present exact values. Rationalize the denominator.

   a) $\sin\left(\frac{5\pi}{3}\right) - 2\tan\left(-\frac{\pi}{6}\right) + \sin\left(\frac{7\pi}{6}\right)$ b) $\cos 1° + \cos 2° + \cos 3° + \ldots.. + \cos 179° + \cos 180°$

6. Solve each of the following equations. You may present your answer in degrees.

   a) $\sin x = -\frac{1}{\sqrt{2}}$ c) $\tan x = -\sqrt{3}$ f) $\tan x = 0$ h) $\cos\beta = -1$

   d) $\sin x = -\frac{3}{2}$

   b) $\cos x = -\frac{\sqrt{3}}{2}$ e) $\cos x = -1$ g) $\sin\alpha = -\frac{\sqrt{3}}{2}$ i) $\tan\gamma = \frac{2}{3}$

7. Solve each of the following equations. Present exact values of all answers.

   a) $\log_3(5x + 1) = 4$ b) $3^{2x-1} = 18$ c) $\log_2(3x - 2) = 4$ d) $5^{0.5x-3} = 10$

8. State the domain for each of the following functions.

   a) $f(x) = \frac{x-2}{x^2-9}$

   b) $g(x) = \sqrt{x-8} - \sqrt{15-x} + \frac{1}{x-10}$

   c) $h(x) = \frac{2x+1}{5x-1} - \sqrt{x+2} - \frac{1}{x^2+2x+7}$

   d) $m(a) = \frac{2a-7}{-5+\sqrt{a+4}}$

   e) $f(x) = \log_3(6x - x^2)$

   f) $p(x) = \log_2 x + \log_2(x - 3)$

   g) $f(\theta) = \tan\theta$

9. Circle C_1 has a radius 5 unit long. Circle C_2 has a radius 11 unit long. The centers are at a distance of 12 units from each other. We draw the common tangent lines drawn to the circles.

   a) Find an approximation of the angle formed by the two tangent lines.

   b) Compute the distance between the two points of tangency on one of the common tangent lines.

10. Consider the parabola $y = 4x^2 - 3x - 8$.

    a) Find both coordinates of the vertex. b) Find all $x-$intercepts. Present exact values.

11. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are 47.5° N and 37.4° N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

12. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

    a) $y = x + x^2 - 24$ and $y = 3x - 16$

    b) $(x+3)^2 + (y+2)^2 = 10$ and $(x-1)^2 + (y-2)^2 = 10$

    c) $(x+3)^2 + (y+2)^2 = 8$ and $x^2 + (y-1)^2 = 2$

13. a) Find all values of m for which the equation $(x-3)^2 + m - 7 = 0$ has exactly one solution.

    b) $m + 10x + x^2 + 25 = m^2$ has exactly one solution.

    c) Find all values of b for which the equation $x^2 - 8x - b^2 + 16 = 5b$ has exactly one real solution for x.

14. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

15. Prove each of the following identities.

    a) $\tan x + \dfrac{\cos x}{1 + \sin x} = \sec x$ b) $\dfrac{\cos x}{1 - \sin x} = \sec x + \tan x$

16. Compute an approximate value for each of the angles in a triangle with sides 7 cm, 7 cm, and 10 cm.

17. One number a is 10 greater than twice another number b. Find each of the following.

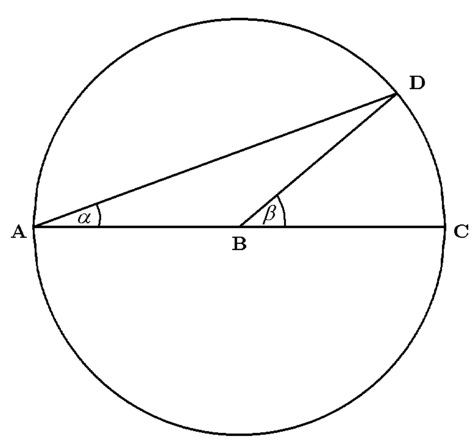
    a) The minimal value of $a^2 + b^2$. c) The maximal value of $b^2 - a^2$

    b) The minimal value of ab

18. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

    a) the perimeter of the polygon b) the area of the polygon

19. Consider the picture below. Given that B is the center of the circle, prove that $\beta = 2\alpha$.



Answers

1. a) $-\dfrac{3}{2}$ b) 8 c) 24 d) $-\dfrac{1}{2}$ e) -6

2. a) $\cos\alpha = \dfrac{\sqrt{8}}{3} = \dfrac{2\sqrt{2}}{3}$ $\tan\alpha = \dfrac{\sqrt{2}}{4}$ b) $\sin\beta = \dfrac{4\sqrt{17}}{17}$ and $\cos\beta = \dfrac{\sqrt{17}}{17}$

   c) $\sin\gamma = \dfrac{\sqrt{21}}{5}$, $\cos\gamma = \dfrac{2}{5}$, and $\tan\gamma = \dfrac{\sqrt{21}}{2}$

3. a) $\sin\alpha$ b) $-\cos\alpha$ c) $\tan\alpha$ d) $\sin\alpha$ e) $\cos\alpha$ f) $-\tan\alpha$ g) $-\sin\alpha$ h) $-\tan\alpha$

4. D

5. a) $\dfrac{\sqrt{3}}{6} - \dfrac{1}{2}$ b) -1

6. a) $x = -45° + k \cdot 360°$ or $x = -135° + k \cdot 360°$ where $k \in \mathbb{Z}$

   $x = -\dfrac{\pi}{4} + 2k\pi$ or $x = -\dfrac{3\pi}{4} + 2k\pi$ where $k \in \mathbb{Z}$

   b) $x = \pm 150° + k \cdot 360°$ where $k \in \mathbb{Z}$ $x = \pm\dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

   c) $x = -60° + k \cdot 180°$ where $k \in \mathbb{Z}$ $x = -\dfrac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

   d) no solution e) $x = 180° + k \cdot 360°$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$

   f) $x = k \cdot 180°$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$

   g) $\alpha = -60° + k \cdot 360°$ or $\alpha = -120° + k \cdot 360°$ where $k \in \mathbb{Z}$

   h) $\beta = 180° + k \cdot 360°$ where $k \in \mathbb{Z}$ i) $\gamma = 33.690\,07° + k \cdot 180°$ where $k \in \mathbb{Z}$

7. a) 16 b) $\dfrac{1}{2}(1 + \log_3 18)$ c) 6 d) $6 + 2\log_5 10$

8. a) $x \neq \pm 3$ b) $8 \leq x \leq 15$ and $x \neq 10$ c) $x \geq -2$ and $x \neq \dfrac{1}{5}$ d) $a \geq -4$ and $a \neq 21$

   e) $0 < x < 6$ f) $x > 3$ g) $\theta \neq \dfrac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

9. a) $60°$ b) $\sqrt{108} = 6\sqrt{3}$

10. a) $\left(\dfrac{3}{8}, -\dfrac{137}{16}\right)$ b) $\left(\dfrac{3-\sqrt{137}}{8}, 0\right)$ and $\left(\dfrac{3+\sqrt{137}}{8}, 0\right)$

11. 698 miles

12. a) $(-2,-22)$ and $(4,-4)$ b) $(-2,1)$ and $(0,-1)$ c) $(-1,0)$

13. a) 7 b) $0, 1$ c) $-5, 0$

14. $71.670\,246\,2°$

15. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

16. $44.415\,3^\circ$, $44.415\,3^\circ$, and $91.169\,4^\circ$

17. a) 20 b) $-\dfrac{25}{2}$ c) $\dfrac{100}{3}$

18. a) $2nR\sin\left(\dfrac{180^\circ}{n}\right)$ b) $nR\sin\left(\dfrac{180^\circ}{n}\right)R\cos\left(\dfrac{180^\circ}{n}\right) = nR^2\sin\left(\dfrac{180^\circ}{n}\right)\cos\left(\dfrac{180^\circ}{n}\right)$

19. Line segments AB and BD are both radii in the circle, and so they are equal. So ABD triangle is isosceles and so the angles opposite AB and BD are also equal to each other. Thus $\measuredangle ADB = \alpha$. The third angle in triangle ADB is $180^\circ - 2\alpha$. Angles ABD and DBC are supplementary beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ABD + \measuredangle DBC &= 180^\circ \\ 180^\circ - 2\alpha + \beta &= 180^\circ \qquad \text{subtract } 180^\circ \\ -2\alpha + \beta &= 0 \qquad \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$