

1. Simplify each of the following.

- a) $\ln\left(\frac{1}{e^2}\right)$ d) $\log_{10} 0.0001$ g) $5^{\log_{25} 20} - (e^2)^{\ln 3} + e^{\ln 5}$
 b) $3^{\log_9 m}$ e) $\log_8\left(\frac{1}{16}\right)$ h) $\frac{\sin 12^\circ}{\cos 78^\circ} + \left(\frac{1}{3}\right)^{-2} + \cos 10^\circ + \cos 170^\circ$
 c) $8^{\log_2 10}$ f) $4^{\log_2 p} \cdot e^{-\ln p} - \log_{10} 100$

2. Simplify each of the following.

- a) $3 \log_{10} x^2 - \frac{1}{2} \log_{10} (10\,000x^{16})$ b) $\log_6 2x - \log_6 12x$ c) $\ln(2e) - \ln\left(\frac{1}{2}e^3\right)$

3. Suppose that $\log_2 5 = a$. Write each of the following in terms of a .

- a) $\log_2 10$ c) $\log_2 50$ e) $\log_2\left(\frac{8}{25}\right)$ f) $\log_5\left(\frac{2}{5}\right)$ h) $\log_{10} 50$
 b) $\log_2 20$ d) $\log_2 500$ g) $\log_2 \sqrt{5}$ i) $\log_{10} 2$

4. Let $a = \log_2 9$ and $b = \log_2 5$. Express $\log_3 5$ in terms of a and b .

5. A radioactive material decays over time. t years from now, the amount in the sample, measured in grams, is $A(t) = 320 \cdot 0.95^t$.

- a) How much is in the sample at $t = 0$?
 b) How long does it take for the sample to decay so that there is exactly half of it left?

6. Which of the following is NOT equivalent to $x^{-8/3}$?

- A) $\frac{1}{(\sqrt[3]{x})^8}$ B) $\frac{1}{\sqrt[3]{x^8}}$ C) $\frac{1}{x^2 \sqrt[3]{x^2}}$ D) $\frac{\sqrt[3]{x}}{x^3}$ E) $\sqrt[3]{-x^8}$

7. a) The number of bacteria increased by 6% overnight. If this morning there are 26 457 600, how many were there last night?

b) The number of bacteria increased by 60% overnight. If this morning there are 26 457 600, how many were there last night?

c) The number of bacteria increased by 160% overnight. If this morning there are 26 457 600, how many were there last night?

8. Find the exact values of all trigonometric functions of β if we know that $\cos \beta = -\frac{12}{13}$ and β is not in the third quadrant.

9. Which of the following is NOT equal to $\sin \alpha$?

- A) $\sin(\alpha + 360^\circ)$ B) $\sin(\alpha + 180^\circ)$ C) $\sin(180^\circ - \alpha)$ D) $\cos(90^\circ - \alpha)$

10. Which of the following is NOT equal to $\cos \alpha$?

- A) $\cos(\alpha + 360^\circ)$ B) $\cos(-\alpha)$ C) $\cos(180^\circ - \alpha)$ D) $\sin(90^\circ - \alpha)$

11. Which of the following is NOT equal to $\tan \alpha$?

- A) $\tan(\alpha + 360^\circ)$ B) $\tan(\alpha + 180^\circ)$ C) $-\tan(180^\circ - \alpha)$ D) $\tan(90^\circ - \alpha)$

12. a) Suppose that α is an angle such that $\tan \alpha = 4$. Compute the exact value of $\cos \alpha$.

b) Suppose that β is an angle such that $\cos \beta = \frac{1}{3}$. Compute the exact value of $\tan \beta$.

13. Solve each of the following equations.

a) $\log_3(4x - 1) = -2$

f) $\log_x 10 = 2$

k) $\cos^2 x = \frac{1}{2}$

b) $\ln(2x + 5) = 3$

g) $\cos x + \cos x \tan x = 0$

c) $10^{3x-1} = 7$

h) $1 - \sin x = 2 \cos^2 x$

l) $\sin x = -\cos x$

d) $3e^{-2x+1} - 4 = 11$

i) $\tan^4 x = 3 \tan^2 x$

e) $2^{x-4} + 5 = 1$

j) $-4 \cos x = 4 + \sin^2 x$

m) $2^{3x-1} = 10$

n) $\log_2(x - 1) + \log_2(x + 3) = 5$

o) $\log_3(x - 7) - \log_3(-1 - 2x) = -1$

14. Find the domain for each of the following functions.

a) $f(x) = \ln(\cos^2 x)$

d) $f(x) = \sqrt{x-2} - \sqrt{5-x}$

g) $f(x) = \log_{10}(6x - x^2) + \sqrt{x-3}$

b) $f(x) = \frac{2x-3}{\log_2(x-3)-1}$

e) $f(x) = \ln(x^2 - 3x)$

c) $f(x) = \frac{\sin x}{x^2 + 10}$

f) $f(x) = \cot x$

15. A point P is located 15 units away from the center of a circle with radius 8 units. Find an approximate value of the angle formed by the two tangent lines drawn to the circle from P .

16. Find both coordinates of all points where the following graphs intersect each other.

a) $x^2 + (y - 4)^2 = 25$ and $(x - 6)^2 + (y - 1)^2 = 10$

b) $(x + 2)^2 + (y + 2)^2 = 5$ and $(x - 2)^2 + y^2 = 5$

c) $(x + 5)^2 + (y - 3)^2 = 13$ and $(x - 4)^2 + (y + 3)^2 = 52$

17. Suppose that p and q are real numbers such that p is twenty less than twice q . Find the smallest value of

a) $p^2 + q^2$

b) pq

18. We deposit \$3000 into a bank account with an annual compound interest rate of 4%, compounded annually.

a) How much money is in the account after 25 years?

b) How long do we need to wait until there is \$50 000 in the account?

19. Prove that each of the following are equal. A) $\log_{8/9} 162$ B) $\frac{\ln 162}{\ln 8 - \ln 9}$ C) $\frac{\ln 2 + 4 \ln 3}{3 \ln 2 - 2 \ln 3}$

20. Find all values of the parameter p for which the equation $(x - p)(x + p) + 9 = 2(3x + p)$ has exactly one real solution for x .

21. Find an equation for all tangent lines drawn to the graph of $y = -\frac{1}{2}x^2 - x + 2$ from the point $P(-1, 7)$.

22. Graph each of the following.

a) $f(x) = \sin x$ on the interval $[-2\pi, 2\pi]$.

d) $h(x) = -x^2(x + 1)(x - 2)$

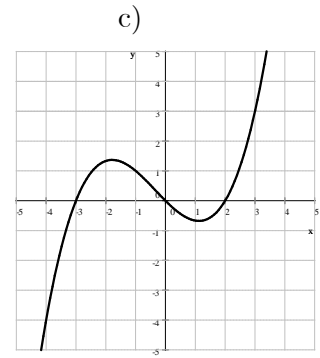
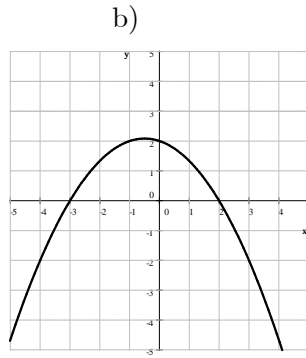
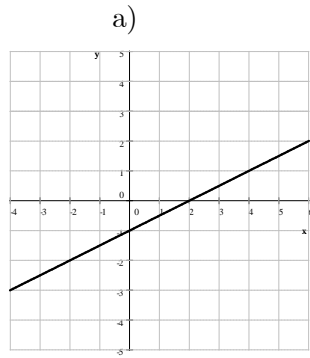
b) $g(x) = \tan x$ on the interval $[-2\pi, 2\pi]$.

e) $j(x) = x(x + 1)^2(x - 2)$

c) $m(x) = \sec x$ on the interval $[-2\pi, 2\pi]$.

f) $p(x) = \frac{1}{x - 3}$

23. The graph below is that of $y = f(x)$. Graph $y = \frac{1}{f(x)}$ in the same coordinate system.



Answers

1.) a) -2 b) \sqrt{m} c) 1000 d) -4 e) $-\frac{4}{3}$ f) $p - 2$ g) $\sqrt{20} - 4$ h) 10

2.) a) $-2 \log_{10} x - 2$ b) -1 c) $2 \ln 2 - 2$

3.) a) $a + 1$ b) $a + 2$ c) $2a + 1$ d) $3a + 2$ e) $3 - 2a$ f) $\frac{1}{a} - 1$ g) $\frac{a}{2}$ h) $\frac{2a + 1}{a + 1}$ i) $\frac{1}{a + 1}$

4.) $\log_3 5 = \frac{\log_2 5}{\log_2 3} = \frac{b}{\left(\frac{a}{2}\right)} = \frac{2b}{a}$ 5.) a) 320 g b) $\log_{0.95} 0.5 = \frac{\ln 0.5}{\ln 0.95} \approx 13.51341$ (years)

6.) E 7.) a) $24\,960\,000$ b) $16\,536\,000$ c) $10\,176\,000$

8.) $\sin \beta = \frac{5}{13}$, $\cos \beta = -\frac{12}{13}$, $\tan \beta = -\frac{5}{12}$, $\csc \beta = \frac{13}{5}$, $\sec \beta = -\frac{13}{12}$, $\cot \beta = -\frac{12}{5}$

9.) B 10.) C 11.) D 12.) a) $\pm \frac{\sqrt{17}}{17}$ b) $\pm 2\sqrt{2}$

13.) a) $\frac{5}{18}$ b) $\frac{1}{2}(e^3 - 5)$ c) $\frac{1}{3}(1 + \log_{10} 7)$ d) $\frac{1}{2}(1 - \ln 5)$ e) no solution f) $\sqrt{10}$

g) $\frac{\pi}{2} + k\pi$, $-\frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$ h) $\frac{\pi}{2} + 2k\pi$, $-\frac{\pi}{6} + 2k\pi$, $-\frac{5\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$ i) $k\pi$, $\pm \frac{\pi}{3} + k\pi$, $k \in \mathbb{Z}$

j) $\pi + 2k\pi$, $k \in \mathbb{Z}$ k) $\pm \frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$ l) $-\frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$ m) $\frac{1}{3}(1 + \log_2 10)$

n) 5 (-7 doesn't work) o) no solution (4 does not work)

14.) a) $x \neq \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$ b) $x > 3$ and $x \neq 5$ c) \mathbb{R} d) $2 \leq x \leq 5$ e) $x < 0$ or $x > 3$

f) $x \neq k\pi$, $k \in \mathbb{Z}$ g) $3 \leq x < 6$ 15.) 64.46191° 16.) a) $(5, 4)$ and $(3, 0)$ b) $(0, -1)$ c) $(-2, 1)$

17.) a) 80 when $q = 8$ and $p = -4$ b) -50 when $q = 5$ and $p = -10$

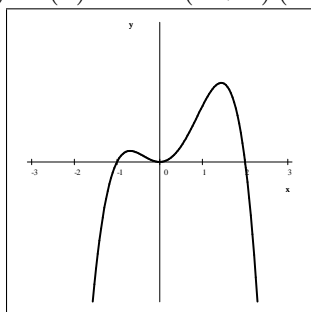
18.) a) \$7997.51 b) 72 years $(\log_{1.04} \left(\frac{50}{3}\right) = \frac{\ln \left(\frac{50}{3}\right)}{\ln 1.04} \approx 71.73278)$

19.)
$$\log_{8/9} 162 = \frac{\ln 162}{\ln(8/9)} = \frac{\ln 162}{\ln 8 - \ln 9} = \frac{\ln 2 \cdot 3^4}{\ln 2^3 - \ln 3^2} = \frac{\ln 2 + \ln 3^4}{\ln 2^3 - \ln 3^2} = \frac{\ln 2 + 4 \ln 3}{3 \ln 2 - 2 \ln 3}$$

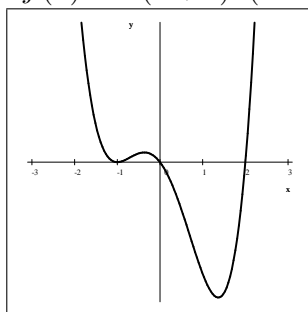
20.) 0 and -2 21.) $y = 3x + 10$ and $y = -3x + 4$

22.) a) b) c) see handout

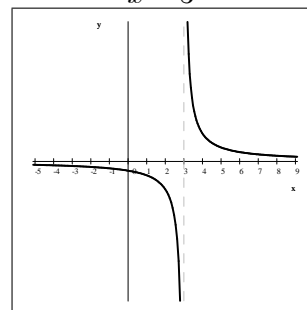
d) $h(x) = -x^2(x+1)(x-2)$



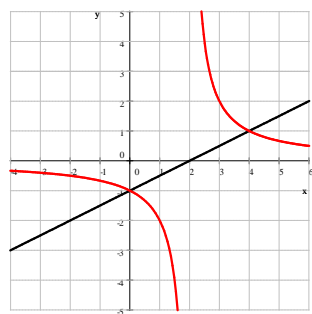
e) $j(x) = x(x+1)^2(x-2)$



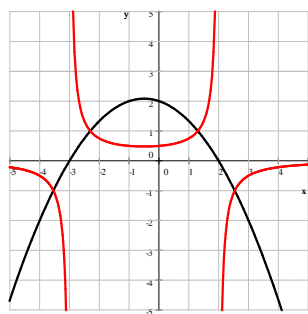
f) $p(x) = \frac{1}{x-3}$



23.) a)



b)



c)

