1. Write each of the following as a single logarithm.

   a) $\ln(x+1) + \ln(x-1)$ c) $\frac{1}{2}\ln\left(100x^{100}\right)$ e) $(\log_3 4)(\log_4 7)$

   b) $\log_{10}\left(24x^3\right) - \log_{10}(3x)$ d) $\frac{1}{2}\log_2\left(36x^{16}\right) - 2\log_2(3x) + \log_2 6 - 6\log_2 x$

2. Let $a = \log_5 3$. Write each of the following in terms of a.

   a) $\log_5 45$ b) $\log_5 75$ c) $\log_3 5$ d) $\log_5 675$ e) $\log_{45} 675$ f) $\log_3\left(\frac{3}{25}\right)$

3. Graph each of the following.

   a) $f(x) = \frac{1}{x} - 2$ b) $f(x) = \sqrt[3]{x} + 2$ c) $f(x) = \log_2 x$

4. Find the domain for each of the following functions.

   a) $f(x) = \frac{\sqrt{5x - x^2}}{x-5}$

   b) $f(x) = \log_2\left(x^2 - 6x + 5\right) + \sqrt{x+3}$

   c) $h(x) = \frac{1}{-1 + \ln x}$

   d) $h(t) = \sqrt{3 - \sin t}$

5. We place \$1000 into a bank account with an annual compound interest rate of 8%, compounded annually. How long does it take for the money in the account to reach \$1000 000?

6. Suppose that α is an angle in the first quadrant with $\sin\alpha = \frac{3}{5}$ and β is an angle in the second quadrant with $\cos\beta = -\frac{5}{13}$. Compute the exact value of each of the following.

   a) $\cos\alpha$ d) $\cos 2\alpha$ g) $\tan\beta$ j) $\tan 2\beta$ m) $\tan(\alpha+\beta)$

   b) $\tan\alpha$ e) $\tan 2\alpha$ h) $\sin 2\beta$ k) $\sin(\alpha+\beta)$

   c) $\sin 2\alpha$ f) $\sin\beta$ i) $\cos 2\beta$ l) $\cos(\alpha+\beta)$

7. Compute the exact value of each of the following. Rationalize all denominators.

   a) $\sin 75°$ b) $\tan 255°$ c) $\frac{\tan 25° + \tan 20°}{1 - (\tan 25°)(\tan 20°)}$ d) $\cos 105°$

8. Prove, using the sum formula that for all x,

   a) $\sin(x + 180°) = -\sin x$ b) $\cos(x + 180°) = -\cos x$ c) $\tan(x + 180°) = \tan x$

9. Suppose that α is an angle with $\cos\alpha = -\frac{2}{3}$. Compute the exact value of $\cos 2\alpha$.

10. Suppose that α and β are acute angles with $\cot\alpha = 7$ and $\cos\beta = \frac{3}{\sqrt{10}}$. Compute the exact value of each of the following.

    a) $\tan\beta$ b) $\tan 2\beta$ c) $\tan(\alpha + 2\beta)$

11. Find the exact value of $\tan\alpha$ if $\tan\beta = -\frac{1}{5}$ and $\tan(\alpha+\beta) = \frac{7}{17}$.

12. Find the exact value of $\tan\alpha$ if $\tan 2\alpha = \frac{4}{3}$.

13. Find a formula for $\cot(x+y)$.

14. Solve each of the following equations.

a) $\cos 2x = \sin x$

b) $\sin 2x = \cos x$

c) $\cos 2x = \cos x$

d) $\sin x + \sin 2x = \tan x$

e) $2^{3x-1} = 5$

f) $\sin x + 1 = \cos^2 x + 2$

g) $\sqrt{x-2} + \sqrt{x+3} = 5$

h) $\frac{3}{8}\ln(2x-5) = 21$

i) $e^{2x} - 3 = 12$

j) $4^x - 2^{x+1} = 8$

k) $\log_2(1-3x) + \log_2(3-x) = 7$

l) $\log_3(3x-5) - \log_3(x-6) = -2$

m) $\log_2(2x-7) - \log_2(5x-1) = -3$

n) $\cos x + \frac{\sin^2 x}{\cos x} + \sin x + \sin 2x = \frac{1}{\cos x}$

15. Find all values of the parameter m for which the equation given has exactly one solution for x.

a) $m + m^2 + x^2 - 2x + 4 = 2mx$

b) $10x - 2mx + x^2 + 20 = 6m - 5$

16. Find an equation for all tangent lines drawn to $y = \frac{1}{4}x^2 - 3x + 1$ from the point $(-1, 2)$.

17. Consider the exponential function $f(x) = 20e^{3x}$.

a) Compute $f(0)$.

b) Find the value of x for which $f(x) = 2f(0)$.

18. Graph each of the following.

a) $y = x^3 - 9x$

b) $y = -2\left(x^3 - 4x^2 + 4x\right)$

c) $y = \tan x$

d) $y = 9x^2 - x^4$

e) $y = (x+4)^2 x^4 (x-4)^2$

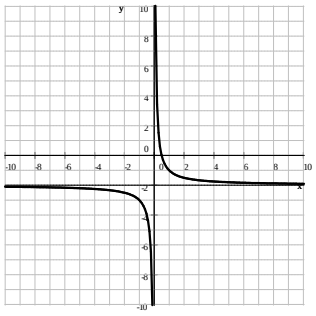
f) $y = (x+2)x^2(x-2)^3$

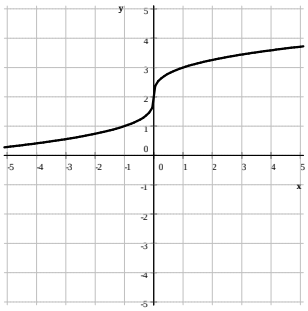
Answers

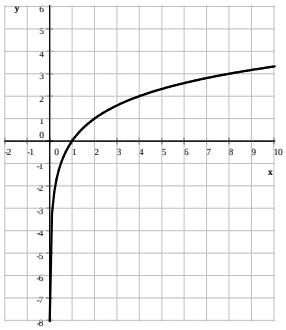
1.) a) $\ln\left(x^2-1\right)$ b) $\log_{10}\left(8x^2\right)$ c) $\ln\left(10x^{50}\right)$ d) 2 e) $\log_3 7$

2.) a) $2a+1$ b) $a+2$ c) $\dfrac{1}{a}$ d) $3a+2$ e) $\dfrac{3a+2}{2a+1}$ f) $1-\dfrac{2}{a}$

3.) a) $f(x)=\dfrac{1}{x}-2$ b) $f(x)=\sqrt[3]{x}+2$ c) $f(x)=\log_2(x+1)$







4.) a) $[0,5)$ b) $[-3,1)\cup(5,\infty)$ c) $(0,\infty)\setminus\{e\}$ d) $\mathbb{R}$ 5.) $t=\dfrac{\ln 1000}{\ln 1.08}\approx 89.756\,521$ years

6.) a) $\dfrac{4}{5}$ b) $\dfrac{3}{4}$ c) $\dfrac{24}{25}$ d) $\dfrac{7}{25}$ e) $\dfrac{24}{7}$ f) $\dfrac{12}{13}$ g) $-\dfrac{12}{5}$ h) $-\dfrac{120}{169}$ i) $-\dfrac{119}{169}$ j) $\dfrac{120}{119}$

k) $\dfrac{33}{65}$ l) $-\dfrac{56}{65}$ m) $-\dfrac{33}{56}$ 7.) a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ b) $\sqrt{3}+2$ c) 1 d) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

8.) a) $\sin(x+180^\circ)=\sin x\cos 180^\circ+\cos x\sin 180^\circ=\sin x\,(-1)+\cos x\,(0)=-\sin x$

b) $\cos(x+180^\circ)=\cos x\cos 180^\circ-\sin x\sin 180^\circ=\cos x\,(-1)-\sin x\,(0)=-\cos x$

c) $\tan(x+180^\circ)=\dfrac{\tan x+\tan 180^\circ}{1-\tan x\tan 180^\circ}=\dfrac{\tan x+0}{1-\tan x\cdot 0}=\dfrac{\tan x}{1}=\tan x$

9.) $-\dfrac{1}{9}$ 10.) a) $\dfrac{1}{3}$ b) $\dfrac{3}{4}$ c) 1 11.) $\dfrac{2}{3}$ 12.) -2 or $\dfrac{1}{2}$

13.) $\cot(x+y)=\dfrac{\cot x\cot y-1}{\cot x+\cot y}$

$$\cot(x+y)=\frac{\cos(x+y)}{\sin(x+y)}=\frac{\cos x\cos y-\sin x\sin y}{\sin x\cos y+\cos x\sin y}$$

divide both numerator and denominator by $\sin x\sin y$

$$\cot(x+y)=\frac{\cos x\cos y-\sin x\sin y}{\sin x\cos y+\cos x\sin y}=\frac{\dfrac{\cos x\cos y}{\sin x\sin y}-\dfrac{\sin x\sin y}{\sin x\sin y}}{\dfrac{\sin x\cos y}{\sin x\sin y}+\dfrac{\cos x\sin y}{\sin x\sin y}}=\frac{\cot x\cot y-1}{\cot y+\cot x}$$

14.) a) $-\frac{\pi}{2}+2k\pi,\ \frac{\pi}{6}+2k\pi,\ \frac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$ b) $\frac{\pi}{2}+2k\pi,\ \frac{\pi}{6}+2k\pi,\ \frac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$

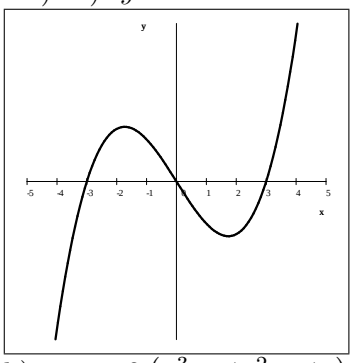
c) $2k\pi,\quad \pm\frac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$ d) $k\pi,\quad \pi+2k\pi,\quad \pm\frac{1}{3}\pi+2k\pi$ where $k\in\mathbb{Z}$

e) $\frac{1}{3}\left(1+\log_2 5\right)$ f) $\frac{\pi}{2}+2k\pi$ where $k\in\mathbb{Z}$ g) 6 h) $\frac{1}{2}e^{56}+\frac{5}{2}$ i) $\frac{1}{2}\left(\ln 15\right)$

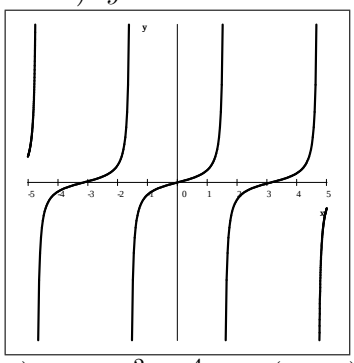
j) 2 k) -5 l) no solution m) 5 n) $k\pi,\quad \pm\frac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

15.) a) 3 b) $0, 4$ 16.) $y=-5x-3$ and $y=-2x$ 17.) a) 20 b) $\frac{\ln 2}{3}$

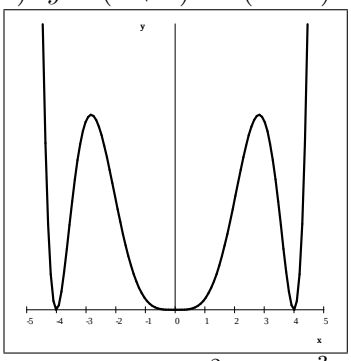
18.) a) $y=x^3-9x$



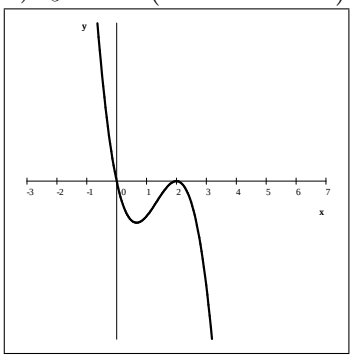
c) $y=\tan x$



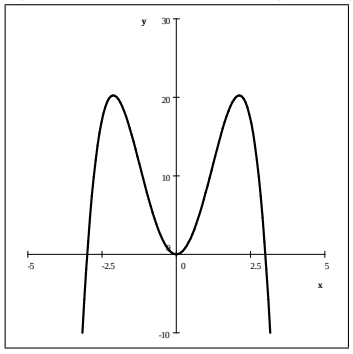
e) $y=(x+4)^2\,x^4\,(x-4)^2$



b) $y=-2\left(x^3-4x^2+4x\right)$



d) $y=9x^2-x^4=-\left(x+3\right)x^2\left(x-3\right)$



f) $y=(x+2)\,x^2\,(x-2)^3$