

1. Solve each of the following.

a) $\frac{2x-3}{x+5} \leq -1$ b) $\frac{2x+1}{x-3} < 3$ c) $\frac{x+1}{3x-2} \leq \frac{3}{4}$ d) $\frac{5x-2}{2x+1} < \frac{5}{2}$

2. Perform the following divisions on the polynomials.

a) $(x^4 - 4x^3 + 2x - 5) \div (x - 2)$ b) $(x^5 - 1) \div (x^2 - 2x - 3)$ c) $(2x^3 - 5) \div (x + 2)$

3. A used car went on a 20% sale. The sale price is \$5760. Find the original price for the car.

4. This morning the sample contained 26 928 000 bacteria. How many were there last night if the sample

a) grew 2% overnight? b) grew 20% overnight? c) grew 120% overnight?

5. What is the value of A if $x^A = \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x}}}}}}$?

6. Compute the exact value of each of the following. Rationalize the denominator in your answer.

a) $\sin 75^\circ$ b) $\cos 75^\circ$ c) $\tan 75^\circ$ d) $\sin 105^\circ$

7. Suppose that α is an angle with $90^\circ < \alpha < 180^\circ$ and $\sin \alpha = \frac{3}{5}$. Compute the exact value of each of the following.

a) $\cos \alpha$ c) $\cos 2\alpha$ e) $\sin 3\alpha$ g) $\sin 4\alpha$
 b) $\sin 2\alpha$ d) $\tan 2\alpha$ f) $\cos 3\alpha$ h) $\cos 4\alpha$

8. Suppose that α is an angle such that $\tan \alpha = 4$. Compute the exact value of

a) $\cos \alpha$ b) $\sin 2\alpha$

9. Express each of the following in terms of $\sin \alpha$ or $\cos \alpha$.

a) $\cos(-\alpha)$ b) $\cos(\alpha + 90^\circ)$ c) $\cos(180^\circ - \alpha)$

10. Which of the following is equal to $\sin \alpha$? There may be more than one correct answer.

A) $\sin(\alpha + 180^\circ)$ B) $\sin(-\alpha)$ C) $\cos(\alpha - 90^\circ)$ D) $\sin(180^\circ - \alpha)$

11. If simplified, the expression $1 - \sin 2x \tan x$ is equivalent to which one of the following?

A) $\cos 2x$ B) $\cos^2 x$ C) $\sin x \cos x$ D) $2 \cos x$ E) $\sin^2 x - \cos^2 x$

12. Compute the exact value of each of the following. Pay special attention to the sign of the results.

a) $\cos x$ if $\sin x = -\frac{2}{3}$ c) $\cos 2\beta$ if $\sin \beta = -\frac{1}{2}$ e) $\sin 2A$ if $\tan A = 4$
 b) $\tan \alpha$ if $\cos \alpha = \frac{2}{5}$ d) $\tan 2\gamma$ if $\cos \gamma = -\frac{3}{4}$ f) $\tan x$ if $\tan 2x = -\frac{12}{5}$

13. Let l be the line $y = \frac{20}{21}x$. Find an equation for the line that bisects the angle formed between l and the positive part of the x -axis.

14. Simplify each of the following expressions.

a) $\log_6 1 + \log_6 2 + \log_6 3$ c) $\log_6(12a^3) + \log_6(3a^5)$ e) $\log_2(\log_2(x^{32}))$
 b) $\log_3(x^2 - 1) - \log_3(x - 1)$ d) $\frac{1}{2} \ln(10^{10})$ f) $2 \log_5 \sqrt{a}$

15. Suppose that $\log_3 2 = x$. Express each of the following in terms of x .

- a) $\log_3 6$ b) $\log_3 24$ c) $\log_2 3$ d) $\log_6 48$

16. Solve each of the following equations.

- a) $1 - \cos x = 2 \sin^2 x$ d) $2 \cdot 3^{2x-1} = 3 \cdot 5^{x+2}$ g) $\sin 2x = \cos x$
 b) $\tan^2 x = \tan x$ e) $\log_3 (x - 4) = -2$ h) $e^{-0.3t+2} = 8$
 c) $-2 = \cos^2 x + 2 \sin x$ f) $\ln(x^2 - 1) = -1$ i) $9^x - 3^{x+2} = -18$
 j) $\sqrt{3x-2} + \sqrt{x-1} = 3$ l) $\log_5 (x - 9) - \log_5 (x - 17) = -1$
 k) $\log_2 (1 - x) + \log_2 (5 - x) = 5$ m) $\log_6 (x + 1) + \log_6 2 + \log_6 (x - 2) = 2$

17. Find the domain of each of the following expressions.

- a) $\frac{1}{\log_2 (x - 10)}$ d) $\ln(x + 3) + \ln(x - 3)$ h) $\sqrt{\cos x - 1}$
 b) $\frac{1}{\sin x - \sqrt{3} \cos x}$ e) $\log_9 \left(\frac{x + 2}{x - 5} \right)$ i) $\sqrt[3]{x - 7} + \frac{1}{x^2 + 1}$
 c) $\ln(x^2 - 9)$ g) $\sqrt{9 - x^2} - \frac{x + 3}{1 - \log_2 x}$ j) $\frac{1}{\sin 2x}$

18. Suppose we have a sample of radioactive material. At time t , the amount in the sample is $A(t) = 20e^{-0.003t}$ grams (t is measured in years). How long does it take for the sample to decay until there is only half of it left from its starting amount of $t = 0$? (This is called the half-life of the radioactive material.)

19. Solve each of the following triangles:

- a) $b = 16$ ft, $\alpha = 38^\circ$, and $\beta = 83^\circ$. d) $a = 12$ m, $c = 15$ m, and $\alpha = 32^\circ$
 b) $a = 4$ cm, $b = 7$ cm, and $\alpha = 58^\circ$.
 c) $a = 6$ in, $b = 4\sqrt{3}$ in, and $\alpha = 60^\circ$. e) $a = 4$ m, $c = 7$ m and $\gamma = 70^\circ$

20. Compute each of the following limits.

- a) $\lim_{x \rightarrow -\infty} (-3x^9)$ d) $\lim_{x \rightarrow -\infty} \sqrt{x}$ g) $\lim_{x \rightarrow \infty} \frac{2^{3x-1}}{3^{x+1}}$ i) $\lim_{x \rightarrow \infty} \frac{2^{3x-1}}{3^{2x+1}}$
 b) $\lim_{x \rightarrow \infty} (-3x^9)$ e) $\lim_{x \rightarrow \infty} \sqrt{x}$ j) $\lim_{x \rightarrow -\infty} \log_{0.8} x$
 c) $\lim_{x \rightarrow -\infty} \left(\frac{2}{x^5} \right)$ f) $\lim_{x \rightarrow -\infty} \frac{2^{3x-1}}{3^{x+1}}$ h) $\lim_{x \rightarrow -\infty} \frac{2^{3x-1}}{3^{2x+1}}$ k) $\lim_{x \rightarrow \infty} \log_{0.8} x$

21. Find all values of the parameter p so that the equation $5p^2 - 2p + x^2 = 4px + 3$ has exactly one real solution for x .

22. Find an equation for all tangent lines drawn to $y = \frac{1}{2}x^2 - 3x - 1$ from the point $(2, -13)$.

23. Graph each of the following on the interval $[-2\pi, 2\pi]$

- a) $f(x) = \sin x$ b) $f(x) = \cos x$ c) $f(x) = \tan x$

24. Graph each of the following.

- a) $y = -(x + 3)x^2(x - 3)^2$ b) $y = (x + 1)x^2(x - 2)^3$ c) $y = \log_2 x$

25. a) Graph $y = x - 3$ and $y = \frac{1}{x - 3}$ in the same coordinate system.
 b) Graph $y = (x - 3)^2$ and $y = \frac{1}{(x - 3)^2}$ in the same coordinate system.
 c) Graph $y = (x - 3)^3$ and $y = \frac{1}{(x - 3)^3}$ in the same coordinate system.

Answers

1. a) $\left(-5, -\frac{2}{3}\right]$ b) $(-\infty, 3) \cup (10, \infty)$ c) $\left(-\infty, \frac{2}{3}\right) \cup [2, \infty)$ d) $\left(-\frac{1}{2}, \infty\right)$
2. a) $x^3 - 2x^2 - 4x - 6$ R -17 b) $x^3 + 2x^2 + 7x + 20$ R $61x + 59$ c) $2x^2 - 4x + 8$ R -21
3. \$7200
4. a) 26 400 000 b) 22 440 000 c) 12 240 000
5. $\frac{1}{64}$
6. a) $\frac{\sqrt{6} + \sqrt{2}}{4}$ b) $\frac{\sqrt{6} - \sqrt{2}}{4}$ c) $2 + \sqrt{3}$ d) $\frac{\sqrt{6} + \sqrt{2}}{4}$
7. a) $-\frac{4}{5}$ b) $-\frac{24}{25}$ c) $\frac{7}{25}$ d) $-\frac{24}{7}$ e) $\frac{117}{125}$ f) $\frac{44}{125}$ g) $-\frac{336}{625}$ h) $-\frac{527}{625}$
8. a) $\pm \frac{\sqrt{17}}{17}$ b) $\frac{8}{17}$
9. a) $\cos(-\alpha) = \cos \alpha$ b) $\cos(\alpha + 90^\circ) = \cos(90^\circ - (-\alpha)) = \sin(-\alpha) = -\sin \alpha$
 c) $\cos(180^\circ - \alpha) = -\cos \alpha$
10. C and D
11. A
12. a) $\pm \frac{\sqrt{5}}{3}$ b) $\pm \frac{\sqrt{21}}{2}$ c) $\frac{1}{2}$ d) $\pm 3\sqrt{7}$ e) $\frac{8}{17}$ f) $-\frac{2}{3}$ or $\frac{3}{2}$
13. $y = \frac{2}{5}x$ or $y = -\frac{5}{2}x$
14. a) 1 b) $\log_3(x + 1)$ c) $2 + 8 \log_6 a$ d) $5 \ln 10$ e) $5 + \log_2(\log_2 x)$ f) $\log_5 a$
15. a) $x + 1$ b) $3x + 1$ c) $\frac{1}{x}$ d) $\frac{4x + 1}{x + 1}$
16. a) $x = \pm \frac{2\pi}{3} + 2k\pi$, $x = 2k\pi$, where $k \in \mathbb{Z}$ b) $x = \frac{\pi}{4} + k\pi$, $x = k\pi$, where $k \in \mathbb{Z}$
 c) $x = -\frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$ d) $\log_{9/5}\left(\frac{225}{2}\right) = \frac{\ln 225 - \ln 2}{\ln 9 - \ln 5}$ e) $\frac{37}{9}$ f) $\pm \sqrt{\frac{1}{e} + 1}$
 g) $\frac{\pi}{2} + k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ h) $\frac{2 - \ln 8}{0.3}$ i) 1, $\log_3 6$ j) 2 k) -3
 l) no solution m) 5

17. a) $x > 10$ but $x \neq 11$ b) $x \neq \frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$ c) $x < -3$ or $x > 3$ d) $x > 3$
 e) $0 < x \leq 3$ but $x \neq 2$ f) $x \neq \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$ g) $0 < x \leq 3$ and $x \neq 2$
 h) $x \neq \frac{k\pi}{2}$ where $k \in \mathbb{Z}$ i) $x < -2$ or $x > 5$ j) $x \neq k\pi$ where $k \in \mathbb{Z}$

18. $\frac{\ln 2}{0.003} \approx 231.05$ years

19. a) $\gamma = 59^\circ$, $a \approx 9.92456$ ft, $c \approx 13.81767164$ ft b) no solution
 c) $\beta = 90^\circ$, $\gamma = 30^\circ$, $c = 2\sqrt{3}$ in
 d) $\gamma_1 \approx 41.483098^\circ$ $\beta_1 \approx 106.516902^\circ$ and $b_1 \approx 21.71053535$ m
 $\gamma_2 \approx 138.516902^\circ$ $\beta_2 \approx 9.483098^\circ$ and $b_2 \approx 3.73090755$ m
 e) $\alpha \approx 32.477421^\circ$ $\beta \approx 77.522579^\circ$ $b \approx 7.2733024$ m

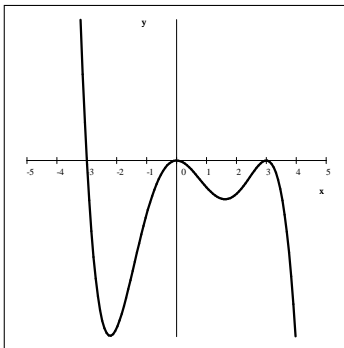
20. a) ∞ b) $-\infty$ c) 0 d) undefined e) ∞ f) 0 g) ∞ h) ∞ i) 0
 j) undefined k) $-\infty$

21. $-1, 3$

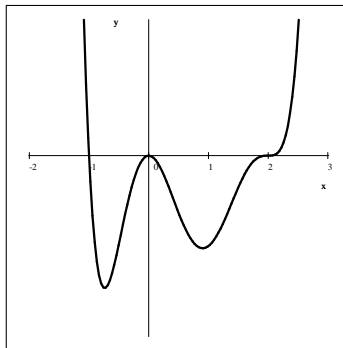
22. $y = 3x - 19$ and $y = -5x - 3$

23. See handout on basic trigonometric functions

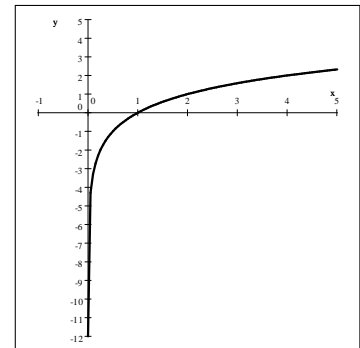
24. a) $y = -(x+3)x^2(x-3)^2$



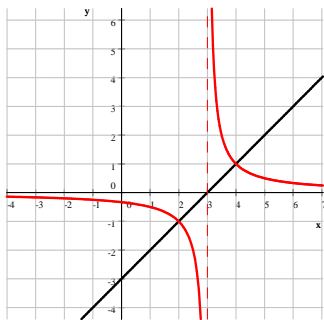
b) $y = (x+1)x^2(x-2)^3$



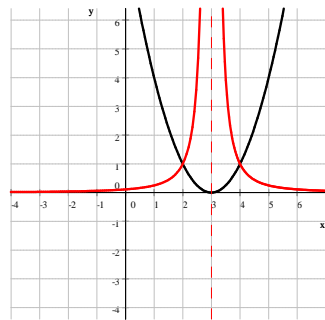
c) $y = \log_2 x$



25. a)



b)



c)

