1. Perfom the following operations on the complex numbers.

   a) $2(3-2i)-3(5-4i)$
   b) $(1-i)^2$
   c) $(3-4i)(3+4i)$
   d) $(5-2i)^2$
   e) $(1+i)^4$
   f) $\dfrac{7+11i}{3-5i}$
   g) $(3-2i)^2(3+2i)^2$
   h) $\dfrac{3i+1}{i-1}$
   i) $|5-12i|$
   j) $|a+bi|$

2. Perform each of the following divisions.

   a) $\left(6x^4-15x^3+x^2\right)\div\left(3x^2-1\right)$
   b) $x^6\div(x-2)$

3. Simplify $\log_{10}33-\dfrac{1}{2}\log_{10}44-\log_{10}15-\log_{10}\sqrt{1100}$

4. Solve each of the following ineqalities.

   a) $\dfrac{x}{2x-1}\le\dfrac{2}{3}$
   b) $\dfrac{1}{2}x^2+18\le 6x$
   c) $x^2+13<6x$

5. Solve each of the equations.

   a) $\log_3(x-1)-\log_3(7x+1)=-2$
   b) $2+\log_2(x+1)-\log_2(x+4)=3$
   c) $9^x-3^{x+1}=10$
   d) $\log_{3x-1}5+\log_{3x-1}\left(x^2+1\right)=2$
   e) $\log_5\left(9^{\sqrt{x+1}}-118\right)-3=0$
   f) $\sqrt{x+2}+\sqrt{1-3x}=0$
   g) $3^{4-x}+3^{x-1}=12$

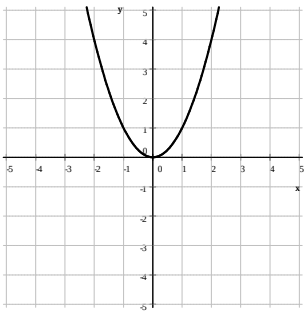
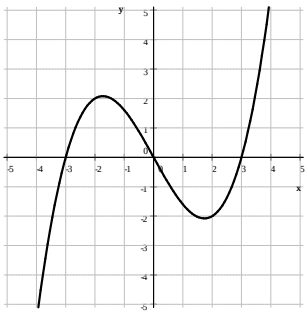
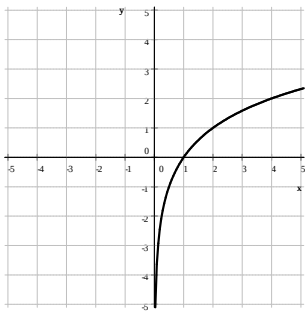
6. For each of the following functions given, find an equation for the inverse.

   a) $f(x)=3x+1$
   b) $f(x)=\sqrt[3]{2x-5}$
   c) $f(x)=e^{3x-1}$
   d) $f(x)=\log_5\left(\dfrac{1}{x}-1\right)$
   e) $f(x)=\dfrac{2x-3}{x+5}$

7. Graph each of the following functions.

   a) $f(x)=-2(x+2)^3x^2(x-2)(x-3)^2$
   b) $f(x)=\dfrac{1}{x}+3$
   c) $f(x)=\dfrac{1}{x+3}$

8. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.

9. Compute each of the following limits.

   a) $\lim\limits_{x\to-\infty}\dfrac{3x^4-2x^2+x-1}{x^4+3x+1}$
   b) $\lim\limits_{x\to\infty}\dfrac{3x^4-2x^2+x-1}{x^4+3x+1}$
   c) $\lim\limits_{x\to-\infty}\dfrac{x^3-2x^2-4x+9}{-x^2+x-6}$
   d) $\lim\limits_{x\to\infty}\dfrac{x^3-2x^2-4x+9}{-x^2+x-6}$
   e) $\lim\limits_{x\to-\infty}\dfrac{8x^2+x+5}{-2x^3+4}$
   f) $\lim\limits_{x\to\infty}\dfrac{8x^2+x+5}{-2x^3+4}$

10. The number of cells in a sample at time t (measured in hours) is $N(t) = 20\,000\left(1.2^{0.5t}\right)$.
    a) How many cells are in the sample at $t = 0$?
    b) How long will it take for the sample to double from the amount that it had at $t = 0$?
    c) How many cells are in the sample at $t = 6$?
    d) How long will it take for the sample to double from the amount that it had at $t = 6$?
    e) What do you observe? Can we make (and perhaps prove) a general statement?

11. Prove the identity $\dfrac{\tan\left(\frac{\pi}{4} - x\right)}{\tan\left(\frac{\pi}{4} + x\right)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$

12. Find the domain for each of the following functions.
    a) $f(x) = \dfrac{\ln(x+4)}{\ln(x-5)}$ b) $g(x) = \ln\left(\dfrac{x+4}{x-5}\right)$ c) $h(x) = \dfrac{1}{\sin x - \cos x}$ d) $h(x) = \dfrac{\cos^4 x - \sin^4 x}{\sin 2x}$

13. Compute the exact value of each of the following.
    a) $\sin x$ if $\cos 2x = -\dfrac{2}{3}$ b) $\cos x$ if $\cos 2x = \dfrac{4}{5}$ c) $\cos x$ if $\sin 2x = \dfrac{5}{13}$ d) $\sin 2x$ if $\tan x = 2$

14. Compute the exact value of $\tan\alpha$ if we know that $90^\circ \le \alpha \le 180^\circ$ and $\cos 2\alpha = \dfrac{3}{5}$.

15. Solve each of the following triangles.
    a) $a = 17$, $b = 13$, and $\beta = 20^\circ$ b) $a = 17$, $b = 13$, and $\alpha = 20^\circ$ c) $a = 17$, $b = 13$, and $\gamma = 20^\circ$

16. Find the exact value of the cosine of the smallest angle in a triangle with sides 3, 5, and 6.

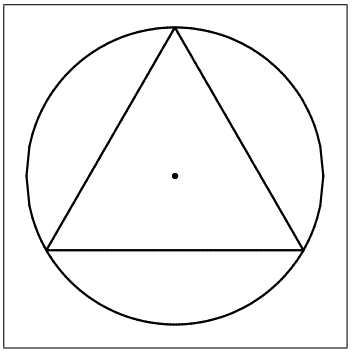
17. Find the **exact value** of the area of a triangle with sides 2, 3, and 4.

18. Triangle SML has sides of length 6, 7, and 8. Find the exact value of $\cos S + \cos M + \cos L$.

19. a) Find the exact value of $\cos\dfrac{\alpha}{2}$ if we know that $\cos\alpha = \dfrac{1}{3}$.
    b) Find the exact value of $\sin\dfrac{\beta}{2}$ if we know that $\sin\beta = \dfrac{2}{3}$.

20. A triangle has sides of length a, b, and c, which are consecutive integers in increasing order, and $\cos\gamma = \dfrac{5}{16}$. Find $\cos\alpha$.

21. Consider an equilateral triangle with sides 1 unit long, inscribed in a circle. Let O be the center of the circle.
    a) Find the radius of the circle.
    b) Find the distance between O and the side of the triangle.



Answers

1. a) $-9+8i$ b) $-2i$ c) 25 d) $21-20i$ e) -4 f) $-1+2i$ g) 169 h) $1-2i$ i) 13 j) $\sqrt{a^2+b^2}$

2. a) $2x^2-5x+1$ R $-5x+1$ b) $x^5+2x^4+4x^3+8x^2+16x+32$ R 64

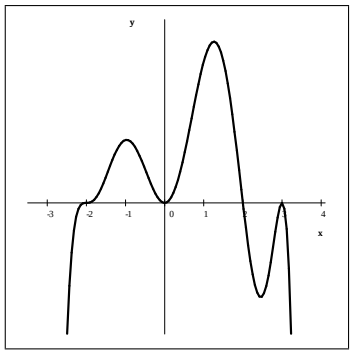
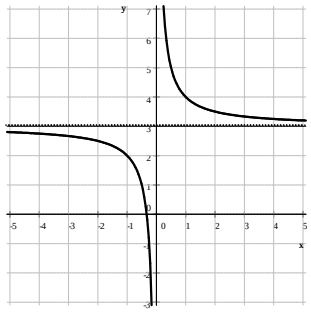
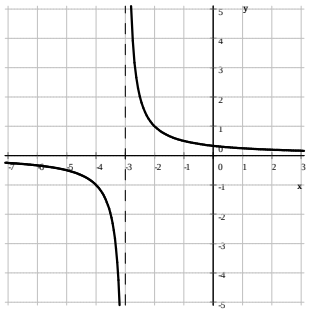
3. -2

4. a) $\left(-\infty,\frac{1}{2}\right)\cup[2,\infty)$ b) $\{6\}$ c) no solution

5. a) 5 b) no solution (-7 does not work) c) $\log_3 5$ d) 2 e) $\frac{21}{4}$ f) no solution g) $2,3$

6. a) $f^{-1}(x)=\frac{1}{3}x-\frac{1}{3}$ b) $f^{-1}(x)=\frac{1}{2}\left(x^3+5\right)$ c) $f^{-1}(x)=\frac{1}{3}(1+\ln x)$ d) $f^{-1}(x)=\frac{1}{5^x+1}$
   e) $f^{-1}(x)=\frac{5x+3}{-x+2}$

7. a) $f(x)=-2(x+2)^3x^2(x-2)(x-3)^2$ b) $f(x)=\frac{1}{x}+3$ c) $f(x)=\frac{1}{x+3}$

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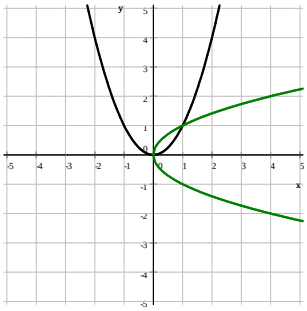
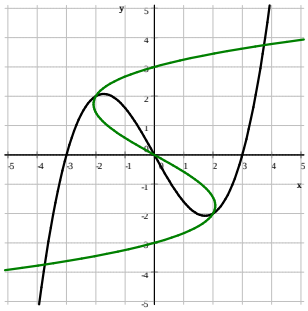
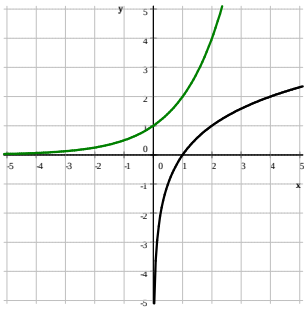

9. a) 3 b) 3 c) ∞ d) $-\infty$ e) 0 f) 0

10. a) $20\,000$ b) $7.603\,568$ hours c) $34\,560$ d) $\frac{\ln 2\left(1.2^3\right)}{0.5\ln 1.2}-6\approx 7.603\,568$ hours

    e) For any time t, we will need to wait until $t+\frac{2\ln 2}{\ln 1.2}$ for the sample to double. So, the doubling time is independent of t and is constant. Let t_1 be any time and t_2 is the time when the amount is doubled from $A(t_1)$. In other words, $A(t_2)=2A(t_1)$.

$$\begin{aligned} A(t_2) &= 2A(t_1) \\ 20\,000\left(1.2^{0.5t_2}\right) &= 2\cdot 20\,000\left(1.2^{0.5t_1}\right) \\ 1.2^{0.5t_2} &= 2\left(1.2^{0.5t_1}\right) \\ \frac{1.2^{0.5t_2}}{1.2^{0.5t_1}} &= 2 \\ 1.2^{0.5(t_2-t_1)} &= 2 \\ 0.5(t_2-t_1)\ln 1.2 &= \ln 2 \\ t_2-t_1 &= \frac{\ln 2}{0.5\ln 1.2} = \frac{2\ln 2}{\ln 1.2} \approx 7.603\,568 \text{ hours} \end{aligned}$$

11. Claim: $\dfrac{\tan\left(\frac{\pi}{4}-x\right)}{\tan\left(\frac{\pi}{4}+x\right)} = \dfrac{1-\sin 2x}{1+\sin 2x}$

Proof:

$$\begin{aligned} \text{LHS} &= \frac{\tan\left(\frac{\pi}{4}-x\right)}{\tan\left(\frac{\pi}{4}+x\right)} = \frac{\frac{1-\tan x}{1+\tan x}}{\frac{\tan x+1}{1-\tan x}} = \left(\frac{1-\tan x}{1+\tan x}\right)^2 = \left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)^2 = \left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\cdot\frac{\cos x}{\cos x}\right)^2 \\ &= \left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)^2 = \frac{\cos^2 x+\sin^2 x-2\sin x\cos x}{\cos^2 x+\sin^2 x+2\sin x\cos x} = \frac{1-\sin 2x}{1+\sin 2x} = \text{RHS} \end{aligned}$$

12. a) $x>5$ and $x\neq 6$ b) $x<-4$ or $x>5$ c) $x\neq\dfrac{\pi}{4}+k\pi$ where $k\in\mathbb{Z}$ d) $x\neq\dfrac{k\pi}{2}$ where $k\in\mathbb{Z}$

13. a) $\pm\dfrac{\sqrt{30}}{6}$ b) $\pm\dfrac{3\sqrt{10}}{10}$ c) $\pm\dfrac{\sqrt{26}}{26}, \pm\dfrac{5\sqrt{26}}{26}$ d) $\dfrac{4}{5}$

14. $-\dfrac{1}{2}$

15. a) $\alpha_1 \approx 26.567\,838^\circ$ $\gamma_1 \approx 133.432\,162^\circ$ $c_1 \approx 27.602\,045$
$\alpha_2 \approx 153.432\,162^\circ$ $\gamma_2 \approx 6.567\,838^\circ$ $c_2 \approx 4.347\,503\,459$
b) $\beta \approx 15.161\,745\,79^\circ$ $\gamma \approx 144.838\,254\,21^\circ$ $c \approx 28.624\,256\,733\,067\,1$
c) $c = 6.531\,146$ $\alpha = 117.095\,52^\circ$ $\beta = 42.904\,48^\circ$

16. $\dfrac{13}{15}$

17. $\dfrac{3\sqrt{15}}{4}$

18. $\dfrac{47}{32}$

19. a) $\pm\dfrac{\sqrt{6}}{3}$ b) $\pm\dfrac{\sqrt{6}}{6}$ or $\pm\dfrac{\sqrt{30}}{6}$

20. $\dfrac{13}{20}$

21. a) $\dfrac{\sqrt{3}}{3}$ b) $\dfrac{\sqrt{3}}{6}$