Please note that Quiz 7 will also cover topics covered on Quizzes 1-6 and Exam 1. Please review those topics as well, even if they do not appear in this document.

1. Simplify each of the following. Present all answers using only positive exponents.

   a) $\dfrac{2^{-1}-3^{-2}}{-2^{-2}+1}$ b) $\dfrac{2b^{-2}\left(-a^{3}\right)^{-2}b^{0}}{\left(-b^{2}\right)^{-3}a^{-5}}$ c) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ d) $\left(\dfrac{1-\sqrt{5}}{2}\right)^{2}-1$ e) $\dfrac{\sqrt{500}-\sqrt{20}}{\sqrt{45}-\sqrt{5}}$

2. Solve each of the following equations.

   a) $\sqrt{2x-1}-\sqrt{x-1}=1$ b) $\sqrt{x-4}-\sqrt{2x-10}=1$ c) $\sqrt{2x+1}=\sqrt{3x+4}-1$

3. A right triangle has sides 5 ft, 12 ft, and 13 ft long.

   a) State the value of all six trigonometric functions of α if α is the angle opposite the 5 ft long side.

   b) State the value of all six trigonometric functions of β if β is the angle opposite the 12 ft long side.

4. The hypotenuse of a right triangle is 31 cm long. One angle in the triangle measures 42°.

   a) Find the exact value of the length of the missing sides in the triangle.

   b) Find an approximate value of the length of the missing sides in the triangle. Round your answer to four or more decimal places.

5. Draw a square with sides 1 unit long and draw in one of the diagonals. Use the isosceles right triangle you obtained to find all trigonometric function values of 45°.

6. Use your calculator to find an approximate value of the smallest angle in a right triangle with sides 84 unit, 85 unit, and 13 units long. Present your answer in degrees, accurate up to four or more decimals. (That means at least four digits after the decimal point).

7. The hypotenuse of a right triangle is 74 units. The difference between the lengths of the other two sides is 46 units.

   a) How long are the sides of this triangle?

   b) Use your calculator to find an approximate value of the smallest angle in the triangle. Present your answer in degrees, accurate up to four or more decimals.

8. Solve the equation $5\left(x-2\right)^{2}-3x+2=x-2$. Check your solution(s) using exact values.

9. Solve each of the following the inequalities.

   a) $\left(2x-1\right)^{2}\leq 3\left(x-1\right)^{2}$ b) $x^{2}-5x+14\geq x+3$

10. Find the exact value of the length of the main diagonal in a rectangular prism with sides 4, 7, and 10 units.

11. Consider a circle of radius 12 units and with a center C. P is a point located 30 units away from C. We draw a tangent line from P to the circle. The point of tangency is Q.

    a) Compute the exact value of $d\left(P,Q\right)$ (that is the distance between P and Q).

    b) Let α denote the angle CPQ. Compute the exact value of all six trigonometric functions of α.

12*. Compute the sum $1+2+3+....+2014$

Answers

1. a) $\dfrac{14}{27}$ b) $-\dfrac{2b^4}{a}$ c) $2-\sqrt{3}$ d) $\dfrac{1-\sqrt{5}}{2}$ e) 4

2. a) $1, 5$ b) 5 (13 doesn't work) c) $0, 4$

3. a) $\sin\alpha = \dfrac{5}{13}$ $\cos\alpha = \dfrac{12}{13}$ $\tan\alpha = \dfrac{5}{12}$ $\csc\alpha = \dfrac{13}{5}$ $\sec\alpha = \dfrac{13}{12}$ $\cot\alpha = \dfrac{12}{5}$

   b) $\sin\beta = \dfrac{12}{13}$ $\cos\beta = \dfrac{5}{13}$ $\tan\beta = \dfrac{12}{5}$ $\csc\beta = \dfrac{13}{12}$ $\sec\beta = \dfrac{13}{5}$ $\cot\beta = \dfrac{5}{12}$

4. a) $(31\sin 42^\circ)$ cm and $(31\cos 42^\circ)$ cm b) $20.743\,0488$ cm and $23.037\,49$ cm

5. $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$ $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$ $\tan 45^\circ = 1$ $\csc 45^\circ = \sqrt{2}$ $\sec 45^\circ = \sqrt{2}$ $\cot 45^\circ = 1$

6. $8.797\,41^\circ$

7. a) $24, 70$, and 74 units long b) $18.924\,644\,42^\circ$

8. $x_{1,2} = \dfrac{12 \pm 2\sqrt{6}}{5}$

   Check: If $x = \dfrac{12+2\sqrt{6}}{5}$, then

$$
\begin{aligned}
\text{LHS} &= 5\left(\frac{12+2\sqrt{6}}{5}-2\right)^2 - 3\left(\frac{12+2\sqrt{6}}{5}\right)+2 = 5\left(\frac{12+2\sqrt{6}}{5}-\frac{10}{5}\right)^2 - 3\left(\frac{12+2\sqrt{6}}{5}\right)+2 \\
&= 5\left(\frac{2+2\sqrt{6}}{5}\right)^2 - 3\left(\frac{12+2\sqrt{6}}{5}\right)+2 = 5\left(\frac{4+8\sqrt{6}+4\cdot 6}{25}\right) - 3\left(\frac{12+2\sqrt{6}}{5}\right)+2 \\
&= \frac{28+8\sqrt{6}}{5} - \frac{3\left(12+2\sqrt{6}\right)}{5} + \frac{10}{5} = \frac{28+8\sqrt{6}-36-6\sqrt{6}+10}{5} = \frac{2+2\sqrt{6}}{5} \\
\text{RHS} &= \frac{12+2\sqrt{6}}{5} - 2 = \frac{12+2\sqrt{6}}{5} - \frac{10}{5} = \frac{2+2\sqrt{6}}{5}
\end{aligned}
$$

   Checking the other solution is a similar computation.

9. a) $-1-\sqrt{3} \le x \le -1+\sqrt{3}$ b) $\mathbb{R}$

10. $\sqrt{165}$

11. a) $\sqrt{756} = 6\sqrt{21}$

    b) $\sin\alpha = \dfrac{2}{5}$ $\cos\alpha = \dfrac{\sqrt{21}}{5}$ $\tan\alpha = \dfrac{2}{\sqrt{21}}$ $\csc\alpha = \dfrac{5}{2}$ $\sec\alpha = \dfrac{5}{\sqrt{21}}$ $\cot\alpha = \dfrac{\sqrt{21}}{2}$

12. $2029\,105$