Please note that Quiz 8 will also cover topics covered on Quizzes 1-7 and Exam 1. Please review those topics as well, even if they do not appear in this document.

1. Simplify each of the following expressions. Use exact values.
   a) $\sin 30^\circ - 2\cos 30^\circ$ b) $\tan 60^\circ \cot 30^\circ - 2\sin 45^\circ \sin 60^\circ$ c) $\sin^2 30^\circ - \cos^2 30^\circ$

2. Factor by completing the square $18x^2 - 24x - 2$

3. Consider the parabola $y = x^2 + 7x - 6$.
   a) Find both coordinates of the vertex. b) Find all $x-$intercepts. Present exact values.

4. Solve each of the following inequalities.
   a) $12x - 2x^2 \le 20$ c) $10x + 5x^2 \le -25$ e) $x^2 + x > 1$
   b) $3x^2 - 12x < 3$ d) $10x + 5x^2 \ge 25$

5. Solve each of the following equations.
   a) $(2x-1)^2 - x(3x-5) = 5x + 1$ b) $2x^3 = 8x$ c) $\sqrt{4x+1} - 1 = \sqrt{x+2}$

6. For each of the following pairs of graphs, find the coordinates of all points where they intersect.
   a) $y = x^2 + 7x - 6$ and $y = 3x - 1$
   b) $y = -\frac{1}{2}x^2 + 5x - 3$ and $y = 3x - 1$
   c) $y = 3x^2 - x + 7$ and $y = x - 4$
   d) $x^2 + (y+4)^2 = 20$ and $x - 3y = 2$
   e) $(x-2)^2 + (y+5)^2 = 10$ and $3y - x = -7$
   f) $x^2 + (y+1)^2 = 16$ and $y + 2x = 15$

7. Find an equation for the line connecting the points of intersections of the circles $(x+1)^2 + (y-2)^2 = 10$ and $(x-1)^2 + (y-6)^2 = 50$.

8. Find an equation of the tangent line drawn
   a) to $2y - 6x + x^2 + y^2 = 40$ at the point $(10, -2)$.
   b) to $2y + x^2 + y^2 = 3(2x+5)$ at the point $(7, -4)$.

9. Find the height of the pyramid $ABCDE$ if its base is rectangle $ABCD$ with sides 6 m and 10 m and edges $AE = BE = CE = DE = 14$ m.

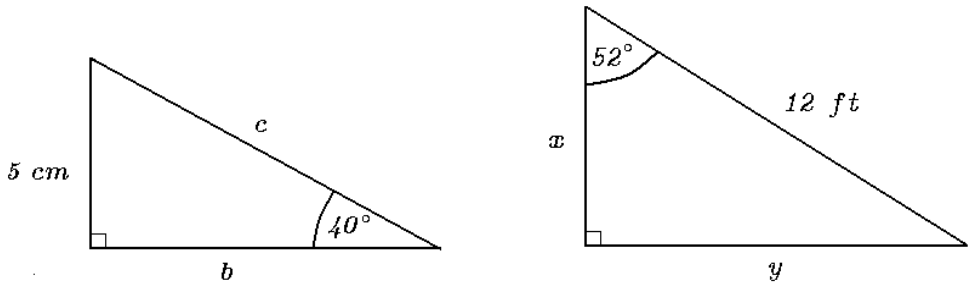
10. The sum of two numbers is 20. Find each of the following.
    a) the maximal value of their product. b) the minimal value of the sum of their squares.

11. When we price a ticket at \$50, then we can sell 2000 tickets For every dollar increase in the price, we can sell 10 less tickets. Find the maximal revenue possible, and find the price that guarantees the maximal income.

12. Side $AB$ of a rectangle $ABCD$ is located on the $x-$ axis, within the interval $[-4, 4]$. Another horizontal side is defined by points $C$ and $D$, lying on the straight lines $y = 3x + 12$ and $y = -3x + 12$, respectively. Find the maximal possible area of the rectangle.

13. Consider the triangle with sides 16 ft, 17 ft, and 17 ft long.
    a) Compute the exact value of the area of the triangle.
    b) Compute the approximatie value of the angles in the triangle. Present your answer in degrees, accurate to four or more decimal places.

14. Based on the pictures below, compute the exact and approximate values of $b$, $c$, $x$ and $y$.

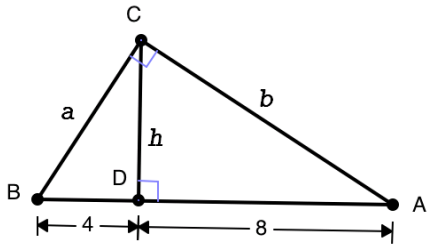



a) b)

15. Consider the right triangle $ABC$ in which the hypotenuse is 37 and the shortest side is 12 units long.
    a) Find the exact value of all six trigonometric functions of $\alpha$ if $\alpha$ is the smallest angle in the triangle.
    b) Find the exact value of all six trigonometric functions of $\beta$ if $\beta$ is the second smallest angle in the triangle.
    c) Find an approximate value for $\alpha$ and $\beta$.

16. $P$ is a point 10 units away from the center of a circle with radius 4. Find the angle formed by the tangent lines dran to the circle from point $P$. Express your answer as
    a) an exact value b) an approximation, accurate up to 4 decimal places.

17. Consider the right triangle $ABC$ shown on the picture below. Find the exact values of $a$, $b$, and $h$.




18. Two real numbers $a$ and $b$ are such that $a + 3b = 10$. Find
    a) the exact values of $a$ and $b$ if $ab = -32$.
    b) the maximal possible value of $ab$.
    c) the smallest possible value of $a^2 + b^2$.

19. Let $f$ be the function defined by $f(x) = -x^2 + 4x + 1$.
    a) Compute $f(5)$
    b) Is it true that $f(2+3) = f(2) + f(3)$
    c) Is it true that $3f(4) = f(3 \cdot 4)$?
    d) Simplify $f(2a)$
    e) Compute $f(f(5))$
    f) Compute $f(f(f(1)))$

# Answers

1.) a) $\frac{1}{2} - \sqrt{3}$ b) $3 - \frac{\sqrt{6}}{2}$ c) $-\frac{1}{2}$ 2.) $18\left(x - \frac{2+\sqrt{5}}{3}\right)\left(x - \frac{2-\sqrt{5}}{3}\right)$

3.) a) $\left(-\frac{7}{2}, -\frac{73}{4}\right)$ b) $\left(\frac{-7-\sqrt{73}}{2}, 0\right)$ and $\left(\frac{-7+\sqrt{73}}{2}, 0\right)$

4.) a) all real numbers b) $\left(2-\sqrt{5}, 2+\sqrt{5}\right)$ c) no solution d) $\left(-\infty, -1-\sqrt{6}\right] \cup \left[-1+\sqrt{6}, \infty\right)$

e) $\left(-\infty, -\frac{1+\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

5.) a) $0, 4$ b) $-2, 0, 2$ c) $2$ $\left(-\frac{2}{9}\right.$ does not work)

6.) a) $(1, 2)$ and $(-5, -16)$ b) $(2, 5)$ c) no intersection point d) $(-4, -2)$ and $(2, 0)$

e) $(1, -2)$ f) no intersection point 7.) $y = -\frac{1}{2}x - 1$

8.) a) $y + 2 = 7(x - 10)$ or $y = 7x - 72$ b) $\frac{4}{3}(x - 7) = y + 4$ or $y = \frac{4}{3}x - \frac{40}{3}$

9.) $\sqrt{162}\,\text{m} = 9\sqrt{2}\,\text{m}$ 10.) a) 100 b) 200 11.) \$156 250 with a price of \$125

12.) 24 13.) a) $120\,\text{ft}^2$ b) $61.927\,513°$, $61.927\,513°$, and $56.144\,974°$

14.) a) $c = \frac{5}{\sin 40°}\,\text{cm} \approx 7.778\,62\,\text{cm}$ $b = \frac{5}{\tan 40°}\,\text{cm} \approx 5.958\,768$

b) $x = 12\cos 52°\,\text{ft} \approx 7.387\,94\,\text{ft}$ $y = 12\sin 52°\,\text{ft} \approx 9.456\,13\,\text{ft}$

15.) a) $\sin\alpha = \frac{12}{37}$ $\cos\alpha = \frac{35}{37}$ $\tan\alpha = \frac{12}{35}$ $\csc\alpha = \frac{37}{12}$ $\sec\alpha = \frac{37}{35}$ $\cot\alpha = \frac{35}{12}$

b) $\sin\beta = \frac{35}{37}$ $\cos\beta = \frac{12}{37}$ $\tan\beta = \frac{35}{12}$ $\csc\beta = \frac{37}{35}$ $\sec\beta = \frac{37}{12}$ $\cot\beta = \frac{12}{35}$

c) $\alpha \approx 18.924\,644°$ $\beta \approx 71.075\,356°$

16.) a) $2\sin^{-1}\left(\frac{4}{10}\right)$ b) $47.156\,357°$ 17.) $a = 4\sqrt{3}$ $b = 4\sqrt{6}$ $h = 4\sqrt{2}$

18.) a) $a = -6, b = \frac{16}{3}$ or $a = 16, b = -2$ b) $\frac{25}{3}$ if $a = 5, b = \frac{5}{3}$ c) 10 if $a = 1, b = 3$

19.) a) $-4$ b) false: $f(2) = 5$, $f(3) = 4$, and $f(5) = -4$ and so $5 + 2 \neq -4$

c) false: $f(4) = 1$, $3f(4) = 3$, and $f(12) = -95$ and so $3 \neq -95$ d) $-4a^2 + 8a + 1$

e) $f(f(5)) = f(-4) = -31$ f) $f(f(f(1))) = f(f(4)) = f(1) = 4$