1. Find the radius of the circle if
   a) an arc subtended by a central angle of 30° has a length of 20 cm.
   b) a sector of 30° has an area of $20\,\text{cm}^2$.

2. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
   a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
   b) Compute the distance between the two points of tangency on a common tangent line.

3. Seattle, WA and Los Angeles, CA have approximately the same longitude. The latitude of Seattle is 47.6° and that of Los Angeles is 34.1°. Find the distance to the nearest mile between the two cities. (The radius of the earth is approximately 3960 miles.)

4. A satellite is 1000 miles above the surface of the equator. The radius of Earth is 3960 miles. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?

5. Solve each of the following equations.
   a) $\sqrt{2x-1} = 2 + \sqrt{x-4}$ b) $\sqrt{x+1} + \sqrt{5x+1} = 6$ c) $\sqrt{2x+1} = \sqrt{3x+4} - 1$

6. Completely factor each of the following.
   a) $x^3 + x^2 - 2x$ b) $x^5 - 9x^3$ c) $6x^2 - 11x - 10$ d) $x^4 - 1$

7. Solve each of the following inequalities.
   a) $x^2 > x$ b) $x^2 + 13 < 6x$ c) $8x + x^2 \le -9$ d) $\frac{1}{4}x^2 + 9 \le -3x$

8. Simplify each of the following.
   a) $-16^{-3/4}$ c) $(-16)^{-1/4}$ e) $\log_2 \sqrt{8}$ g) $\log_5\left(5^{100}\right)$ i) $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$
   b) $(-8)^{-1/3}$ d) $(-5)^0$ f) $\log_4\left(\frac{1}{\sqrt{8}}\right)$ h) $2^{\log_2 16}$ j) $\log_{0.1} 1000$

9. Simplify each of the following.
   a) $e^{-\ln A} + e^{-\ln B}$
   b) $e^{-\ln A} e^{-\ln B}$
   c) $3^{\log_3 A}$
   d) $9^{\log_2 P}$
   e) $3^{\log_9 Q}$
   f) $\left(\sqrt{8}\right)^{\log_2 T}$
   g) $\log_2\left(16^a\right)$
   h) $\log_{16}\left(2^a\right)$
   i) $\log_2(\sec 45^\circ)$
   j) $\log_3(\tan 60^\circ)$
   k) $\log_5\left(5^{143}\right)$
   l) $\ln\left(e^{123}\right)$
   m) $e^{-2\ln 5}$

10. Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$. Compute each of the following.
    a) $f(g(-2))$ b) $g(f(-2))$ c) $f(f(2))$ d) $f(f(f(2)))$ e) $f(g(f(3)))$

11. Let $f(x) = \dfrac{2x-1}{x+8}$ and $g(x) = \dfrac{8x+1}{-x+2}$. Compute each of the following.
    a) $f(3)$ b) $f(-7)$ c) $g(2)$ d) $g(1)$ e) $f(g(1))$ f) $f(2)$ g) $g(f(2))$

12. One number a is five more than twice another number b. Find the smallest value of $a = 2b + 5$
    a) $a^2 + b^2$ b) ab c) $a^2 - b^2$

13. Compute the exact value of each of the following.

a) $\sin 30^\circ \cos 45^\circ - \tan 120^\circ$

b) $\sec 45^\circ - \tan 45^\circ + \sin^2 30^\circ \cos^2 30^\circ$

c) $\dfrac{2 \sin 30^\circ \cos 30^\circ - 3 \sin 60^\circ \cos 60^\circ}{(\tan 30^\circ)(\tan 60^\circ)}$

d) $\sin^2 18^\circ + \cos^2 18^\circ$

e) $\sin 28^\circ - \cos 62^\circ$

f) $\dfrac{\tan 60^\circ - \tan 45^\circ}{1 + (\tan 60^\circ)(\tan 45^\circ)}$

14. Prove each of the following identities.

a) $\tan^2 x + 1 = \sec^2 x$

b) $\tan x + \dfrac{\cos x}{1 + \sin x} = \sec x$

c) $\dfrac{\cos x}{1 - \sin x} = \sec x + \tan x$

d) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

e) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

15. Consider a circle with radius 10 m. Let P be a point 35 m away from the center of the circle. Compute the angle formed between the two tangent lines drawn from P to the circle.

16. Compute the area of a triangle with sides 15 m, 15 m, and 22 m.

17. Suppose that α is an acute angle. (An acute angle is one between 0 and 90°)

a) Find the exact value of $\tan \alpha$ if $\cos \alpha = \dfrac{1}{3}$.

b) Find the exact value of $\cos \alpha$ if $\tan \alpha = 2$.

c) Find the value of $\sin \alpha$ and $\cos \alpha$ in terms of M if we know that $\tan \alpha = M$.

18. Solve each of the following equations.

a) $\log_3 (x - 2) = 1$

b) $\ln (3x - 1) = 2$

c) $\dfrac{\log_3 (x + 1) - 1}{2} = 3$

d) $\dfrac{\log_3 (x - 4)}{2} - 1 = 3$

e) $\dfrac{2}{3} \ln (x - 1) + 6 = 4$

f) $\dfrac{2}{3} (\ln (x - 1) + 6) = 4$

g) $3 - 5 \log_2 (x + 1) = 8$

h) $11 - 8 \ln (3x + 1) = 7$

i) $7 - 2 \log_2 (5x - 1) = 3$

19. Solve each of the following equations.

a) $3^{2x-1} = \dfrac{1}{27}$

b) $2^{\frac{1}{3}x+1} = 32$

c) $3^{2x-1} = 10$

d) $e^{3x-1} = 10$

e) $2^{5x+1} = -1$

f) $3^{2x-10} = 3^{x-2}$

g) $3^{3x-1} = 9^{x-1}$

h) $e^{-\ln x} = 5$

20. Consider circles C_1 and C_2 with the following property. An arc subtended in C_1 by a central angle of 20° has the same length as an arc subtended in C_2 by a central angle of 15°. Find the ratio between the areas of the two circles.

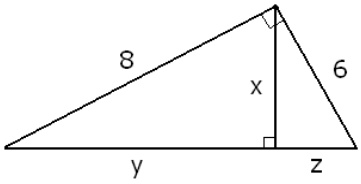
21. a) Solve the equation $9x^2 - 12x = 1$. b) Check your solution using exact values.

22. My friend is 6.4 ft tall! How long is his shadow if he stands 10 feet from a street light that is 24 feet tall?

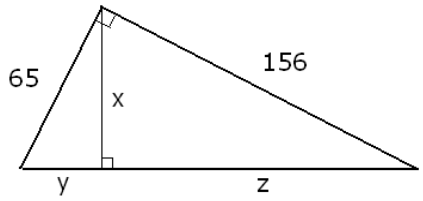
23. Graph each of the following functions. State the basic properties of each of the functions.

a) $f(x) = 2^x$ b) $f(x) = \left(\dfrac{1}{2}\right)^x$ c) $f(x) = \log_2 x$ d) $f(x) = \log_{1/2} x$

24. Find the exact value of x, y, and z, based on the figures shown below.

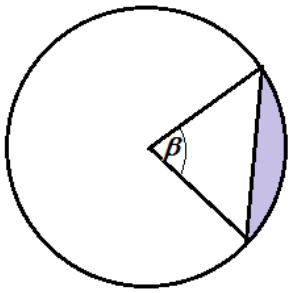


(a)



(b)

25. Compute the area of the shaded region shown on the picture below. The radius of the circle is 3 meters and $\beta = 80°$. Present the exact value and approximate value of the answer.

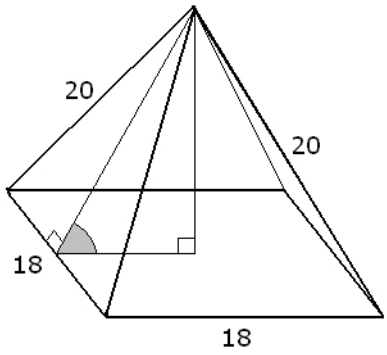


26. A water storage tank has the shape of a cylinder with diameter 10 feet. It is mounted so that the circular cross sections are vertical. If the depth of the water is 7 feet, what percentage of the total capacity is used?

27. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

a) the perimeter of the polygon b) the area of the polygon

28. A straight pyramid has a square base with sides 18 units long. All other edges are 20 units long. Compute the angle formed by the base and a triangular face.



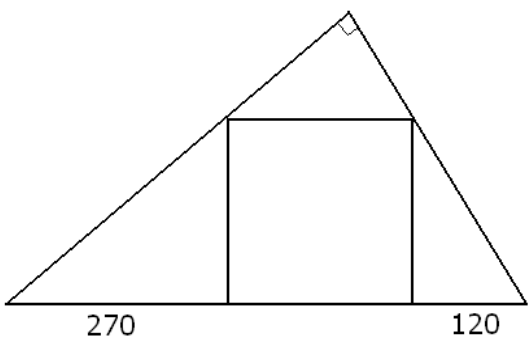
29. Write an equation for the line that passes through the point $(8, -1)$ and forms a $30°$ angle with the positive part of the $x-$axis.

30. Find the smallest possible value of $3a^2 + b^2$ if we know that $3a + b = 12$.

31. The picture shows a square within a right triangle.

a) Find the length of the sides in the square.

b) Compute the angles in the triangle.



32. * Find the equation of the circle that passes through $A\,(1,-3)$, $B\,(3,3)$, and $C\,(7,-5)$.

33. * Solve the equation $\sqrt{x^2+x+3}+x^2+x-7=2$

Answers

1.) a) $r=\dfrac{120}{\pi}\,\text{cm}\approx 38.197\,186\,342\,\text{cm}$ b) $r=\sqrt{\dfrac{240}{\pi}}\,\text{cm}\approx 8.740\,387\ \text{cm}$ 2.) a) $2\sin^{-1}\left(\dfrac{3}{20}\right)\approx 17.253\,853^\circ$

b) $\sqrt{391}$ unit 3.) 933 miles 4.) $1299\dfrac{\text{mi}}{\text{h}}$ 5.) a) $13,5$ b) 3 (24 doesn't work) c) $0,4$

6.) a) $x\,(x+2)\,(x-1)$ b) $x^3\,(x+3)\,(x-3)$ d) $(2x-5)\,(3x+2)$ d) $(x+1)\,(x-1)\,\left(x^2+1\right)$

7.) a) $(-\infty,0)\cup(1,\infty)$ b) no real solution c) $\left[-4-\sqrt{7},-4+\sqrt{7}\right]$ d) $\{-6\}$

8.) a) $-\dfrac{1}{8}$ b) $-\dfrac{1}{2}$ c) undefined d) 1 e) $\dfrac{3}{2}$ f) $-\dfrac{3}{4}$ g) 100 h) 16 i) -4 j) -3

9.) a) $\dfrac{1}{A}+\dfrac{1}{B}$ b) $\dfrac{1}{AB}$ c) A d) P^2 e) $\sqrt{Q}$ f) $\sqrt{T^3}$ g) $4a$ h) $\dfrac{a}{4}$ i) $\dfrac{1}{2}$

j) $\dfrac{1}{2}$ k) 143 l) 123 m) $\dfrac{1}{25}$ 10.) a) 7 b) 50 c) -1 d) -5 e) 17

11.) a) $\dfrac{5}{11}$ b) -15 c) undefined d) 9 e) 1 f) $\dfrac{3}{10}$ g) 2 12.) a) 5 b) $-\dfrac{25}{8}$ c) $-\dfrac{25}{3}$

13.) a) $\dfrac{1}{4}\sqrt{2}+\sqrt{3}$ b) $\sqrt{2}-\dfrac{13}{16}$ c) $-\dfrac{\sqrt{3}}{4}$ d) 1 e) 0 f) $2-\sqrt{3}$

14.) a) $\tan^2 x+1=\sec^2 x$

$$\text{LHS}=\tan^2 x+1=\left(\frac{\sin x}{\cos x}\right)^2+1=\frac{\sin^2 x}{\cos^2 x}+1=\frac{\sin^2 x}{\cos^2 x}+\frac{\cos^2 x}{\cos^2 x}=\frac{\sin^2 x+\cos^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\text{RHS}$$

b) $\tan x+\dfrac{\cos x}{1+\sin x}=\sec x$

$$\begin{aligned}\text{RHS} &= \tan x+\frac{\cos x}{1+\sin x}=\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=\frac{\sin x\,(1+\sin x)+\cos^2 x}{\cos x\,(1+\sin x)}\\ &= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x\,(1+\sin x)}=\frac{\sin x+1}{\cos x\,(1+\sin x)}=\frac{1}{\cos x}=\sec x=\text{LHS}\end{aligned}$$

c) $\dfrac{\cos x}{1-\sin x}=\sec x+\tan x$

$$\begin{aligned}\text{LHS} &= \frac{\cos x}{1-\sin x}=\frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}=\frac{\cos x\,(1+\sin x)}{1-\sin^2 x}=\frac{\cos x\,(1+\sin x)}{\cos^2 x}=\frac{1+\sin x}{\cos x}\\ &= \frac{1}{\cos x}+\frac{\sin x}{\cos x}=\sec x+\tan x=\text{RHS}\end{aligned}$$

d) $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

$$\text{LHS} = \frac{\csc^2 x - 1}{\csc^2 x} = \frac{\dfrac{1}{\sin^2 x} - 1}{\dfrac{1}{\sin^2 x}} = \frac{\dfrac{1}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}} = \frac{\dfrac{1 - \sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}}$$

To divide is to multiply by the reciprocal:

$$\frac{\dfrac{1 - \sin^2 x}{\sin^2 x}}{\dfrac{1}{\sin^2 x}} = \frac{1 - \sin^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{1} = 1 - \sin^2 x = \cos^2 x = \text{RHS}$$

e) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\dfrac{\cos x}{\sin x} - 1}{\dfrac{\cos x}{\sin x} + 1}$$

Multiply numerator and denominatior by $\sin x$

$$\frac{\dfrac{\cos x}{\sin x} - 1}{\dfrac{\cos x}{\sin x} + 1} = \frac{\cos x - \sin x}{\cos x + \sin x}$$

Divide numerator and denominator by $\cos x$

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\dfrac{\cos x - \sin x}{\cos x}}{\dfrac{\cos x + \sin x}{\cos x}} = \frac{\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{\cos x}} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

15.) $2\sin^{-1}\left(\dfrac{10}{35}\right) \approx 33.203\,1°$ 16.) $A = 22\sqrt{26}\,\text{m}^2 \approx 112.178\,43\,\text{m}^2$

17.) a) $2\sqrt{2}$ b) $\dfrac{1}{\sqrt{5}}$ c) $\sin\alpha = \dfrac{M}{\sqrt{M^2+1}}$ $\cos\alpha = \dfrac{1}{\sqrt{M^2+1}}$

18.) a) 5 b) $\dfrac{1}{3}e^2 + \dfrac{1}{3}$ c) 2186 d) 6565 e) $e^{-3} + 1$ f) 2 g) $-\dfrac{1}{2}$ h) $\dfrac{1}{3}\sqrt{e} - \dfrac{1}{3}$ i) 1

19.) a) -1 b) 12 c) $\dfrac{1}{2}(1 + \log_3 10)$ d) $\dfrac{1}{3}(1 + \ln 10)$ e) no solution f) 8 g) -1 h) $\dfrac{1}{5}$

20.) $\dfrac{A_1}{A_2} = \dfrac{9}{16}$ 21.) a) $\dfrac{2 \pm \sqrt{5}}{3}$ b) If $x = \dfrac{2+\sqrt{5}}{3}$, then

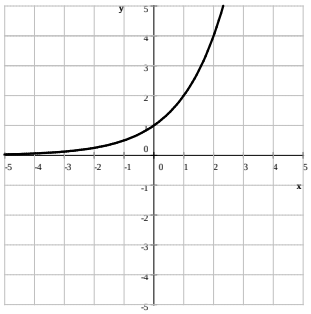
$$\begin{aligned}\text{LHS} &= 9\left(\frac{2+\sqrt{5}}{3}\right)^2 - 12\left(\frac{2+\sqrt{5}}{3}\right) = 9\frac{\left(2+\sqrt{5}\right)^2}{3^2} - 4\left(2+\sqrt{5}\right)\\ &= \left(2+\sqrt{5}\right)^2 - 4\left(2+\sqrt{5}\right) = 4 + 5 + 4\sqrt{5} - 8 - 4\sqrt{5} = 1 = \text{RHS}\end{aligned}$$

and if $x = \dfrac{2-\sqrt{5}}{3}$, then

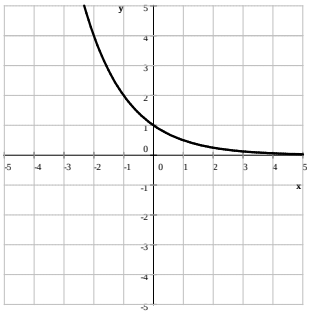
$$\begin{aligned}\text{LHS} &= 9\left(\frac{2-\sqrt{5}}{3}\right)^2 - 12\left(\frac{2-\sqrt{5}}{3}\right) = 9\frac{\left(2-\sqrt{5}\right)^2}{3^2} - 4\left(2-\sqrt{5}\right)\\ &= \left(2-\sqrt{5}\right)^2 - 4\left(2-\sqrt{5}\right) = 4 + 5 - 4\sqrt{5} - 8 + 4\sqrt{5} = 1 = \text{RHS}\end{aligned}$$

22.) $3.\overline{63}\,\text{ft}$

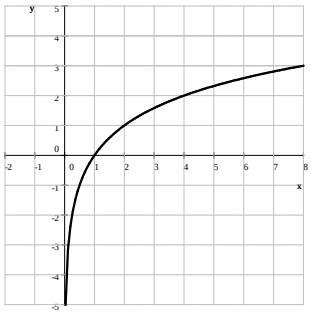
23.) a) $f(x) = 2^x$ b) $f(x) = \left(\frac{1}{2}\right)^x$ c) $f(x) = \log_2 x$ d) $f(x) = \log_{1/2} x$



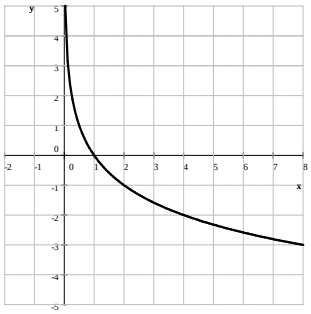
domain: $\mathbb{R}$
range: $(0, \infty)$
increasing
one-to-one



domain: $\mathbb{R}$
range: $(0, \infty)$
decreasing
one-to-one



domain: $(0, \infty)$
range: $\mathbb{R}$
increasing
one-to-one



domain: $(0, \infty)$
range: $\mathbb{R}$
decreasing
one-to-one

24.) a) $x = \frac{24}{5} = 4.8$, $y = \frac{32}{5} = 6.4$, $z = \frac{18}{5} = 3.6$ b) $x = 60$, $y = 25$, $z = 144$

25.) $(2\pi - 9 \sin 40^\circ \cos 40^\circ)\ \text{m}^2 \approx 1.851\,55\ \text{m}^2$ 26.) 74.77%

27.) a) $2nR \sin\left(\frac{180^\circ}{n}\right)$ b) $nR \sin\left(\frac{180^\circ}{n}\right) R \cos\left(\frac{180^\circ}{n}\right) = nR^2 \sin\left(\frac{180^\circ}{n}\right) \cos\left(\frac{180^\circ}{n}\right)$

28.) $\tan^{-1}\left(\frac{\sqrt{238}}{9}\right) \approx 59.741\,424\,7^\circ$ 29.) $\frac{\sqrt{3}}{3}(x - 8) = y + 1$ 30.) 36

31.) a) 180 b) $\alpha = \tan^{-1}\left(\frac{180}{270}\right) \approx 33.6900675^\circ$ $\beta = \tan^{-1}\left(\frac{180}{120}\right) \approx 56.3099325^\circ$

32.) $(x - 5)^2 + (y + 1)^2 = 20$ 33.) Hint: try introducing a new variable!

Last revised: March 2, 2015