Quiz 12 will also cover material from Quizzes 1-11 and Exams 1 and 2. Review those topics even if they do not appear in this review.

1. Graph each of the following rotational angles.

   a) $180°$ c) $210°$ e) $270°$ g) $90°$
   b) $-60°$ d) $135°$ f) $-135°$ h) $150°$

2. Which of the following angles are co-terminal? Group co-terminal angles together.

   $-585°$ $630°$ $270°$ $-225°$ $-180°$ $660°$ $180°$ $-540°$
   $135°$ $300°$ $540°$ $-90°$ $495°$ $-780°$ $-60°$

3. Find the exact value of each of the following.

   a) $\sin 210°$ f) $\tan 315°$ k) $\cos 180°$ p) $\sin 600°$ u) $\tan(-90°)$
   b) $\cos 210°$ g) $\sin(-60°)$ l) $\tan 180°$ q) $\cos 600°$ v) $\sin(-225°)$
   c) $\tan 210°$ h) $\cos(-60°)$ m) $\sin(-150°)$ r) $\tan 600°$ w) $\cos(-225°)$
   d) $\sin 315°$ i) $\tan(-60°)$ n) $\cos(-150°)$ s) $\sin(-90°)$ x) $\tan(-225°)$
   e) $\cos 315°$ j) $\sin 180°$ o) $\tan(-150°)$ t) $\cos(-90°)$

4. Convert each of the following to radians. Use exact values.

   a) $120°$ c) $45°$ e) $900°$ g) $225°$ i) $210°$
   b) $600°$ d) $20°$ f) $-50°$ h) $-200°$ j) $-135°$

5. Convert each of the following to degrees.

   a) $\frac{5\pi}{4}$ c) $-\frac{\pi}{6}$ e) $-\frac{2}{3}\pi$ g) $-\frac{3}{2}\pi$ i) $\frac{11}{4}\pi$
   b) $\frac{7\pi}{10}$ d) $\frac{3}{5}\pi$ f) $\frac{4}{15}\pi$ h) 4π j) 1

6. Simplify each of the following.

   a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\cos\left(\frac{7\pi}{4}\right)$ c) $\tan(-3\pi)$ d) $\tan\left(\frac{\pi}{2}\right)$ e) $\sec\left(-\frac{7\pi}{3}\right)$ f) $\csc\left(\frac{3\pi}{4}\right)$

7. Prove each of the following identities.

   a) $1+\tan^2 x = \sec^2 x$
   b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$
   c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$
   d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$
   e) $\cos x\,(\sec x - \cos x) = \sin^2 x$
   f) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$
   g) $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

8. Use your calculator to decide whether the following statement is true or false.

$$\log_{10} 2 \cdot \log_{10} 5 = \log_{10} 10$$

9. Solve each of the following equations.

   a) $\sqrt{2x+6} - \sqrt{x+1} = 2$

   b) $\sqrt{2x} - \sqrt{x+1} = 1$

   c) $\frac{2}{3}\log_2(5x-1) = 4$

   d) $\frac{2}{3}\log_2(5x-1) = 1$

   e) $4 \cdot 5^{3x-1} = 100$

   f) $4 \cdot 5^{3x-1} = 80$

   g) $2e^{5x-1} = 100$

   h) $\frac{1}{2}\ln(3x-2) = -4$

   i) $\frac{2}{3}\log_2(x^2-4) = 4$

   j*) $\sqrt{x^2-3x-1} + x^2 - 3x = 13$

10. Compute the exact value of all six trigonometric function values of α if α is an acute angle with $\cos\alpha = \frac{3}{7}$

11. Compute the exact value of all six trigonometric function values of α if α is an acute angle with $\tan\alpha = M$

12. Simplify each of the following.

   a) $\log_3(\tan 60°) + \log_2(\sin 45°) - \ln(\tan 45°)$

   b) $\sec 30° - \csc 45° + \cot 60°$

   c) $\dfrac{\tan 120° - \tan 135°}{1 + \tan 120° \tan 135°}$

   d) $\tan^2 120° - \tan^2 150°$

13. Prove that if α is any acute angle, then $\sin\alpha + \cos\alpha > 1$.

14. Simplify each of the following.

   a) $2^{\log_2 4} + 2^{\log_2 8}$

   b) $2^{\log_2 4 + \log_2 8}$

   c) $\log_{25}\left(\frac{1}{\sqrt{125}}\right)$

   d) $\ln\left(e^{-7}\right)$

   e) $e^{-\ln 7}$

   f) $\log_{81} 3 \cdot \log_3 81$

   g) $5^{\log_5 M}$

   h) $25^{\log_5 M}$

   i) $5^{\log_{25} M}$

   j) $\log_{1/4}(8)$

   k) $3^{\log_3 9} + 3^{\log_3 27}$

   l) $3^{\log_3 9 + \log_3 27}$

   m) $\log_8(-64)$

   n) $e^{\ln A} + e^{\ln B}$

   o) $e^{\ln A + \ln B}$

   p) $\log_2(\sin 45°)$

   q) $\log_2\left(\sec^{10} 45°\right)$

   r) $\log_3(\tan 60°)$

   s) $\log_9\left(\frac{1}{\sqrt{27}}\right)$

   t) $\log_{\sqrt{2}} 32$

   u) $\log_A 1 - \log_A \sqrt{A} + \log_B \frac{1}{\sqrt[3]{B^2}}$

   v) $\ln\left(\frac{1}{\sqrt[3]{e}}\right) - 2\log_{10}(0.001) + \log_7 \frac{1}{49}$

   w) $2^{\log_2 x} + 2^{\log_2 y}$

   x) $2^{\log_2 x + \log_2 y}$

15. a) Compute the exact value of $\tan 30° + \tan 30°$

   b) Compute the exact value of $\tan 60°$

   c) Based on your findings, determine whether the following statement is true or false:

$$\tan(\alpha + \beta) = \tan\alpha + \tan\beta$$

16. Suppose that C is a circle with radius 12 feet. Compute the exact and approximate values of each of the following.

   a) The length of an arc subtended by a central angle of 42°.

   b) The area of a sector determined by a central angle of 42°.

17. a) Find the radius if a circle if we know that the length of an arc subtended by a central angle of 85° is 28 cm.

   b) Find the radius if a circle if we know that the area of a sector defined by a central angle of 35° is 75 cm^2.

18. a) Find the equation of the line connecting the points where the circles $(x+2)^2+(y-1)^2=5$ and $(x+3)^2+(y-3)^2=10$ intersect each other.

    b) Find an equation of the tangent line drawn to $(x+3)^2+(y-3)^2=10$ at the point $P(-2,6)$.

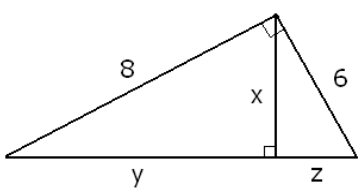
19. a) Find the speed of the sattelite that orbits the Earth above the equator at a height of 1200 miles and appears at the same point in the sky at all times. Round your answer to the nearest miles per hour. (Assume that Earth is a sphere with radius 3960 mi)

    b*) Find the speed of the sattelite that orbits the Earth above Chicago at a height of 1200 miles and appears at the same point in the sky at all times. Assume that Earth is a sphere with radius 3960 mi and that the latitude of Chicago is $42°N$. Round your answer to the nearest miles per hour.

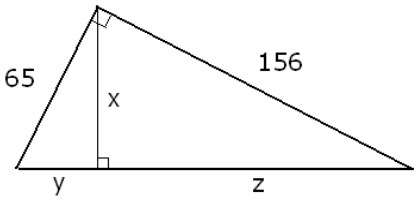
20. How fast is the endpoint of the minute pointer moving on my watch if the pointer is 1.2 centimeters long? Express your answer in centimeters per minute.

21. Berlin, Germany and Copenhagen, Denmark have nearly the same longitude. Use this fact to find the distance between the two cities if the latitude of Berlin is 52.5° N and the latitude of Copenhagen is 55.7° N. Assume that Earth is a sphere with radius 3960 mi. Round your answer to the nearest mile.

22. Find the exact value of x, y, and z, based on the figures shown below.



(a)



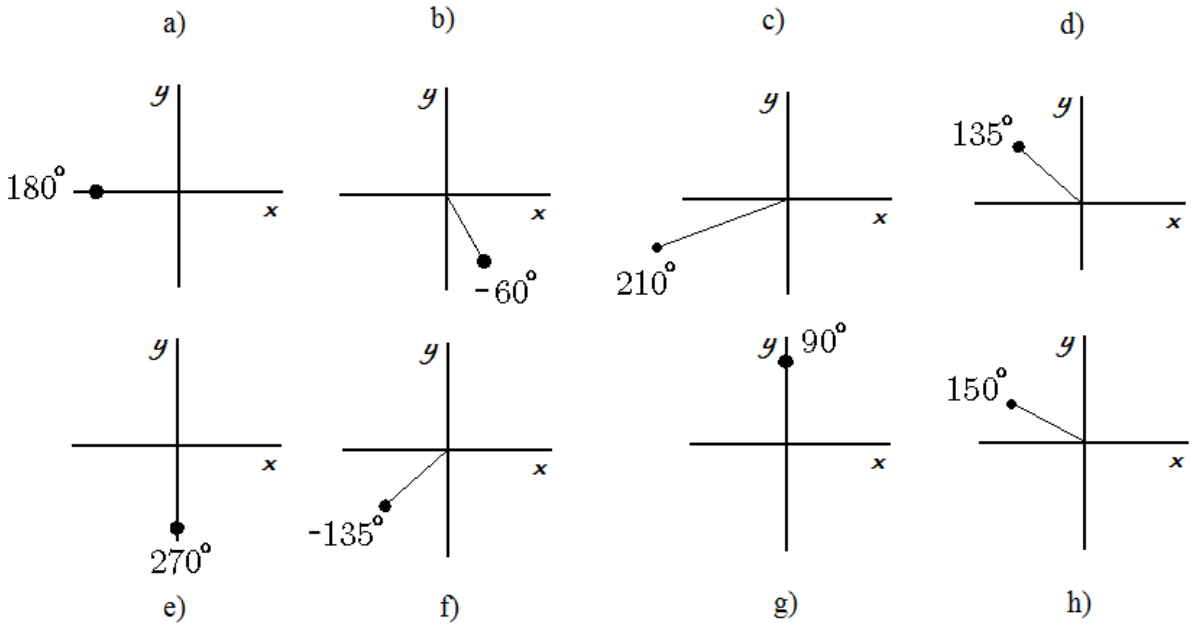
(b)

23. Compute the exact and approximate values for the perimeter and area of an 18−sided regular polygon that was written in a circle with radius 7 centimeters.

24. We are driving away from a tower. First, the angle of elevation from the car to the top of the tower is 52°. After we drove 300 feet away from the tower, the angle of elevation from the car to the top of the tower is 32°. How tall is the tower?

Answers

1.



2. Group 1: 135°, 495°, −225°, −585° Group 2: −60°, 300°, 660°, −780°
   Group 3: 180°, −180°, 540°, −540° Group 4: −90°, 270°, 630°

3. a) $-\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{3}$ d) $-\frac{\sqrt{2}}{2}$ e) $\frac{\sqrt{2}}{2}$ f) -1 g) $-\frac{\sqrt{3}}{2}$ h) $\frac{1}{2}$ i) $-\sqrt{3}$
   j) 0 k) -1 l) 0 m) $-\frac{1}{2}$ n) $-\frac{\sqrt{3}}{2}$ o) $\frac{\sqrt{3}}{3}$ p) $-\frac{\sqrt{3}}{2}$ q) $-\frac{1}{2}$ r) $\sqrt{3}$
   s) -1 t) 0 u) undefined v) $\frac{\sqrt{2}}{2}$ w) $-\frac{\sqrt{2}}{2}$ x) -1

4. a) $\frac{2\pi}{3}$ b) $\frac{10\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{9}$ e) 5π f) $-\frac{5\pi}{18}$ g) $\frac{5\pi}{4}$ h) $-\frac{10\pi}{9}$ i) $\frac{7\pi}{6}$ j) $-\frac{3\pi}{4}$

5. a) 225° b) 126° c) −30° d) 108° e) −120° f) 48° g) −270° h) 720°
   i) 495° j) $\left(\frac{180}{\pi}\right)^\circ \approx 57.295\,78°$

6. a) -1 b) $\frac{\sqrt{2}}{2}$ c) 0 d) undefined e) 2 f) $\sqrt{2}$

7. a) $1+\tan^2 x = \sec^2 x$

$$\text{LHS} = 1+\tan^2 x = 1+\frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\dfrac{1}{\tan x} - 1}{\dfrac{1}{\tan x} + 1} \cdot \frac{\tan x}{\tan x} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right) \cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right) \cos x = \sin^2 x \left(1 - \sin^2 x\right) \cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

d) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = \text{RHS}$$

e) $\cos x \left(\sec x - \cos x\right) = \sin^2 x$

$$\text{LHS} = \cos x \left(\sec x - \cos x\right) = \cos x \left(\frac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x \left(1 + \cos x\right)}{1 - \cos^2 x} = \frac{\sin x \left(1 + \cos x\right)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{\left(1 + \sin^2 x + 2 \sin x\right) - \left(1 + \sin^2 x - 2 \sin x\right)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2 \sin x - 1 - \sin^2 x + 2 \sin x}{\cos^2 x} = \frac{4 \sin x}{\cos^2 x} = 4 \frac{\sin x}{\cos x} \frac{1}{\cos x} = 4 \tan x \sec x = \text{RHS} \end{aligned}$$

8. False. $\log_{10} 2 \approx 0.301\,03$ and $\log_{10} 5 \approx 0.698\,97$ and so $\log_{10} 2 \cdot \log_{10} 5 \approx 0.210\,4$ while the other side is 1.

9. a) $-1, 15$ b) 8 (0 does not work) c) 13 d) $\dfrac{2}{5}\sqrt{2} + \dfrac{1}{5}$ e) 1 f) $\dfrac{1}{3}\left(1 + \log_5 20\right)$ g) $\dfrac{1}{5}\left(1 + \ln 50\right)$ h) $\dfrac{1}{3}e^{-8} + \dfrac{2}{3}$ i) $\pm 2\sqrt{17}$ j) $-2, 5$

10. $\sin \alpha = \dfrac{2\sqrt{10}}{7}$ $\cos \alpha = \dfrac{3}{7}$ $\tan \alpha = \dfrac{2\sqrt{10}}{3}$ $\csc \alpha = \dfrac{7\sqrt{10}}{20}$ $\sec \alpha = \dfrac{7}{3}$ $\cot \alpha = \dfrac{3\sqrt{10}}{20}$

11. $\sin \alpha = \dfrac{M}{\sqrt{1 + M^2}}$ $\cos \alpha = \dfrac{1}{\sqrt{1 + M^2}}$ $\tan \alpha = M$ $\csc \alpha = \dfrac{\sqrt{1 + M^2}}{M}$ $\sec \alpha = \sqrt{1 + M^2}$ $\cot \alpha = \dfrac{1}{M}$

12. a) 0 b) $\sqrt{3} - \sqrt{2}$ c) $2 - \sqrt{3}$ d) $\dfrac{8}{3}$

13. Claim: if α is any acute angle, then $\sin \alpha + \cos \alpha > 1$.

Proof: Let a, b, and c be sides of a right triangle that contains α as an angle. If α is the angle opposite side a, then $\sin \alpha = \dfrac{a}{c}$ and $\cos \alpha = \dfrac{b}{c}$. To prove:

$$\begin{aligned} \sin \alpha + \cos \alpha &> 1 \\ \frac{a}{c} + \frac{b}{c} &> 1 \\ \frac{a + b}{c} &> \frac{c}{c} \end{aligned}$$

Since $c > 0$, we can multiply both sides by c.

$$a + b > c$$

This is true for any triangle by the triangle inequality.

14. a) 12 b) 32 c) $-\frac{3}{4}$ d) -7 e) $\frac{1}{7}$ f) 1 g) M h) M^2 i) $\sqrt{M}$ j) $-\frac{3}{2}$ k) 5
l) 243 m) undefined n) $A + B$ o) AB p) $-\frac{1}{2}$ q) 5 r) $\frac{1}{2}$ s) $-\frac{3}{4}$ t) 10
u) $-\frac{7}{6}$ v) $\frac{11}{3}$ w) $x + y$ x) xy

15. a) $\frac{2}{3}\sqrt{3}$ b) $\sqrt{3}$ c) false

16. a) $\frac{14}{5}\pi$ ft ≈ 8.79646 ft b) $\frac{168}{5}\pi$ ft $\approx 105.557\,513$ ft

17. a) $\frac{1008}{17\pi}$ cm ≈ 18.87390384 cm b) $\frac{30}{7}\sqrt{\frac{42}{\pi}}$ cm ≈ 15.6701417 cm

18. a) $y = \frac{1}{2}x + 2$ b) $-\frac{1}{3}(x + 2) = y - 6$

19. a) $1351\frac{\text{mi}}{\text{h}}$ b*) $1004\frac{\text{mi}}{\text{h}}$

20. $0.04\pi\frac{\text{cm}}{\text{min}} \approx 0.1256637\,\frac{\text{cm}}{\text{min}}$

21. 221 mi

22. a) $x = \frac{24}{5} = 4.8$, $y = \frac{32}{5} = 6.4$, $z = \frac{18}{5} = 3.6$ b) $x = 60$, $y = 25$, $z = 144$

23. $P = 252 \sin 10°$ cm ≈ 43.759341 cm $A = 882 \sin 10° \cos 10°$ cm$^2 \approx 150.830883$ cm^2

24. 366.2785 ft

Last revised: March 9, 2015