Please note that Quiz 13 will cover topics covered on Quizzes 1-12 and Exams 1 and 2. Review those topics even if the do not appear in this document.

1. Convert to radians.

   a) $45°$ b) $-120°$ c) $10°$ d) $135°$ e) $270°$

2. Convert to degrees.

   a) $\frac{\pi}{6}$ b) $-\frac{\pi}{9}$ c) $\frac{5\pi}{12}$ d) -3π e) $\frac{7\pi}{4}$

3. Use the unit circle to find the exact value of each of the following expressions.

   a) $\sin 120°$ c) $\tan \frac{7\pi}{4}$ e) $\cos(-90°)$ g) $\sec\left(-\frac{5\pi}{3}\right)$ i) $\tan(-270°)$

   b) $\cos 120°$ d) $\sin \frac{5\pi}{6}$ f) $\cos \frac{5\pi}{6}$ h) $\sin\left(-\frac{\pi}{2}\right)$ j) $\sec\left(\frac{5\pi}{3}\right)$

4. The following are all equal to each other, except for one. Which one?

   A) $\cos 150°$ B) $\sin(-60°)$ C) $\cos 210°$ D) $\cos(-120°)$ E) $\sin 300°$

5. Simplify each of the following expressions. Use exact values and rationalize denominators.

   a) $\dfrac{\tan 120° - \tan 45°}{1 + \tan 120° \tan 45°}$ b) $\dfrac{\sin(-240°)\cos 135° - \sin 30° \sin 315° \tan 300°}{\sin 150° \sec 225° \tan(-30°)}$ c) $\cot 300° \sec 210° \tan^2 240°$

6. Simplify each of the following, i.e. re-write in terms of a trigonometric function of θ.

   a) $\sin(\theta + 360°)$ c) $\tan(\pi - \theta)$ e) $\sin\left(\frac{\pi}{2} - \theta\right)$ g) $\sin(\theta - \pi)$ i) $\tan(-\theta)$

   b) $\cos(\pi - \theta)$ d) $\sin(180° - \theta)$ f) $\cos(\theta + \pi)$ h) $\tan(\theta + \pi)$ j) $\cos(360° - \theta)$

7. Solve each of the following equations. You may present your answer in degrees.

   a) $\sin x = -\frac{1}{\sqrt{2}}$ c) $\tan x = -\sqrt{3}$ f) $\tan x = 0$ h) $\cos \beta = -1$

   d) $\sin x = -\frac{3}{2}$

   b) $\cos x = -\frac{\sqrt{3}}{2}$ e) $\cos x = -1$ g) $\sin \alpha = -\frac{\sqrt{3}}{2}$ i) $\tan \gamma = \frac{2}{3}$

8. a) Suppose that α is an acute angle. Compute the exact value of $\sin \alpha$ if we know that $\tan \alpha = \frac{4}{7}$.

   b) Suppose that β is an acute angle. Compute the exact value of $\tan \beta$ if we know that $\sin \beta = \frac{4}{7}$.

   c) Suppose that γ is an acute angle. Compute the exact value of $\sin \gamma$ and $\cos \gamma$ if we know that $\tan \gamma = M$.

9. Find all angles coterminal with $120°$. Express your answer

   a) in degrees b) in radians

   c) Find all coterminal angles α such that $-500° < \alpha < 500°$.

10. True or false?

    a) If an angle α is co-terminal with $40°$, then 2α is co-terminal with $80°$.

    b) If twice an angle 2β is co-terminal with $20°$, then β is co-terminal with $10°$.

11. Find an equation for the tangent line drawn to the graph of $6y-14x+x^2+y^2+18=0$ to the point $P(1,-5)$.

12. Find both coordinates of the point where the parabola $y=-\frac{1}{2}x^2+4x-7$ and the line $y=6x-5$ intersect each other.

13. Solve each of the following inequalities.

a) $x^2<4x$ b) $x^2\geq 4$ c) $x^2+8\geq 6x+1$

14. Simplify each of the following.

a) $-16^{-3/4}$ b) $(-8)^{-1/3}$ c) $(-16)^{-1/4}$ d) $(-5)^0$ e) $\log_2\sqrt{8}$ f) $\log_4\left(\frac{1}{\sqrt{8}}\right)$ g) $\log_5\left(5^{100}\right)$ h) $2^{\log_2 16}$ i) $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$ j) $\log_{0.1}1000$

k) $3^{\log_3 9}+3^{\log_3 27}$
l) $3^{\log_3 9+\log_3 27}$
m) $e^{\ln 5}+e^{\ln 8}$
n) $e^{\ln 5+\ln 8}$
o) $\log_{10}(0.001)$
p) $e^{-2\ln 5}$
q) $2^{\log_2 8}+2^{\log_2 16}$
r) $2^{\log_2 8+\log_2 16}$
s) $2^{\log_2 3}+2^{\log_2 7}$
t) $2^{\log_2 3+\log_2 7}$

15. Simplify $\log_2\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}\right)$

16. Solve each of the following equations.

a) $4+\log_2(5x)=7$
b) $2+\log_3\left((x-1)^2\right)=6$
c) $\ln\left((x+1)^2\right)=4$
d) $\frac{2}{3}\log_2\left(\frac{1}{2}x-2\right)-1=3$
e) $\dfrac{2\log_2\left(\frac{1}{2}x-2\right)-1}{3}=3$
f) $2\log_3(8x+1)-2=6$
g) $3\ln(2x-7)=8$
h) $\log_2\left(x^2-1\right)=5$
i) $\ln\left(x^2-1\right)=-5$

17. Solve each of the following equations.

a) $5^x=\frac{1}{25}$
b) $10^{1-5x}=1000$
c) $3^{2x-1}=243$
d) $3^{x^2-3x}=81$
e) $2\cdot 5^{3x-1}=50$
f) $2\cdot 5^{3x-1}=20$
g) $2\cdot 3^x+5\cdot 3^x=63$
h) $2\cdot 3^x+5\cdot 3^x=56$

18. Graph each of the following functions.

a) $f(x)=x^2-3x+1$
b) $f(x)=x^3$
c) $f(x)=|x|$
d) $f(x)=\sqrt{x}$
e) $f(x)=\sqrt[3]{x}$
f) $f(x)=\frac{1}{x}$
g) $f(x)=\frac{1}{x^2}$
h) $f(x)=2^x$

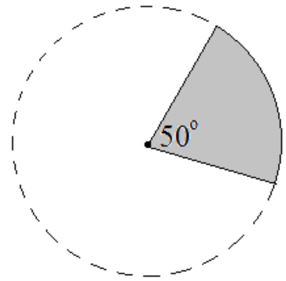
19. Which of the functions from the previous problem are one-to-one?

20. Prove each of the following identities.

a) $\frac{\sin x}{\csc x}+\frac{\cos x}{\sec x}=1$
b) $\tan x-\csc x\sec x\left(1-2\cos^2 x\right)=\cot x$
c) $\frac{1-\cos x}{1+\cos x}=\frac{\sec x-1}{\sec x+1}$
d) $\frac{\sec x+\csc x}{\tan x+\cot x}=\sin x+\cos x$

21. Prove that the area of any triangle can be computed as $A = \frac{1}{2}ab\sin\gamma$. (Recall that γ is the angle between sides a and b.)

22. Find the perimeter and area of the sector shown on the picture. We know that the circle has a radius of 12 cm.



23. A water storage tank has the shape of a cylinder with diameter 10 feet. It is mounted so that the circular cross sections are vertical. If the depth of the water is 7 feet, what percentage of the total capacity is used?

24. Find the smallest value of $a^2 + 6b^2$ given that a is ten less than twice b.

Answers

1. a) $\frac{\pi}{4}$ b) $-\frac{2\pi}{3}$ c) $\frac{\pi}{18}$ d) $\frac{3\pi}{4}$ e) $\frac{3\pi}{2}$

2. a) $30°$ b) $-20°$ c) $75°$ d) $-540°$ e) $315°$

3. a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) -1 d) $\frac{1}{2}$ e) 0 f) $-\frac{\sqrt{3}}{2}$ g) 2 h) -1 i) undefined j) 2

4. D

5. a) $\sqrt{3}+2$ b) -3 c) 2

6. a) $\sin\theta$ b) $-\cos\theta$ c) $-\tan\theta$ d) $\sin\theta$ e) $\cos\theta$ f) $-\cos\theta$ g) $-\sin\theta$ h) $\tan\theta$ i) $-\tan\theta$ j) $\cos\theta$

7. a) $x = -45° + k\cdot 360°$ or $x = -135° + k\cdot 360°$ where $k \in \mathbb{Z}$

   $x = -\frac{\pi}{4} + 2k\pi$ or $x = -\frac{3\pi}{4} + 2k\pi$ where $k \in \mathbb{Z}$

   b) $x = \pm 150° + k\cdot 360°$ where $k \in \mathbb{Z}$ $x = \pm\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

   c) $x = -60° + k\cdot 180°$ where $k \in \mathbb{Z}$ $x = -\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

   d) no solution e) $x = 180° + k\cdot 360°$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$

   f) $x = k\cdot 180°$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$

   g) $\alpha = -60° + k\cdot 360°$ or $\alpha = -120° + k\cdot 360°$ where $k \in \mathbb{Z}$

   h) $\beta = 180° + k\cdot 360°$ where $k \in \mathbb{Z}$ i) $\gamma = 33.690\,07° + k\cdot 180°$ where $k \in \mathbb{Z}$

8. a) $\dfrac{4\sqrt{65}}{65}$ b) $\dfrac{4\sqrt{33}}{33}$ c) $\sin\gamma = \dfrac{M}{\sqrt{M^2+1}}$ $\cos\gamma = \dfrac{1}{\sqrt{M^2+1}}$

9. a) $120° + k \cdot 360°$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$

b) $\dfrac{2\pi}{3} + 2k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$ c) $-240°, 120°, 480°$

10. True or false?

a) If an angle α is co-terminal with $40°$, then 2α is co-terminal with $80°$.

True: $\alpha = 40° + k \cdot 360° \implies 2\alpha = 80° + k \cdot 720°$

b) If twice an angle 2β is co-terminal with $20°$, then β is co-terminal with $10°$.

False: Let $\beta = 190°$. Clearly $10°$ and $190°$ are not co-terminal, but $2\beta = 380°$ which is co-terminal with $20°$.

11. $y = -3x - 2$

12. $(-2, -17)$

13. a) $(0, 4)$ b) $(-\infty, -2] \cup [2, \infty)$ c) $\left(-\infty, 3 - \sqrt{2}\right] \cup \left[3 + \sqrt{2}, \infty\right)$

14. a) $-\dfrac{1}{8}$ b) $-\dfrac{1}{2}$ c) undefined d) 1 e) $\dfrac{3}{2}$ f) $-\dfrac{3}{4}$ g) 100 h) 16 i) -4 j) -3 k) 36

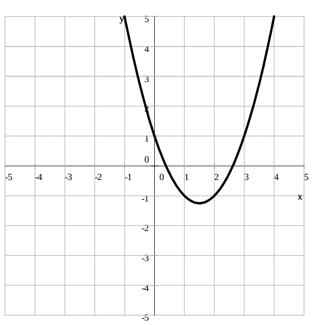
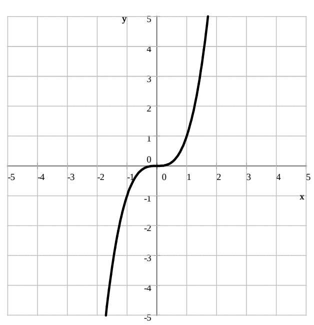
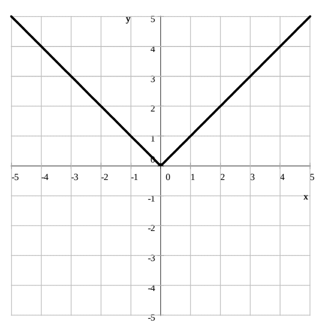
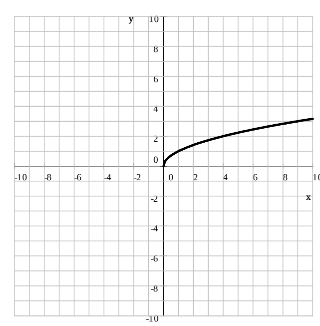
l) 243 m) 13 n) 40 o) -3 p) $\dfrac{1}{25}$ q) 24 r) 128 s) 10 t) 21 15. $\dfrac{1}{32}$

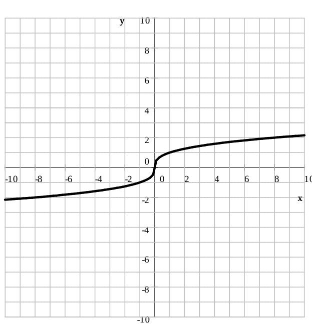
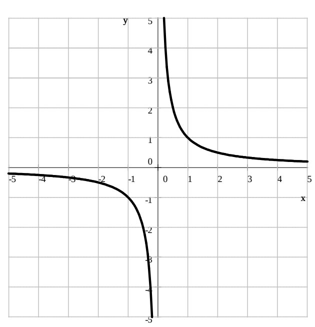
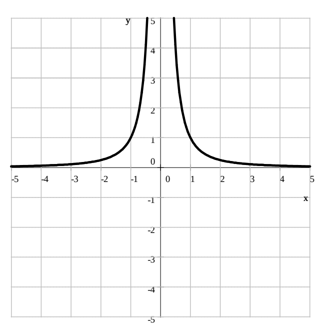
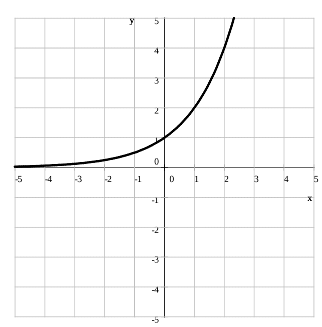
16. a) $\dfrac{8}{5}$ b) $10, -8$ c) $-e^2 - 1, e^2 - 1$ d) 132 e) 68 f) 10 g) $\dfrac{1}{2}e^{8/3} + \dfrac{7}{2}$ h) $\pm\sqrt{33}$

i) $\pm\sqrt{\dfrac{1}{e^5} + 1}$

17. a) -2 b) $-\dfrac{2}{5}$ c) 3 d) $-1, 4$ e) 1 f) $\dfrac{1}{3}\left(1 + \log_5 10\right)$ g) 2 h) $\log_3 8$

18. a) $f(x) = x^2 - 3x + 1$ b) $f(x) = x^3$ c) $f(x) = |x|$ d) $f(x) = \sqrt{x}$

e) $f(x) = \sqrt[3]{x}$ f) $f(x) = \dfrac{1}{x}$ g) $f(x) = \dfrac{1}{x^2}$ h) $f(x) = 2^x$

19. b) $f(x) = x^3$ d) $f(x) = \sqrt{x}$ e) $f(x) = \sqrt[3]{x}$ f) $f(x) = \dfrac{1}{x}$ and h) $f(x) = 2^x$

20. a) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$$\text{LHS} = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\left(\dfrac{1}{\sin x}\right)} + \frac{\cos x}{\left(\dfrac{1}{\cos x}\right)} = \sin^2 x + \cos^2 x = 1 = \text{RHS}$$

b) $\tan x - \csc x \sec x \left(1 - 2\cos^2 x\right) = \cot x$

$$\begin{aligned} \text{LHS} &= \tan x - \csc x \sec x \left(1 - 2\cos^2 x\right) = \frac{\sin x}{\cos x} - \frac{1}{\sin x}\left(\frac{1}{\cos x}\right)\left(1 - 2\cos^2 x\right) \\ &= \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos^2 x}{\sin x \cos x} = \frac{\sin x}{\cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos x}{\sin x} \\ &= \frac{\sin^2 x}{\sin x \cos x} - \frac{1}{\sin x \cos x} + \frac{2\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x - 1 + 2\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x - 1 + \cos^2 x}{\sin x \cos x} \\ &= \frac{1 - 1 + \cos^2 x}{\sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x = \text{RHS} \end{aligned}$$

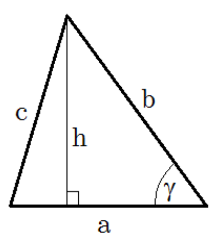
c) $\dfrac{1 - \cos x}{1 + \cos x} = \dfrac{\sec x - 1}{\sec x + 1}$

$$\text{RHS} = \frac{\sec x - 1}{\sec x + 1} = \frac{\dfrac{1}{\cos x} - 1}{\dfrac{1}{\cos x} + 1} = \frac{\dfrac{1}{\cos x} - \dfrac{\cos x}{\cos x}}{\dfrac{1}{\cos x} + \dfrac{\cos x}{\cos x}} = \frac{\dfrac{1 - \cos x}{\cos x}}{\dfrac{1 + \cos x}{\cos x}} = \frac{1 - \cos x}{\cos x} \cdot \frac{\cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} = \text{LHS}$$

d) $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

$$\begin{aligned} \text{LHS} &= \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} = \frac{\dfrac{\sin x}{\cos x \sin x} + \dfrac{\cos x}{\sin x \cos x}}{\dfrac{\sin^2 x}{\cos x \sin x} + \dfrac{\cos^2 x}{\sin x \cos x}} = \frac{\dfrac{\sin x + \cos x}{\cos x \sin x}}{\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}} = \frac{\dfrac{\sin x + \cos x}{\cos x \sin x}}{\dfrac{1}{\cos x \sin x}} \\ &= \frac{\sin x + \cos x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{1} = \sin x + \cos x = \text{RHS} \end{aligned}$$

21. The area is $A = \dfrac{1}{2}ah$ and h, the height belonging to side a is $h = b \sin \gamma$.



22. $P = \left(24 + \dfrac{10}{3}\pi\right)$ cm $\approx 34.471\,975\,512$ cm
$A = 20\pi$ cm$^2 \approx 62.831\,853\,0718$ cm^2

23. 74.77%

24. 60