Please note that Quiz 15 may cover topics from Quizzes 1-14 and Exams 1 and 2, even if they do not appear here.

1. Express each of the following in terms of x if $x = 2^{2015}$.
   a) $2^{2015} + 2^{2016} + 2^{2017}$ b) $2^{2015} + 4^{2015}$

2. We put \$2000 in a bank account with an annual compound interest rate of 7%. If we do not withdraw or add money to this account, how long do we need to wait until there is \$4000 in the account?

3. Compute the exact value of each of the following. Rationalize denominators and simplify your answer.
   a) $\cos 15°$ b) $\sin 15°$ c) $\tan 75°$

4. Prove each of the following co-function identities. For parts a) and b) , use an appropriate compund angle formula. Why is that not an option for part c)?
   a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ c) $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$

5. Suppose that α and β are acute angles with $\sin\alpha = \frac{1}{3}$ and $\cos\beta = \frac{5}{13}$. Compute the exact value of each of the following.

   a) $\cos\alpha$ d) $\tan\beta$ g) $\sin(\alpha+\beta)$ j) $\cos(\alpha-\beta)$
   b) $\tan\alpha$ e) $\sin 2\alpha$ h) $\sin(\alpha-\beta)$ k) $\tan 2\alpha$
   c) $\sin\beta$ f) $\cos 2\beta$ i) $\cos(\alpha+\beta)$ l) $\tan 2\beta$

6. Suppose that $\sin A = -\frac{3}{5}$. Compute the exact value of each of the following.
   a) $\cos A$ b) $\tan A$ c) $\sin 2A$ d) $\cos 2A$

7. Suppose that $\sin B = \frac{1}{3}$ and B is not in the first quadrant. Compute the exact value of each of the following.
   a) $\sec B$ b) $\tan 2B$ c) $\cos 2B$ d) $\tan\left(B - \frac{\pi}{4}\right)$

8. The expression $\dfrac{2\sin x}{\cos x - \sin x \tan x}$ is equivalent to which of the following?
   A) $\tan 2x$ B) $\cot 2x$ C) $\tan x$ D) $\cot x$ E) $\sec x$

9. Compute the exact value of $\dfrac{\tan\left(\frac{4\pi}{3}\right) - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{4\pi}{3}\right)\tan\left(\frac{\pi}{12}\right)}$. (Hint: there is an easy way and also a difficult way to do this.)

10. Graph each of the following functions.

    a) $f(x) = \log_2 x$ d) $f(x) = \frac{1}{x}$ g) $f(x) = -|x|$
    b) $f(x) = 2^x$ e) $f(x) = \sqrt[3]{x}$ h) $f(x) = \left(\frac{1}{2}\right)^x$
    c) $f(x) = x^2 + 6x + 5$ f) $f(x) = \sqrt{4 - x^2}$

11. Find the value of $\tan\alpha$ if we know that $\tan\beta = \frac{2}{3}$ and $\tan(\alpha+\beta) = \frac{11}{10}$.

12. Prove that $\log_{3/4}\left(\frac{15}{8}\right) = \dfrac{\ln 3 + \ln 5 - 3\ln 2}{\ln 3 - 2\ln 2}$.

13. Find the exact value of each of the following expressions.

a) $\sin\left(-\frac{5\pi}{2}\right) - \cos\left(\frac{7\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right) - \cos(7\pi)$

b) $\log_{10}(0.01) + \log_8\left(\frac{1}{2}\right) - \ln\left(\frac{1}{e^5}\right)$

c) $\cos 420° - \tan 210° + \sec 240° + \cot 135°$

d) $\cos 100° \cos 40° + \sin 100° \sin 40°$

14. Simplify each of the following.

a) $\ln\left(e^{-5}\right)$
b) $e^{-\ln 3}$
c) $3^{\log_9 A}$
d) $\log_m\left(m^4\right)$
e) $\log_{27} 9$
f) $\ln\left(-e^3\right)$
g) $\ln\left(e^{-3}\right)$
h) $9^{\log_3 7}$
i) $e^{3\ln 2}$
j) $\ln 1$
k) $25^{\log_5 10}$
l) $5^{\log_{25} 10}$
m) $2^{\log_8 x}$

15. Simplify each of the following expressions.

a) $\log_5\left(5^b\right)$
b) $\log_5\left(125^m\right)$
c) $\log_2\left[\left(\frac{1}{8}\right)^p\right]$
d) $\log_{16}\left(2^x\right)$
e) $3^{\log_3 4}$
f) $9^{\log_3 5}$
g) $3^{\log_9 2}$
h) $2^{\log_8 10}$

16. Simplify each of the following.

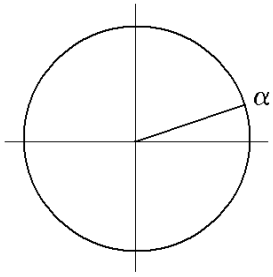
a) $\log_{10} 2 + \log_{10} 5$
b) $\log_6 180 - \log_6 5$
c) $\log_2 24 - \log_2 3$
d) $\log_{10} 40 - 2\log_{10} 2$
e) $\log_3 18 + \log_3 24 - 4\log_3 2$
f) $\log_5 0.4 + \log_5 2.5$

17. Write each of the following as a single logarithm. Assume that all variables represent positive numbers.

a) $\log_{10} a + 2\log_{10} b$
b) $\frac{1}{3}\log_2 a - 3\log_2 b$
c) $1 + \log_3 2$
d) $2 + \log_5 3$
e) $\frac{1}{2} - \log_7 x$
f) $2 + \ln 3 + \ln x - 2\ln y$

18. The picture shows an angle α on the unit circle. Draw each of the following angles in the same circle.

a) $-\alpha$ b) $\alpha - 180°$ c) $\alpha + 90°$



19. Suppose that x is an acute angle.

a) Find the exact value of $\cos x$ if $\sin x = \frac{2}{5}$.

b) Find the exact value of $\sin x$ if $\cot x = 2$.

20. Simplify each of the following. (i.e. write it in terms of trigonometric functions of α.)

a) $\sin(90° - \alpha)$
b) $\sin(180° - \alpha)$
c) $\sin(-\alpha)$
d) $\sin(\alpha + 180°)$
e) $\cos(90° - \alpha)$
f) $\cos(180° - \alpha)$
g) $\cos(-\alpha)$
h) $\cos(\alpha + 180°)$
i) $\tan(90° - \alpha)$
j) $\tan(180° - \alpha)$
k) $\tan(-\alpha)$
l) $\tan(\alpha + 180°)$

21. Find all angles β so that twice β is coterminal with $120°$. Express your answer
    a) in degrees b) in radians
    c) Find all coterminal angles β such that $-500° < \beta < 500°$.

22. Solve each of the following equations.
    a) $\tan x = -\dfrac{1}{\sqrt{3}}$
    b) $\sin^2 x = \dfrac{1}{2}$
    c) $\sin x = 2\sin x \cos x$
    d) $\sin x + 2\cos^2 x = 1$
    e) $\cos x - 2\sin^2 x = 1$
    f*) $\tan x = \tan 2x$

23. Solve each of the following equations for x. Present the exact value of each solution. (Hint: re-write the exponential statements as logarithmic statements!)
    a) $2^{3x-1} = 4$ b) $3^{2x-1} = 5$ c) $e^{t-2} = 5$ d) $2 + 5e^{x-1} = 2012$

24. Solve each of the following equations for x. Present the exact value of each solution. (Hint: re-write the logarithmic statements as exponential statements!)
    a) $\log_5 x = 3$
    b) $\log_x 16 = 2$
    c) $\dfrac{1}{2}\log_2(3x+1) = 3$
    d) $\ln(x-1) = -2$
    e) $\log_3(2x^2 - 5) = 3$
    f) $\log_{10}(x-2) = -1$
    g) $\dfrac{2}{3}\ln(x-1) - 1 = 5$
    h) $\dfrac{\log_2(5x+1) - 1}{3} = -1$

25. Solve each of the following equations.
    a) $\log_6(x-3) + \log_6 3 + \log_6(x+1) = 2$
    b) $\log_2(x-5) - \log_2(2x-14) = -2$
    c) $\log_2(1-x) + \log_2(9-x) = 7$

26. Compute the exact value of $\dfrac{1-\tan 15°}{1+\tan 15°}$. (Hint: there is a really smart way to do this that involves very little computation.)

27. Compare the domains of $f(x) = \log_3(x^2-4)$ and $g(x) = \log_3(x+2) + \log_3(x-2)$.

28. Graph each of the following pairs of functions together, in the same coordinate system.
    a) $f(x) = x^2$ and $g(x) = (x+3)^2$
    b) $f(x) = x^3$ and $g(x) = (x+3)^3$
    c) $f(x) = 2^x$ and $g(x) = \left(\dfrac{1}{2}\right)^x$
    d) $f(x) = 2^x$ and $g(x) = \log_2 x$

29. a) Samantha got a 4% raise. Now her monthly salary is \$2496. How much was her salary before the raise?
    b) A TV set went on a 15% off sale. The sale price is \$1020. What was the original price of the TV set?

30*. Which is greater: $2^{\log_{10} 3}$ or $3^{\log_{10} 2}$?

Answers

1. a) $7x$ b) $x + x^2$

2. It will happen sometime during the 11th year. $\log_{1.07} 2 = \dfrac{\ln 2}{\ln 1.07} \approx 10.244\,77$

3. a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ c) $\sqrt{3}+2$

4. a) $\sin\left(\dfrac{\pi}{2}-\alpha\right) = \cos\alpha$

$$\sin\left(\frac{\pi}{2}-\alpha\right) = \sin\frac{\pi}{2}\cos\alpha - \cos\frac{\pi}{2}\sin\alpha = 1\cdot\cos\alpha - 0\cdot\sin\alpha = \cos\alpha$$

b) $\cos\left(\dfrac{\pi}{2}-\alpha\right) = \sin\alpha$

$$\cos\left(\frac{\pi}{2}-\alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0\cdot\cos\alpha + 1\cdot\sin\alpha = \sin\alpha$$

c) $\tan\left(\dfrac{\pi}{2}-\alpha\right) = \cot\alpha$ We can not use the difference formula for tangent here, because $\tan\dfrac{\pi}{2}$ is undefined. Instead, we need to use a compound angle formula separately for sine and cosine (see parts a and b).

$$\tan\left(\frac{\pi}{2}-\alpha\right) = \frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)} = \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

5. a) $\dfrac{2\sqrt{2}}{3}$ b) $\dfrac{\sqrt{2}}{4}$ c) $\dfrac{12}{13}$ d) $\dfrac{12}{5}$ e) $\dfrac{4\sqrt{2}}{9}$ f) $-\dfrac{119}{169}$ g) $\dfrac{5+24\sqrt{2}}{39}$ h) $\dfrac{5+24\sqrt{2}}{39}$

   i) $\dfrac{10\sqrt{2}-12}{39}$ j) $\dfrac{10\sqrt{2}+12}{39}$ k) $\dfrac{4\sqrt{2}}{7}$ l) $-\dfrac{120}{119}$

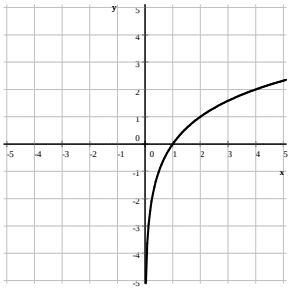
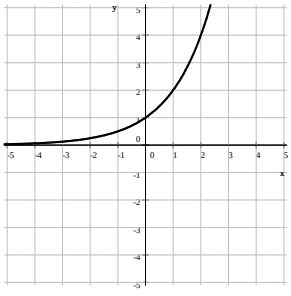
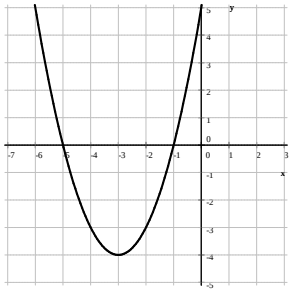
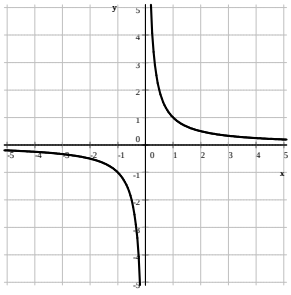
6. a) $\pm\dfrac{4}{5}$ b) $\pm\dfrac{3}{4}$ c) $\pm\dfrac{24}{25}$ d) $\dfrac{7}{25}$

7. a) $-\dfrac{3\sqrt{2}}{4}$ b) $-\dfrac{4\sqrt{2}}{7}$ c) $\dfrac{8}{9}$ d) $-\dfrac{1}{2}$

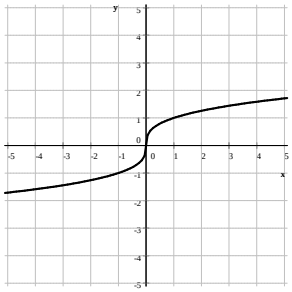
8. A

9. 1

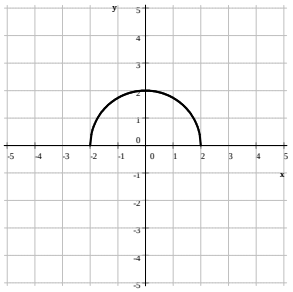
10. a) $f(x) = \log_2 x$ b) $f(x) = 2^x$ c) $f(x) = x^2 + 6x + 5$ d) $f(x) = \dfrac{1}{x}$

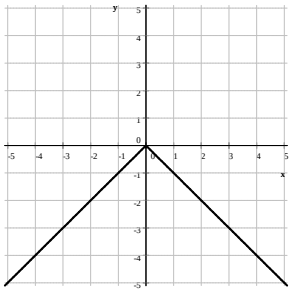
e) $f(x) = \sqrt[3]{x}$



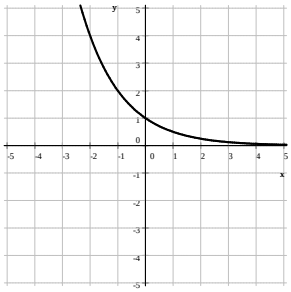
f) $f(x) = \sqrt{4 - x^2}$



g) $f(x) = -|x|$



h) $f(x) = \left(\frac{1}{2}\right)^x$



11. $\frac{1}{4}$

12. $\log_{3/4}\left(\frac{15}{8}\right) = \frac{\ln\left(\frac{15}{8}\right)}{\ln\left(\frac{3}{4}\right)} = \frac{\ln 15 - \ln 8}{\ln 3 - \ln 4} = \frac{(\ln 3 + \ln 5) - \ln\left(2^3\right)}{\ln 3 - \ln\left(2^2\right)} = \frac{\ln 3 + \ln 5 - 3\ln 2}{\ln 3 - 2\ln 2}$

13. a) $-\frac{3}{2}$ b) $\frac{8}{3}$ c) $-\frac{\sqrt{3}}{3} - \frac{5}{2}$ d) $\frac{1}{2}$

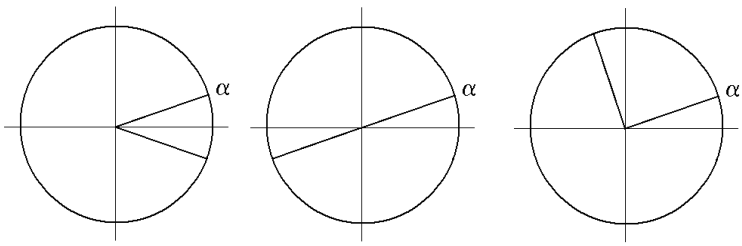
14. a) -5 b) $\frac{1}{3}$ c) $\sqrt{A}$ d) 4 e) $\frac{2}{3}$ f) undefined g) -3 h) 49 i) 8 j) 0 k) 100
l) $\sqrt{10}$ m) $\sqrt[3]{x}$

15. a) b b) $3m$ c) $-3p$ d) $\frac{1}{4}x$ e) 4 f) 25 g) $\sqrt{2}$ h) $\sqrt[3]{10}$

16. a) 1 b) 2 c) 3 d) 1 e) 3 f) 0

17. a) $\log_{10} ab^2$ b) $\log_2\left(\frac{\sqrt[3]{a}}{b^3}\right)$ c) $\log_3 6$ d) $\log_5 75$ e) $\log_7\left(\frac{\sqrt{7}}{x}\right)$ f) $\ln\left(\frac{3e^2x}{y^2}\right)$

18. a) b) c)



19. a) $\frac{\sqrt{21}}{5}$ b) $\frac{\sqrt{5}}{5}$

20. a) $\cos\alpha$ b) $\sin\alpha$ c) $-\sin\alpha$ d) $-\sin\alpha$ e) $\sin\alpha$ f) $-\cos\alpha$ g) $\cos\alpha$ h) $-\cos\alpha$ i) $\cot\alpha$
j) $-\tan\alpha$ k) $-\tan\alpha$ l) $\tan\alpha$

21. a) $60° + k \cdot 180°$ where $k = 0, 1, -1, 2, -2, 3, -3, \ldots..$ b) $\frac{\pi}{3} + k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, \ldots.$
c) $-480°, -300°, -120°, 60°, 240°, 420°$

22. a) $-\frac{1}{6}\pi + k\pi$ where $k \in \mathbb{Z}$ b) $\frac{\pi}{4} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$ c) $k\pi$ or $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

d) $x = \frac{\pi}{2} + 2k\pi$ or $x = -\frac{\pi}{6} + 2k\pi$ or $x = -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

e) $x = 2k\pi$ where $k \in \mathbb{Z}$ f) $x = k\pi$ where $k \in \mathbb{Z}$

23. a) 1 b) $\frac{1}{2}(1 + \log_3 5)$ or $\log_9 15$ c) $2 + \ln 5$ d) $1 + \ln 402$

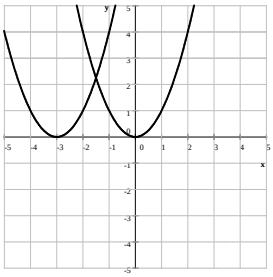
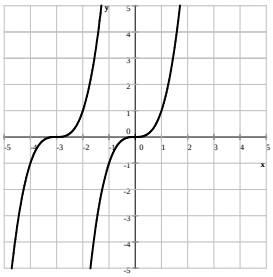
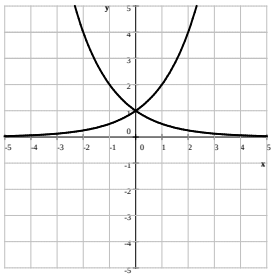
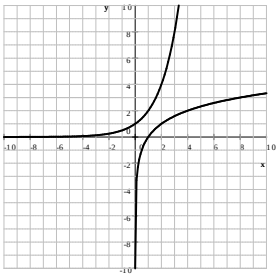
24. a) 125 b) 4 c) 21 d) $\frac{1}{e^2} + 1$ e) $4, -4$ f) 2.1 g) $e^9 + 1$ h) $-\frac{3}{20}$

25. a) 5 (-3 does not work) b) no solution (3 does not work) c) -7 (16 does not work)

26. $\frac{\sqrt{3}}{3}$

27. The domain of f is $(-\infty, -2) \cup (2, \infty)$ and the domain of g is $(2, \infty)$

28. a) b) c) d)

29. a) \$2400 b) \$1200

30. not tellin'