Please note that Quiz 7 may also cover topics from Quiz Reviews for Quizzes 1-6 or from the Exam 1 Review.

1. Rationalize the denominator in $\dfrac{\sqrt{x}-\sqrt{2}}{\sqrt{x}+\sqrt{2}}$.

2. Consider the circle $2x-6y+x^2+y^2=10$. Find an equation for the tangent line that can be drawn to this circle at the point $P(-3,7)$.

3. Simplify each of the following.

   a) $2^{-3}+5^{-2}$

   b) $\left(x^{-1}y^{-1}\right)^{-1}$

   c) $\left(x^{-1}+y^{-1}\right)^{-1}$

   d) $\left(\dfrac{2a^3b^{-2}\left(-ab^{-2}\right)^{-3}ba^0}{b^{-1}\left(-2b^2a^{-2}\right)^3 b}\right)$

   e) $\dfrac{2a^{-2}b^3}{a^5b^{-1}}$

   f) $\dfrac{2a^{-2}+b^3}{a^5-b^{-1}}$

4. a) Solve $3x^2+x=3x+2$.
   b) Check your solutions using exact values.

5. The budget has increased. The new budget is \$1377 152. How much was the budget before the increase if this new number represents an increase of

   a) 6% b) 16% c) 60% d) 600%?

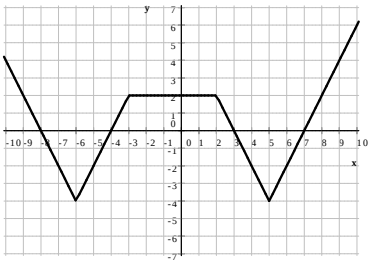
6. The value of a stock first increased by 10%. Then it lost 4% of its value. Express the two cahnges as a single change in the percentage. (Hint: it is NOT an increase of 6%.)

7. Find an equation for the line that

   a) is parallel to $5x+3y=-15$ and passes through the point $(-6,2)$.
   b) is perpendicular to $5x+3y=-15$ and passes through the point $(-6,2)$.
   c) passes through the points $(3,8)$ and $(-1,2)$.

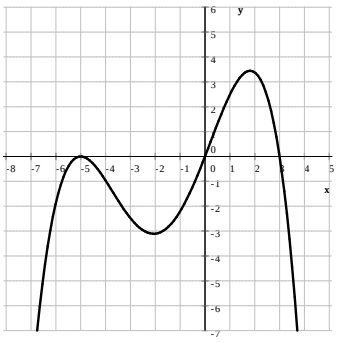
8. Based on the picture given, solve each of the following inequalities for x. Assume that $-10\le x\le 10$.

   a) $y\le 0$
   b) $y>2$
   c) $y\ge 2$
   d) $y\le 2$
   e) $y<-2$



9. Based on the picture given, solve each of the following inequalities for x. Assume that $-7\le x\le 4$.

   a) $y>0$
   b) $y<0$
   c) $y\ge 0$
   d) $y\le 0$



10. Solve each of the following inequalities.

    a) $6x + x^2 \leq 16$
    b) $6x + x^2 \leq 15$
    c) $x^2 > 4x$
    d) $x^2 > 4x + 1$
    e) $16x^2 \leq 8x - 1$

11. A company finds that if it prices its product at \$30, then it can sell 1000 items. For every dollar increase in the price, the company will sell 5 less items.
    a) What is the maximum revenue possible, and what price guarantees that maximal revenue?
    b) What price range will guarantee a revenue greater than \$56 000?

12. Solve each of the following systems over the real numbers.

    a) $\begin{cases} x + y = 8 \\ 2xy = 30 \end{cases}$
    b) $\begin{cases} x + y = -1 \\ \frac{1}{x} + \frac{1}{y} = \frac{1}{6} \end{cases}$
    c) $\begin{cases} x^2 + y^2 = 10 \\ xy = 3 \end{cases}$
    d) $\begin{cases} x^2 + 4x + 4 = -2 \\ x + y = 3 \end{cases}$

13. Suppose that m and n are real number such that m is 10 less than three times n. Find each of the following.
    a) the smallest value of $n^2 + m^2$
    b) the smallest value of nm
    c) the greatest value of $n^2 - m^2$

14. Suppose that m and n are real number such that m is 10 less than three times n. For what values of n will the value of $m^2 + n^2$ be greater than 260?

15. Find where the graphs of the given equations intersect each other.

    a) $\begin{cases} (x-1)^2 + (y+3)^2 = 10 \\ 3y = -x - 8 \end{cases}$
    b) $\begin{cases} (x+8)^2 + (y-2)^2 = 50 \\ y + 7x = -4 \end{cases}$
    c) $\begin{cases} (x-4)^2 + (y+6)^2 = 20 \\ x + 2y = 7 \end{cases}$
    d) $\begin{cases} (x+8)^2 + (y-2)^2 = 50 \\ 7y = x + 22 \end{cases}$

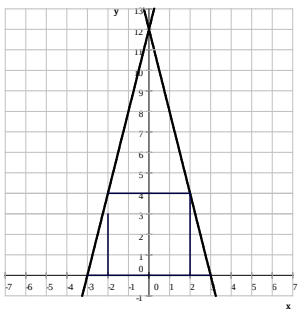
16. An arch is in the shape of a semicircle. At a point along the base 4 foot from an end of the arch, the height of the arch is 10 feet. Find the maximum height of the arch.

17. How many milliliters of an 8% and a 40% solutions need to be mixed in order to produce 640 milliliters of a 20% solution?

18. The hypotenuse of a right triangle is 50 feet long. Find the other two sides, given that the perimeter of the triangle is 112 feet?

19. Prove that there are no real values for x and y that would satisfy the equation $x^2 - 4x + 5 = -6y - y^2 - 11$.

20. Let l_1 and l_2 denote the lines $y = 4x + 12$ and $y = -4x + 12$, respectively. Let R be a rectangle with vertical and horizontal sides, where one horizontal side is on the $x-$axis and the vertices connecting the other horizontal side lie on the lines l_1 and l_2, above the $x-$axis. What is the maximal value of the area of such a rectangle?



Answers

1. $\dfrac{x+2-2\sqrt{2x}}{x-2}$ 2. $\dfrac{1}{2}(x+3)=y-7$

3. a) $\dfrac{33}{200}$ b) xy c) $\dfrac{xy}{x+y}$ d) $\dfrac{a^6}{4b}$ e) $\dfrac{2b^4}{a^7}$ f) $\dfrac{2b+a^2b^4}{a^7b-a^2}$ or $\dfrac{b\left(2+a^2b^3\right)}{a^2\left(a^5b-1\right)}$

4. a) $\dfrac{1\pm\sqrt{7}}{3}$ b) If $x=\dfrac{1-\sqrt{7}}{3}$, then

$$\text{LHS} = 3\left(\frac{1-\sqrt{7}}{3}\right)^2+\frac{1-\sqrt{7}}{3}=3\cdot\frac{8-2\sqrt{7}}{9}+\frac{1-\sqrt{7}}{3}=\frac{8-2\sqrt{7}}{3}+\frac{1-\sqrt{7}}{3}$$

$$= \frac{9-3\sqrt{7}}{3}=\frac{3\left(3-\sqrt{7}\right)}{3}=3-\sqrt{7}$$

$$\text{RHS}=3\left(\frac{1-\sqrt{7}}{3}\right)+2=1-\sqrt{7}+2=3-\sqrt{7}$$

Checking the other solution goes similarly.

5. a) \$1299 200 b) \$1187 200 c) \$860 720 d) \$196 736

6. 5.6% increase

7. a) $y=-\dfrac{5}{3}x-8$ b) $y=\dfrac{3}{5}x+\dfrac{28}{5}$ c) $y=\dfrac{3}{2}x+\dfrac{7}{2}$

8. a) $[-8,-4]\cup[3,7]$ b) $[-10,-9)\cup(8,10]$ c) $[-10,-9]\cup[-3,2]\cup[8,10]$ d) $[-9,8]$ e) $(-7,-5)\cup(4,6)$

9. a) $(0,3)$ b) $[-7,-5)\cup(-5,0)\cup(3,4]$ c) $[0,3]\cup\{-5\}$ d) $[-7,0]\cup[3,4]$

10. a) $[-8,2]$ b) $\left[-3-2\sqrt{6},-3+2\sqrt{6}\right]$ c) $(-\infty,0)\cup(4,\infty)$ d) $\left(-\infty,2-\sqrt{5}\right)\cup\left(2+\sqrt{5},\infty\right)$ e) $\left\{\dfrac{1}{4}\right\}$

11. a) a price of \$115 for a revenue of \$66 125 b) a price between \$100 and \$130

12. a) $(3,5)$ and $(5,3)$ b) $(2,-3)$ and $(-3,2)$ c) $(-1,-3)$, $(3,1)$, $(1,3)$, $(-3,-1)$ d) no real solution

13. a) 10 b) $-\dfrac{25}{3}$ c) $\dfrac{25}{2}$

14. interval notation: $(-\infty,-2)\cup(8,\infty)$ inequality notation: $n<-2$ or $n>8$

15. a) $(-2,-2)$ and $(4,-4)$ b) $(-1,3)$ c) no intersection point d) $(-1,3)$ and $(-15,1)$

16. 14. 5 feet

17. 400 milliliters of the 8% solution and 240 milliliters of the 20% solution

18. 14 and 48 feet

19. We complete the square on both sides. $(x-2)^2+1=-(y+3)^2-2$

Then we see that the value of the right-hand side is 1 or greater for all values of x. Meanwhile, the value of the other side is -2 or less. So the two sides can never be equal.

20. 18

Last revised: February 14, 2015