1. Simplify each of the following.

a) $\left(-2a^3b^2\right)^{-2}\left(-2a^5b^6\right)$

b) $\dfrac{a^3b^{-2}}{a^{-1}b^{-1}}$

c) $\dfrac{a^3-b^{-2}}{a^{-1}+b^{-1}}$

d) $2^{-3}\left(1-3^{-2}\right)$

e) $2^{-3}-\left(1-3^{-2}\right)$

f) $\left(\left(2^{-2}\right)^{-2}\right)^{-2}$

g) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{-3}$

h) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{0}$

i) -2^2

j) $(-2)^{-2}$

k) $32^{-2/5}$

l) $-32^{-2/5}$

m) $(-32)^{-2/5}$

n) $\left(\dfrac{a^2b^0}{a^{-2}b^{12}}\right)^{1/2}$

o) $\dfrac{a^{1/2}b^{2/3}}{a^{-1/2}b^{-1/3}}$

2. Let k denote 2^{1000}. Re-write each of the following in terms of k.

a) 2^{1001} b) 2^{999} c) 2^{1002} d) 2^{3000} e) 4^{1000} f) 2^{500} g) $2^{1000}-2^{1001}+2^{1002}$

3. Let $f(x)=\dfrac{x-2}{x+5}$ and $g(x)=\dfrac{5x+2}{-x+1}$. Compute each of the following.

a) $f(0)$

b) $g(0)$

c) $g(1)$

d) $g\left(\sqrt{2}\right)$

e) $f\left(\sqrt{20}\right)$

f) $f(g(0))$

g) $g(f(3))$

h) $f(1+3)$

i) $f(1)+f(3)$

j) $2f(5)$

k) $f(2\cdot 5)$

4. Simplify each of the following. Assume that x is positive.

a) $(-16)^{-3/4}$

b) $-16^{-3/4}$

c) $\left(x^{2/3}\right)^{3/4}\left(\dfrac{1}{\sqrt{x}}\right)$

d) $\dfrac{2}{\left(\sqrt{5}-1\right)^2}$

e) $\left(1+\sqrt{3}\right)^3\left(1-\sqrt{3}\right)^3$

f) $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$

g) $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}$

5. Simplify each of the following. Assume that a is positive.

a) $\log_4\left(\dfrac{1}{2}\right)$

b) $\log_4 8$

c) $\log_{10}1000$

d) $\log_{100}1000$

e) $\log_2\left(\dfrac{1}{8}\right)$

f) $\log_8 2$

g) $\log_{\sqrt{2}}8$

h) $\log_4 8$

i) $\log_4(-8)$

j) $\log_8\left(\dfrac{1}{4}\right)$

k) $\log_{(1/5)}125$

l) $\log_{0.1}100\,000$

m) $\log_5 1$

n) $\log_1 5$

o) $\log_a\left(a^{17}\right)$

p) $\log_{(-5)}25$

q) $\log_9\left(\dfrac{1}{\sqrt{27}}\right)$

r) $\log_a\left(\sqrt[7]{a^2}\right)$

s) $2^{\log_2 8}$

t) $3^{\log_3 8}$

6. Simplify each of the following.

a) $5^{\log_5 A}$

b) $25^{\log_5 A}$

c) $5^{\log_{25} A}$

d) $5^{-\log_5 A}$

e) $e^{\ln B}$

f) $e^{-\ln B}$

g) $e^{3\ln B}$

h) $e^{-2\ln B}$

i) $4^{-3\log_2 C}$

j) $2^{-3\log_4 C}$

7. Simplify each of the following. Use exact values.

a) $\cos 30°$

b) $\csc 45°$

c) $\sin 60°$

d) $\cot 45°$

e) $\tan 30°$

f) $\sec 60°$

g) $\csc 30°$

h) $\sin 45°$

i) $\sec 30°$

j) $\cot 60°$

k) $\cos 600°$

l) $\csc 450°$

m) $\sin(-120°)$

n) $\cot 390°$

o) $\tan(-330°)$

p) $\sec(-90°)$

q) $\csc(270°)$

r) $\tan(315°)$

s) $\sec(930°)$

t) $\tan(-180°)$

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8. Suppose that a and b are real numbers so that a is eight less than three times b. Find the greatest possible value of $5b^2 - a^2$.

9. Re-write $\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x}}}}}$ using exponential notation.

10. Find the exact value of each of the following expressions and simplify.

a) $\sin 30° - \cos 45° \tan 60°$

b) $\sin 45° \cdot \cos 45° - \cos 30° \cdot \sin 30°$

c) $\dfrac{\sin 10°}{\cos 80°}$

d) $\sin 120° \sin 120° + \cos 120° \cos 120°$

e) $(\sin 45° + \cos 45°)^2$

f) $\sin^2 45° + \cos^2 45°$

g) $(\sin 135° + \cos 135°)^4$

h) $\sin^4 45° + \cos^4 45°$

i) $\log_2(\sec 315°)$

j) $\log_3(\tan 240°)$

k) $\dfrac{\sec 315° \cos(-30°) - \csc 60°}{\sin 120°}$

l) $\dfrac{\cot 30° - \tan^2 60°}{\cot 30° + \tan^2 60°}$

11. Approximate each of the following by placing them between two consecutive integers.

a) $\log_2 100$ b) $\log_2 200$ c) $\log_{10} 2014$ d) $\log_5 2014$ e) $\log_2\left(\dfrac{1}{3}\right)$

12. Re-write each of the following as an exponential statement.

a) $x = \log_7(2y - 1)$

b) $3 = \log_B(A - 1)$

c) $p = \log_q T$

d) $x + 2 = \log_a\left(x^2 + 1\right)$

e) $2 = \log_x 20$

13. Solve each of the following basic logarithmic equations.

a) $\log_3(2x - 7) = 2$

b) $\log_3\left(x^2 - 19\right) = 4$

c) $\ln(1 - 3x) = 28$

d) $\ln(2x + 1) = -3$

e) $\log_2(3x + 4) = -1$

f) $3\log_2(7x - 1) - 2 = 10$

g) $\dfrac{2}{3}\log_2(x + 2) - 1 = 7$

h) $\dfrac{2\log_2(x + 2) - 1}{3} = 7$

14. Re-write each of the following as a logarithmic statement. Assume that all variables represent positive numbers.

a) $3^{x-2} = 60$ b) $10^{2x-5} = 2012$ c) $\left(\dfrac{1}{3}\right)^{a+1} = b - 2$ d) $A^B = C$

15. Solve each of the following basic exponential equations.

a) $2^x = \dfrac{1}{16}$

b) $2^x = -8$

c) $2^x = \dfrac{1}{\sqrt{2}}$

d) $2^x = 21$

e) $2^x = -4$

f) $3^{5x-1} = \sqrt{27}$

g) $3^{5x-1} = -9$

h) $3^{5x-1} = \dfrac{1}{9}$

i) $3^{5x-1} = 40$

j) $e^{2x+8} = 0$

k) $e^{2x+8} = \dfrac{1}{e^5}$

l) $e^{2x+8} = 1$

m) $e^{2x+8} = 10$

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16. Solve each of the following equations. Make sure to check your solutions.

a) $\dfrac{2x+1}{5} - \dfrac{x-1}{3} = 2x + 16$

b) $(2x-1)^2 - (x-2)^2 = x + 1$

c) $\sqrt{x+6} = 1 + \sqrt{x-1}$

d) $\sqrt{x} + \sqrt{x-5} = 5$

e) $x^2 + x = 1$

f) $\sqrt{3x+1} = \sqrt{x+4} + 3$

17. Solve each of the following inequalities.

a) $3x^2 \geq 15x$ b) $28x + 88 \leq -2x^2$ c) $\dfrac{1}{3}x^2 < 4x - 12$ d) $\dfrac{1}{3}x^2 \leq 4x - 12$

18. Find the domain of each of the following functions.

a) $f(x) = \dfrac{1}{5x+3}$

b) $f(x) = \sqrt{5x+3}$

c) $f(x) = \dfrac{1}{x^2 - 10x + 24}$

d) $f(x) = \sqrt{x^2 - 10x + 24}$

e) $f(x) = \sqrt{x-3} - 2$

f) $f(x) = \dfrac{1}{\sqrt{x-3} - 2}$

19. The sum of a and three times b is 100. What is the smallest possible value of $a^2 - (2b)^2$?

20. Find the height of a buiding if the angle of elevation from the ground to its top changes from 15° to 25° as the observer advances 120 ft toward it.

21. Find the radius of the circle if
a) an arc subtended by a central angle of 30° has a length of 20 cm.
b) a sector of 30° has an area of 20 cm^2.

22. Compute the area and perimeter of a regular 12−sided polygon written into a circle with radius 9 cm. Present the exact value and approximation of your answer.

23. Find all points of intersections of $(x+4)^2 + (y-4)^2 = 25$ and $(x-10)^2 + (y-2)^2 = 125$.

24. Consider the circle $(x-10)^2 + (y-2)^2 = 125$. Find the equation of the tangent line drawn to the circle at $(0, 7)$.

25. A circle has a radius of 10 units. A point P is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from P to the circle.

26. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
b) Compute the distance between the two points of tangency on a common tangent line.

27. Seattle, WA and Los Angeles, CA have approximately the same longitude. The latitude of Seattle is 47.6° and that of Los Angeles is 34.1°. Find the distance to the nearest mile between the two cities. (The radius of the earth is approximately 3960 miles.)

28. A satellite is 1000 miles above the surface of the equator. The radius of Earth is 3960 miles. To the nearest miles per hour, how fast is the satellite moving if it can be seen at the same point in the sky?

29. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.

30. a) Suppose that α is an acute angle (acute means between 0° and 90°) with $\tan\alpha = \dfrac{3}{8}$. Compute the exact value of $\sin\alpha$.

b) Suppose that β is an acute angle with $\sec\beta = 4$. Compute the exact value of $\tan\beta$.

c) Suppose that γ is an acute angle with $\tan\gamma = A$. Compute $\cos\gamma$ in terms of A.

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31. Prove each of the following identities.

a) $1+\tan^2 x = \sec^2 x$

b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

e) $\cos x\,(\sec x - \cos x) = \sin^2 x$

f) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

g) $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

32. Graph each of the following.

a) $f(x) = \dfrac{1}{2}x^2 + 2x - \dfrac{5}{2}$

b) $y = 2^x$

c) $(x-2)^2 + (y-1)^2 = 25$

d) $f(x) = \dfrac{1}{x}$

e) $f(x) = \sqrt[3]{x}$

f) $y = \log_2 x$

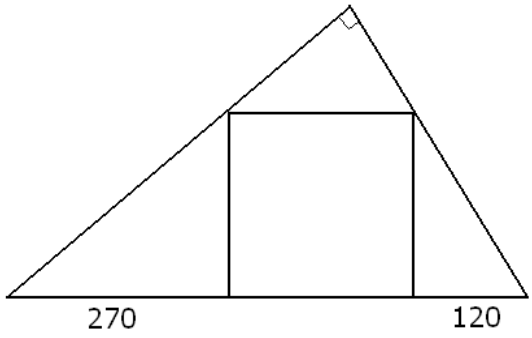
g) $f(x) = \left(\dfrac{1}{2}\right)^x$

h) $f(x) = -|x|$

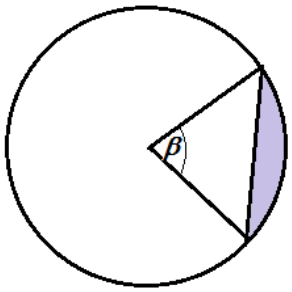
33. a) Find the exact and approximate values of the smaller angle that is formed between the positive part of the $x-$axis and the line $y = 2x - 1$.

b) Find an equation for the straight line that passes through the point $P(4,-2)$ and forms a 60° angle with the positive part of the $x-$axis.

34. The picture shows a square within a right triangle.
a) Find the length of the sides in the square.
b) Compute the angles in the triangle.



35. Compute the area of the shaded region shown on the picture below. The radius of the circle is 3 meters and $\beta = 80^\circ$. Present the exact value and approximate value of the answer.



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Answers

1.) a) $-\frac{b^2}{2a}$ b) $\frac{a^4}{b}$ c) $\frac{a^4b^2-a}{ab+b^2}$ d) $\frac{1}{9}$ e) $-\frac{55}{72}$ f) $\frac{1}{256}$ g) $\frac{a^6}{b^3}$ h) 1 i) -4
j) $\frac{1}{4}$ k) $\frac{1}{4}$ l) $-\frac{1}{4}$ m) undefined n) $\frac{a^2}{b^6}$ o) ab

2.) a) $2k$ b) $\frac{k}{2}$ c) $4k$ d) k^3 e) k^2 f) $\sqrt{k}$ g) $3k$

3.) a) $-\frac{2}{5}$ b) 2 c) undefined d) $-7\sqrt{2}-12$ e) $\frac{14}{5}\sqrt{5}-6$ f) 0 g) 3 h) $\frac{2}{9}$ i) $-\frac{1}{24}$
j) $\frac{3}{5}$ k) $\frac{8}{15}$ 4.) a) undefined b) $-\frac{1}{8}$ c) 1 d) $\frac{\sqrt{5}+3}{4}$ e) -8 f) 2 g) $\sqrt{3}+2$

5.) a) $-\frac{1}{2}$ b) $\frac{3}{2}$ c) 3 d) $\frac{3}{2}$ e) -3 f) $\frac{1}{3}$ g) 6 h) $\frac{3}{2}$ i) undefined j) $-\frac{2}{3}$ k) -3
l) -5 m) 0 n) undefined o) 17 p) undefined q) $-\frac{3}{4}$ r) $\frac{2}{7}$ s) 8 t) 8

6.) a) A b) A^2 c) $\sqrt{A}$ d) $\frac{1}{A}$ e) B f) $\frac{1}{B}$ g) B^3 h) $\frac{1}{B^2}$ i) $\frac{1}{C^6}$ j) $\frac{1}{\sqrt{C^3}}=\frac{1}{\left(\sqrt{C}\right)^3}$

7.) a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1 e) $\frac{\sqrt{3}}{3}$ f) 2 g) 2 h) $\frac{\sqrt{2}}{2}$ i) $\frac{2\sqrt{3}}{3}$ j) $\frac{\sqrt{3}}{3}$
k) $-\frac{1}{2}$ l) 1 m) $-\frac{\sqrt{3}}{2}$ n) $\sqrt{3}$ o) $\frac{\sqrt{3}}{3}$ p) undefined q) -1 r) -1 s) $-\frac{2\sqrt{3}}{3}$ t) 0

8.) 80 9.) $x^{1/32}$

10.) a) $\frac{1-\sqrt{6}}{2}$ b) $\frac{1}{2}-\frac{1}{4}\sqrt{3}$ c) 1 d) 1 e) 2 f) 1 g) 4 h) $\frac{1}{2}$ j) $\frac{1}{2}$ k) $\sqrt{2}-\frac{4}{3}$ l) $\sqrt{3}-2$

11.) a) $6<\log_2 100<7$ b) $7<\log_2 200<8$ c) $3<\log_{10} 2014<4$ d) $4<\log_5 2014<5$
e) $-2<\log_2\left(\frac{1}{3}\right)<-1$

12.) a) $7^x=2y-1$ b) $B^3=A-1$ c) $q^p=T$ d) $a^{x+2}=x^2+1$ e) $x^2=20$

13.) a) 8 b) ± 10 c) $\frac{1}{3}-\frac{e^{28}}{3}$ d) $\frac{1}{2e^3}-\frac{1}{2}$ e) $-\frac{7}{6}$ f) $\frac{17}{7}$ g) 4094 h) 2046

14.) a) $x-2=\log_3 60$ b) $2x-5=\log_{10} 2012$ c) $a+1=\log_{1/3}(b-2)$ d) $B=\log_A C$

15.) a) -4 b) no solution c) $-\frac{1}{2}$ d) $\log_2 21$ e) no solution f) $\frac{1}{2}$ g) no solution
h) $-\frac{1}{5}$ i) $\frac{1}{5}(1+\log_3 40)$ j) no solution k) $-\frac{13}{2}$ l) -4 m) $\frac{1}{2}(-8+\ln 10)$

16.) a) -8 b) $-1, \frac{4}{3}$ c) 10 d) 9 e) $\frac{-1\pm\sqrt{5}}{2}$ f) 21

17.) a) $(-\infty,0]\cup[5,\infty)$ b) $\left[-7-\sqrt{5},-7+\sqrt{5}\right]$ c) no solution d) 6

18.) a) $x\neq -\frac{3}{5}$ b) $x\geq -\frac{3}{5}$ c) $x\neq 4,6$ d) $x\leq 4$ or $x\geq 6$ e) $x\geq 3$ f) $x\geq 3$ but $x\neq 7$

19.) -8000 20.) $75.588\,461$ ft

21.) a) $r=\frac{120}{\pi}\,\text{cm}\approx 38.197\,186\,342\,\text{cm}$ b) $r=\sqrt{\frac{240}{\pi}}\,\text{cm}\approx 8.740\,387$ cm

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22.) $P = 216 \sin 15^\circ \text{ cm} \approx 55.904\,91 \text{ cm}$ $A = 972 \sin 15^\circ \cos 15^\circ \text{ cm}^2 \approx 243 \text{ cm}^2$ 23) $(0, 7)$ and $(-1, 0)$

24.) $y = 2x + 7$ 25.) $2 \sin^{-1}\left(\frac{10}{17}\right) \approx 72.063\,758^\circ$ 26.) a) $2 \sin^{-1}\left(\frac{3}{20}\right) \approx 17.253\,853^\circ$ b) $\sqrt{391}$ unit

27.) 933 miles 28.) $1299 \frac{\text{mi}}{\text{h}}$ 29.) 17 feet 30.) a) $\frac{3\sqrt{73}}{73}$ b) $\sqrt{15}$ c) $\frac{1}{\sqrt{A^2+1}}$

31.) a) $1 + \tan^2 x = \sec^2 x$

$$\text{LHS} = 1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

b) $\frac{\cot x - 1}{\cot x + 1} = \frac{1 - \tan x}{1 + \tan x}$

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\frac{1}{\tan x} - 1}{\frac{1}{\tan x} + 1} \cdot \frac{\tan x}{\tan x} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right) \cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right) \cos x = \sin^2 x \left(1 - \sin^2 x\right) \cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

d) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = \text{RHS}$$

e) $\cos x \left(\sec x - \cos x\right) = \sin^2 x$

$$\text{LHS} = \cos x \left(\sec x - \cos x\right) = \cos x \left(\frac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

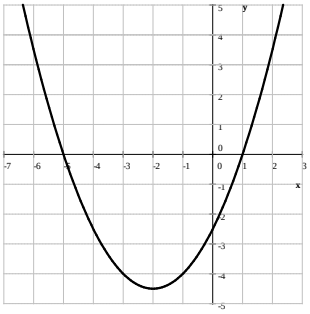
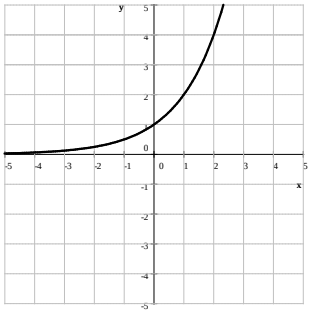
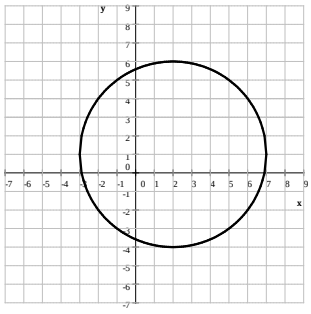
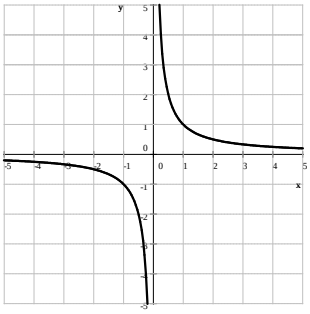
f) $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x \left(1 + \cos x\right)}{1 - \cos^2 x} = \frac{\sin x \left(1 + \cos x\right)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

g) $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{\left(1 + \sin^2 x + 2 \sin x\right) - \left(1 + \sin^2 x - 2 \sin x\right)}{1 - \sin^2 x} \\ &= \frac{1 + \sin^2 x + 2 \sin x - 1 - \sin^2 x + 2 \sin x}{\cos^2 x} = \frac{4 \sin x}{\cos^2 x} = 4 \frac{\sin x}{\cos x} \frac{1}{\cos x} = 4 \tan x \sec x = \text{RHS} \end{aligned}$$

32.) a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x - 2)^2 + (y - 1)^2 = 25$ d) $f(x) = \frac{1}{x}$

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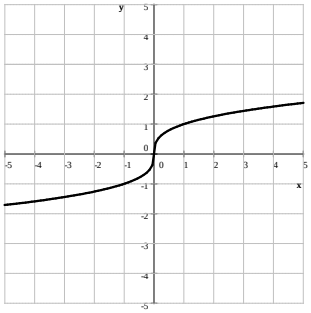
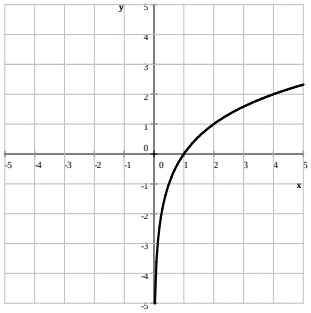
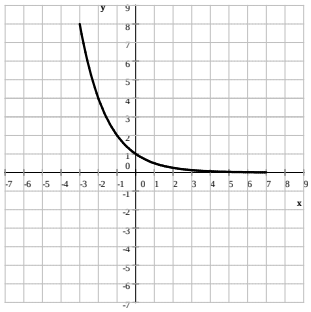
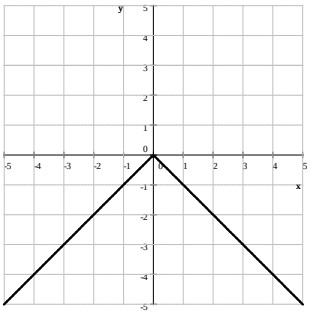
e) $f(x) = \sqrt[3]{x}$ f) $y = \log_2 x$ g) $f(x) = \left(\frac{1}{2}\right)^x$ h) $f(x) = -|x|$

33.) a) $\tan^{-1} 2 \approx 63.434949$ b) $y + 2 = \sqrt{3}(x - 4)$

34.) a) 180 b) $\alpha = \tan^{-1}\left(\frac{180}{270}\right) \approx 33.6900675^\circ$ $\beta = \tan^{-1}\left(\frac{180}{120}\right) \approx 56.3099325^\circ$

35.) $(2\pi - 9\sin 40^\circ \cos 40^\circ)\ \mathrm{m}^2 \approx 1.851\,55\,\mathrm{m}^2$

References

1. Exponents 1, Integer Exponents (for c, see Complex Fractions)
2. Exponents 1
3. Functions
4. Rational Exponents and Radical Expressions
5. Logarithms 1
6. Logarithms 1
7. Unit Circle Definitions
8. Optimization
9. Hint: just try smaller numbers, such as $\sqrt{x}$ and $\sqrt{\sqrt{x}}$ and $\sqrt{\sqrt{\sqrt{x}}}$.
10. Simplifying trigonometric Expressions and Unit Circle Definitions
11. Logarithms 1
12. Logarithms 1
13. Basic Exponential and Logarithmic Equations
14. Basic Exponential and Logarithmic Equations
15. Basic Exponential and Logarithmic Equations

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16. Review of Equations and Radical Equations
17. Quadratic Inequalities
18. Domains
19. Optimization
20. Right Triangle Trigonometry
21. Arcs and Sectors
22. Right Triangle Trigonometry
23. Circles 1, Non-Linear Systems
24. Circles 1
25. Right Triangle Trigonometry
26. Circles 2
27. Arcs and Sectors
28. Arcs and Sectors
29. Pythagorean Theorem
30. Computing Trigonometric Expressions
31. Trigonometric Identities 1
32. a) Graphing Parabolas 2, b) Basic functions c) Circles 1, d-h) Basic Functions
33. Right Triangle Trigonometry
34. No reference, but similar triangles does the trick
35. No reference, but you will need ideas from Right Triangle Trigonometry and Arcs and Sectors

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