1. Simplify each of the following.

a) $\log_{25}\left(\frac{1}{125}\right)$ c) $\ln\left(5e^2\right)$ e) $\csc\left(\frac{13\pi}{4}\right)$ g) $\tan 75°$ i) $5^{\log_{25} A}$

b) $3^{\log_9 5}$ d) $\cos\left(\frac{13\pi}{6}\right)$ f) $\cos(-105°)$ h) $\tan 22.5°$ j) $e^{-2\ln B}$

k) $\dfrac{\csc 30° + \csc 60° + \csc 90°}{\sec 0° + \sec 30° + \sec 60°}$

l) $\sec^2\beta - \tan^2\beta$

m) $\log_6 18 + \log_6 2$

n) $\log_{10} 2 + \log_{10} 50 - 1$

o) $2\log_2\left(6x^3\right) - \log_2\left(18x^5\right)$

p) $\log_3\left(\tan\left(\frac{\pi}{6}\right)\right)$

q) $\cos 48° \cos 3° + \sin 48° \sin 3°$

r) $\dfrac{\tan\frac{2\pi}{15} + \tan\frac{\pi}{5}}{1 - \left(\tan\frac{2\pi}{15}\right)\left(\tan\frac{\pi}{5}\right)}$

s) $\left(\sqrt{6+2\sqrt{5}} - \sqrt{6-2\sqrt{5}}\right)^2$

t) $\cos\left(-\frac{13\pi}{3}\right) + \sin\frac{13\pi}{4} - \tan\frac{9\pi}{4} + \cos\left(\frac{13\pi}{3}\right)$

u) $\sin 120° \cos 150° - 3\tan 330° - \tan 495°$

v) $\sin 20° \cos(-70°) + \cos 380° \cos(-20°)$

w) $\cos 68° \sin 8° - \sin 68° \cos 8°$

x) $\cos\left(-\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right) - \tan(3\pi)$

y) $\sin 143° \cos 82° + \cos 143° \sin 82°$

2. Simplify each of the following.

a) $\dfrac{\log_3 90}{\log_{30} 3} - \dfrac{\log_3 270}{\log_{10} 3}$

b) $\log_{\sec(\pi/4)}(\tan 15° + \tan 60°)$

c) $\log_2\left(\sin^{2015}\left(\frac{\pi}{4}\right)\right)$

d) $\log_3(\tan 45°) \cdot \log_3(\tan 75°)$

e) $\log_5(\tan 20°) + \log_5(\tan 70°)$

f) $1 + \log_2(\sin 15°) + \log_2(\cos 15°)$

3. Solve the equation $3x^2 - x - 1 = 0$ by completing the square. Use exact values Check your solution.

4. Find the domain of each of the following expressions.

a) $\log_5\left(-x^2 + 2x + 20\right)$

b) $\dfrac{1}{\sqrt{2x-6}-6}$

c) $\dfrac{1}{\log_2(x-5) + \log_2(x+5)}$

d) $\dfrac{1}{x^2-4} + \ln(x+10)$

e) $\log_7\left(x^2+3\right)$

f) $\ln\left(x^2 + x - 30\right)$

g) $\ln(x-5) + \ln(x+6)$

h) $\dfrac{\ln(x+1)}{\ln(x-1)}$

j) $\dfrac{\cos x + 1}{2\sin x - 1}$

i) $\cot x$

5. Solve each of the following equations.

a) $\log_3(x-7)(2x+7) = 4$

b) $2^{x+1} \cdot 3 = 2^x - 10$

c) $4 \cdot 3^x + 1 = 3^{x+2}$

d) $\sin x + \cos x = 0$

e) $3 \cdot 2^{3x-2} = 4 \cdot 5^{x-2}$

f) $9^{x+1} - 28 \cdot 3^x = -3$

g) $1 - \sin x = 2\cos^2 x$

h) $\cos x = \cos 2x$

i) $\tan^2 x = 3$

j) $\sin^2 x = \sin x$

k) $3\cos x + 2\cos^2 x = 2$

l) $\sin x = \sin 2x$

m) $\cos 2x + 5\cos x = -3$

n) $2 + 3\sin x = \cos 2x$

o) $2 + 3\sin 6x = \cos 12x$

p) $\sin 2x = 2\cos x$

q) $\sin 3x = \dfrac{\sqrt{3}}{2}$

r) $6 \cdot 2^{x-2} = 10 \cdot 3^{x+1}$

s) $\dfrac{3}{5}\ln(2x-1) = 6$

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t) $\log_2(3-x)+\log_2(-x-4)=3$

u) $\log_3(x-7)+\log_3(2x+7)=4$

v) $\log_3(x-5)-\log_3(2x-11)=-1$

x) $\log_2(2x-3)-\log_2(x+1)=-3$

6. Suppose that $\log_2 3 = x$. Express each of the following in terms of x.

a) $\log_2 6$ c) $\log_2 18$ e) $\log_2\left(\frac{3}{2}\right)$ g) $\log_2\left(\frac{4}{27}\right)$

b) $\log_2 12$ d) $\log_2 72$ f) $\log_3 2$ h) $\log_6 12$

7. Suppose that $\log_2 3 = a$ and $\log_2 5 = b$. Express each of the following in terms of a.and b.

a) $\log_2 15$ b) $\log_2 30$ c) $\log_2 240$ d) $\log_3 15$ e) $\log_6 30$

8. a) Suppose that $a = \log_5 4$ and $b = \log_2 6$. Express $\log_3 5$ in terms of a and b.

b) Suppose that $\log_3 6 = x$ and $\log_5 9 = y$. Express $\log_{18} 100$ in terms of x and y.

9. Compute the area of a regular $n-$sided polygon that is written inside a circle with radius R.

10. a) Find the smallest value of a^2+b^2 if $a+3b=50$.

b) Find the smallest possible value of $2a^2+3b^2$ if $a+b=20$.

11. Find an equation for the tangent line drawn to $14y+x^2+y^2+13=4x$ at the point $P(8,-5)$.

12. Simplify $(\sin 15°)(2\cos 15°)(2\cos 30°)(2\cos 60°)(2\cos 120°)\ldots(2\cos 960°)(2\cos 1920°)$

13. Prove that $\log_{a/b}\left(\frac{c}{d}\right) = \log_{b/a}\left(\frac{d}{c}\right)$

14. a) Solve $\cos 5x = -\cos 10x$. You may present the solution in degrees.

b) List all solutions that fall between $0°$ and $360°$.

15. Consider the line $y=\frac{12}{5}x$. Find an equation of the line that bisects the smaller angle that is formed between the line and the positive part of the $x-$axis.

16. Graph each of the following relations.

a) $y=-\frac{1}{2}x^2-3x+\frac{7}{2}$ b) $y=\sqrt[3]{x}$ c) $f(x)=|x|$ d) $y=1.5^x$

e) $f(x)=\log_2 x$ f) $f(x)=\log_{1/2} x$ g) $f(x)=-\frac{1}{x}$ h) $2y=4x+x^2+y^2$

17. We placed \$3000 into a bank account with an annual compound interest rate of 4%, compounded annually. How long until the account reaches

a) \$10 000 b) \$20 000 c) \$30 000 d) \$40 000 e) \$50 000 f) \$60 000

18. Simplify each of the following.

a) $\sin(x+180°)$ b) $\cos(180°-x)$ c) $\tan(90°-x)$

19. Suppose that we draw both tangent lines to $(x-6)^2+(y+3)^2=25$ from the point $P(-2,2)$.

a) Find the exact value of the sine of the angle formed by the two tangent lines.

b) Find an aproximate value for the angle formed by the two tangent lines.

20. Find an equation for all tangent lines drawn to the given parabola from the given point.

a) $y=-\frac{1}{2}x^2+3x-7$ from $P(1,8)$ b) $y=\frac{1}{2}x^2-x+3$ from $P(3,4)$

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21. Prove each of the following identities.

a) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

b) $4\sin^4 x = 1 - 2\cos 2x + \cos^2 2x$

c) $\cos 3x = 4\cos^3 x - 3\cos x$

d) $\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$

e) $\dfrac{\tan(45° - x)}{\tan(45° + x)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$

f) $\sin(30° + \beta) + \sin(30° - \beta) = \cos\beta$

22. Suppose that $\sin\alpha = \dfrac{3}{5}$ and α is not in the first quadrant; $\cos\beta = -\dfrac{7}{25}$ and β is not in the third quadrant. Find the exact value for each of the following.

a) $\sin 2\beta$ c) $\sin 2\alpha$ e) $\sin 3\alpha$ g) $\tan(\alpha + \beta)$

b) $\cos 2\beta$ d) $\cos 2\alpha$ f) $\cos(\alpha - \beta)$ h) $\tan(\alpha - \beta)$

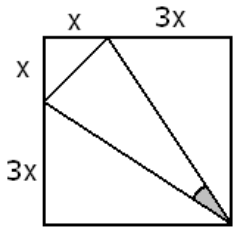
23. Find the exact value of $\tan\alpha$ if α is the acute angle formed by the lines $2x + 3y = -12$ and $2x - 7y = 1$.

24. Find the exact value of $\tan x$ if $\tan y = \dfrac{2}{3}$ and $\tan(x + y) = 4$.

25. Supose that x is an angle with $\cot x = -3$. Compute the exact value of each of the following.

a) $\cos x$ b) $\sin 2x$ c) $\cos 2x$

26. Compute the exact value of the cosine of the angle shaded on the picture below.



27. Two circles, of radii 10 cm and 14 cm are placed so that their centers are 20 cm apart. Compute the exact value of the sine of the angle formed by the common tangent lines drawn to the circles.

28. Find the coordinates of all points where the circles $(x + 2)^2 + (y + 1)^2 = 5$ and $(x - 4)^2 + (y - 1)^2 = 25$ intersect each other.

29. Graph each of the following functions on $[-2\pi, 2\pi]$ and state domain, range, and basic properties for each function.

a) $f(x) = \sin x$ b) $f(x) = \cos x$ c) $f(x) = \tan x$

d) Graph $f(x) = \sin^{-1} x$ and state its basic propertiea.

30. a) Prove that the area of any triangle can be computed as $A = \dfrac{1}{2}ab\sin\gamma$.

b) Find the value of x so that the area of the triangle with sides 12 cm, 12 cm and x, is the greatest?

31. We draw tangent lines to a circle from a point P outside of the circle. The line segment between P and a point of tangency is 3 units long. The line segment connecting P and the center of the circle intersects the circle in point Q. Line segment PQ is $\sqrt{3}$ units long. Compute the angle formed between the two tangent lines.

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32. For each of the following functions given, find an equation for the inverse.

a) $f(x) = 3x + 1$

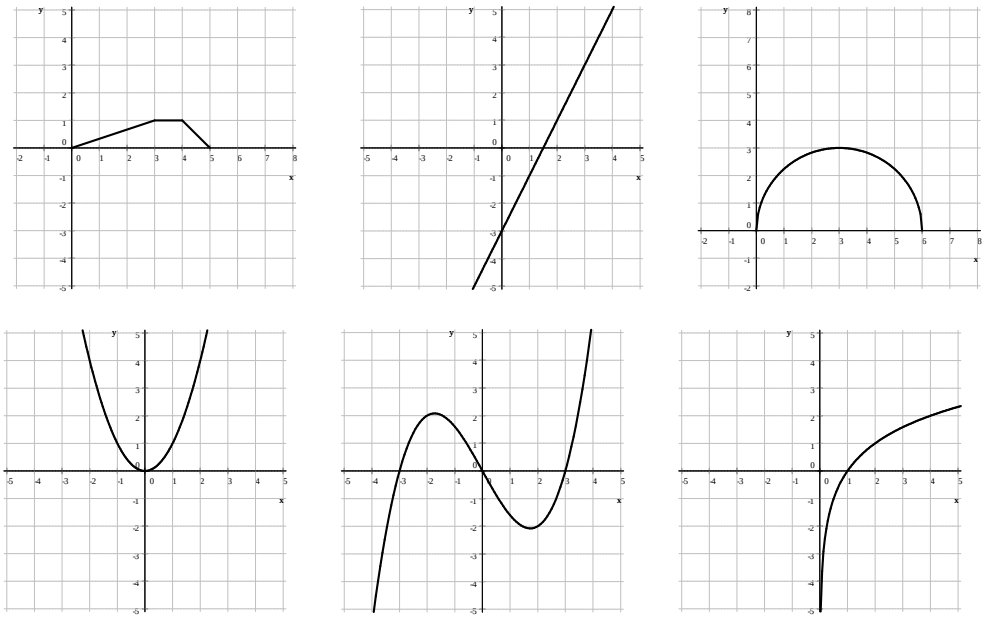
b) $f(x) = \sqrt[3]{2x - 5}$

c) $f(x) = e^{3x-1}$

d) $f(x) = \log_5\left(\frac{1}{x} - 1\right)$

e) $f(x) = \frac{2x - 3}{x + 5}$

33. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.



34. Solve each of the following triangles.

a) $a = 38\,\text{cm}$, $\alpha = 76°$, $\gamma = 41°$ b) $b = 7\,\text{ft}$, $c = 18\,\text{ft}$, $\beta = 73°$

Answers

1.) a) $-\frac{3}{2}$ b) $\sqrt{5}$ c) $2 + \ln 5$ d) $\frac{\sqrt{3}}{2}$ e) $-\sqrt{2}$ f) $\frac{\sqrt{2} - \sqrt{6}}{4}$ g) $\sqrt{3} + 2$ h) $\sqrt{2} - 1$
i) $\sqrt{A}$ j) $\frac{1}{B^2}$ k) 1 l) 1 m) 2 n) 1 o) $1 + \log_2 x$ p) $-\frac{1}{2}$ q) $\frac{1}{\sqrt{2}}$ r) $\sqrt{3}$
s) 4 t) $-\frac{\sqrt{2}}{2}$ u) $\sqrt{3} + \frac{1}{4}$ v) 1 w) $-\frac{\sqrt{3}}{2}$ x) $\sqrt{3}$ y) $-\frac{1}{\sqrt{2}}$

2.) a) 2 b) 2 c) -1007.5 d) 0 e) 0 f) -1 3.) $\frac{1 + \sqrt{13}}{6}, \frac{1 - \sqrt{13}}{6}$

4.) a) $1 - \sqrt{21} < x < 1 + \sqrt{21}$ b) $x \geq 3$, $x \neq 21$ c) $x > 5$, $x \neq \sqrt{26}$ d) $x > -10$ and $x \neq \pm 2$
e) $\mathbb{R}$ f) $x < -6$ or $x > 5$ g) $x > 5$ h) $x > 1$ and $x \neq 2$ i) $x \neq k\pi$ $k \in \mathbb{Z}$
j) $x \neq \frac{\pi}{6} + 2k\pi$, $x \neq \frac{5\pi}{6} + 2k\pi$

5.) a) $-\frac{13}{2}, 10$ b) no solution c) $-\log_3 5 = -\frac{\ln 5}{\ln 3}$ d) $-\frac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$ e) $\log_{5/8}\left(\frac{75}{16}\right)$
f) $1, -2$ g) $\frac{\pi}{2} + 2k\pi, -\frac{\pi}{6} + 2k\pi, -\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ h) $2k\pi, \pm\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$

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i) $\pm\frac{\pi}{3}+k\pi$ where $k\in\mathbb{Z}$ j) $\frac{\pi}{2}+2k\pi,\ k\pi$ where $k\in\mathbb{Z}$ k) $\pm\frac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

l) $k\pi,\pm\frac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$ m) $\pm\frac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$

n) $x=-\frac{\pi}{2}+2k_1\pi,\ -\frac{\pi}{6}+2k_2\pi,\ -\frac{5\pi}{6}+2k_3\pi$ where $k_1,k_2,k_3\in\mathbb{Z}$

o) $x=-\frac{\pi}{12}+k_1\frac{\pi}{3},\ -\frac{\pi}{36}+k_2\frac{\pi}{3},\ -\frac{5\pi}{36}+k_3\frac{\pi}{3}$ where $k_1,k_2,k_3\in\mathbb{Z}$ p) $\frac{\pi}{2}+k\pi$ where $k\in\mathbb{Z}$

q) $\frac{\pi}{9}+\frac{2}{3}k\pi\quad\frac{2\pi}{9}+\frac{2}{3}k\pi$ where $k\in\mathbb{Z}$ r) $\log_{2/3}20$ s) $\frac{1}{2}e^{10}+\frac{1}{2}$ t) -5 u) 10

v) no solution x) $\frac{5}{3}$ 6.) a) $x+1$ b) $x+2$ c) $2x+1$ d) $2x+3$ e) $x-1$ f) $\frac{1}{x}$

g) $2-3x$ h) $\frac{x+2}{x+1}$ 7.) a) $a+b$ b) $a+b+1$ c) $a+b+4$ d) $\frac{a+b}{a}$ e) $\frac{a+b+1}{a+1}$

8.) a) $\frac{2}{a(b-1)}$ b) $\frac{2x-2+\frac{4}{y}}{x+1}$ 9.) $A=2nR^2\sin\left(\frac{180^\circ}{n}\right)\cos\left(\frac{180^\circ}{n}\right)$ 10.) a) 250 b) 480

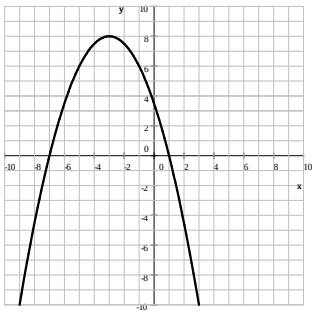
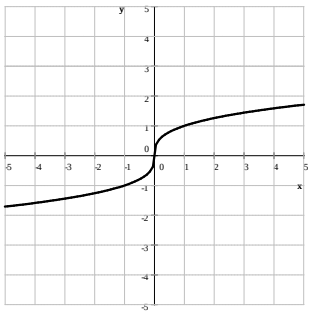
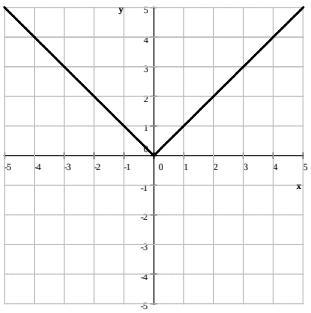
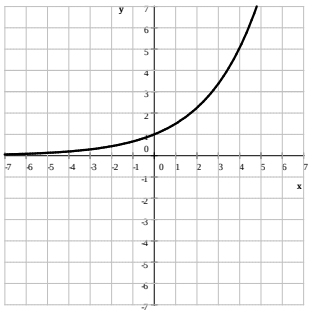
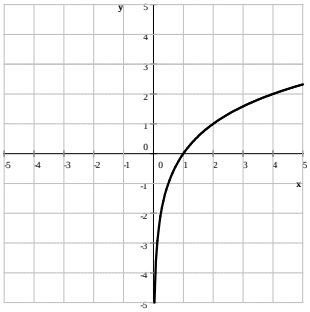
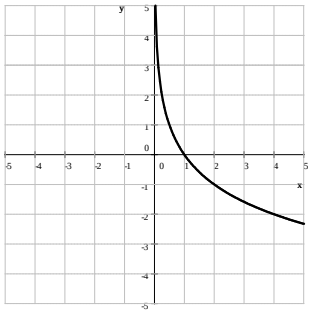
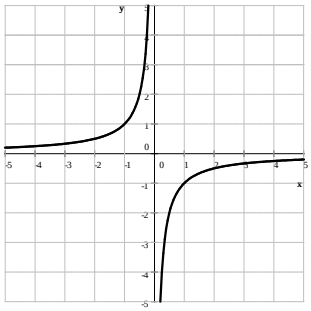
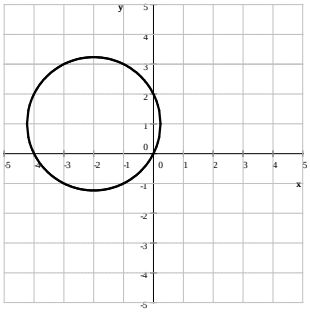
11.) $y=-3x+19$ 12.) $-\frac{\sqrt{3}}{2}$

13.) $\log_{a/b}\left(\frac{c}{d}\right)=\frac{\ln\left(\frac{c}{d}\right)}{\ln\left(\frac{a}{b}\right)}=\frac{\ln c-\ln d}{\ln a-\ln b}=\frac{-1(\ln c-\ln d)}{-1(\ln a-\ln b)}=\frac{\ln d-\ln c}{\ln b-\ln a}=\frac{\ln\left(\frac{d}{c}\right)}{\ln\left(\frac{b}{a}\right)}=\log_{b/a}\left(\frac{d}{c}\right)$

14.) a) $36^\circ+k\cdot72^\circ$ where $k\in\mathbb{Z}\quad\pm12^\circ+k\cdot72^\circ$ where $k\in\mathbb{Z}$

b) $12^\circ, 36^\circ, 60^\circ, 84^\circ\ 108^\circ, 132^\circ, 156^\circ, 180^\circ, 204^\circ, 228^\circ, 252^\circ, 276^\circ, 300^\circ, 324^\circ, 348^\circ$ 15.) $y=\frac{2}{3}x$

16.) a) $y=-\frac{1}{2}x^2-3x+\frac{7}{2}$ b) $y=\sqrt[3]{x}$ c) $f(x)=|x|$ d) $y=1.5^x$

e) $f(x)=\log_2 x$ f) $f(x)=\log_{1/2}x$ g) $f(x)=-\frac{1}{x}$ h) $2y=4x+x^2+y^2$

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17.) a) $\dfrac{\ln 10 - \ln 3}{\ln 1.04} = 30.6974$ during the 31st year b) $\dfrac{\ln 20 - \ln 3}{\ln 1.04} \approx 48.37036$ during the 49th year

c) $\dfrac{\ln 10}{\ln 1.04} \approx 58.7084$ during the 59th year d) $\dfrac{\ln 40 - \ln 3}{\ln 1.04} \approx 66.043347$ during the 67th year

e) $\dfrac{\ln 50 - \ln 3}{\ln 1.04} \approx 71.7328$ during the 72nd year f) $\dfrac{\ln 20}{\ln 1.04} = 76.3814$ during the 77th year

18.) a) $-\sin x$ b) $-\cos x$ c) $\cot x$ 19.) a) $\dfrac{80}{89}$ b) 64.0107664°

20.) a) $y = 7x + 1$ and $y = -3x + 11$ b) $y = 3x - 5$ and $y = x + 1$

21.) a) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

$$\text{RHS} = \frac{\cot^2 x - 1}{2\cot x} = \frac{\dfrac{\cos^2 x}{\sin^2 x} - 1}{2\dfrac{\cos x}{\sin x}} = \frac{\dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}}{2\dfrac{\cos x}{\sin x}} = \frac{\dfrac{\cos^2 x - \sin^2 x}{\sin^2 x}}{\dfrac{2\cos x}{\sin x}} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} \cdot \frac{\sin x}{2\cos x}$$
$$= \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{LHS}$$

b) $\cos 3x = 4\cos^3 x - 3\cos x$ b) $4\sin^4 x = 1 - 2\cos 2x + \cos^2 2x$

$$\text{LHS} = 4\sin^4 x = 4\left(\sin^2 x\right)^2 = \left(2\sin^2 x\right)^2 = \left(2\sin^2 x - 1 + 1\right)^2 = (-\cos 2x + 1)^2 =$$
$$= \cos^2 2x - 2\cos 2x + 1 = \text{RHS}$$

c) $\cos 3x = 4\cos^3 x - 3\cos x$

$$\text{LHS} = \cos 3x = \cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x = \cos x\left(2\cos^2 x - 1\right) - \sin x\left(2\sin x \cos x\right) =$$
$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x = 2\cos^3 x - \cos x - 2\left(1 - \cos^2 x\right)\cos x =$$
$$= 2\cos^3 x - \cos x - 2\cos x\left(1 - \cos^2 x\right) = 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x = \text{RHS}$$

d) $\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$

$$\text{LHS} = \sin\left(\frac{\pi}{2}\right)\cos x - \cos\left(\frac{\pi}{2}\right)\sin x = 1\cos x - 0\sin x = \cos x = \text{RHS}$$

e) $\dfrac{\tan(45^\circ - x)}{\tan(45^\circ + x)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$

$$\text{LHS} = \frac{\tan(45^\circ - x)}{\tan(45^\circ + x)} = \frac{\dfrac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \tan x}}{\dfrac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x}} = \frac{\dfrac{1 - \tan x}{1 + 1 \cdot \tan x}}{\dfrac{1 + \tan x}{1 - 1 \cdot \tan x}} = \frac{1 - \tan x}{1 + \tan x} \cdot \frac{1 - \tan x}{1 + \tan x} = \left(\frac{1 - \dfrac{\sin x}{\cos x}}{1 + \dfrac{\sin x}{\cos x}}\right)^2$$
$$= \frac{\left(1 - \dfrac{\sin x}{\cos x}\right)^2}{\left(1 + \dfrac{\sin x}{\cos x}\right)^2} = \frac{\left(1 - \dfrac{\sin x}{\cos x}\right)^2}{\left(1 + \dfrac{\sin x}{\cos x}\right)^2} \cdot \frac{\cos^2 x}{\cos^2 x} = \frac{\left(\left(1 - \dfrac{\sin x}{\cos x}\right)\cos x\right)^2}{\left(\left(1 + \dfrac{\sin x}{\cos x}\right)\cos x\right)^2} = \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2}$$
$$= \frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{1 - \sin 2x}{1 + \sin 2x} = \text{RHS}$$

f) $\sin(30^\circ + \beta) + \sin(30^\circ - \beta) = \cos\beta$

$$\text{LHS} = \sin(30^\circ + \beta) + \sin(30^\circ - \beta) = \sin 30^\circ \cos\beta + \cos 30^\circ \sin\beta + \sin 30^\circ \cos\beta - \cos 30^\circ \sin\beta$$
$$= 2\sin 30^\circ \cos\beta = 2 \cdot \frac{1}{2}\cos\beta = \cos\beta = \text{RHS}$$

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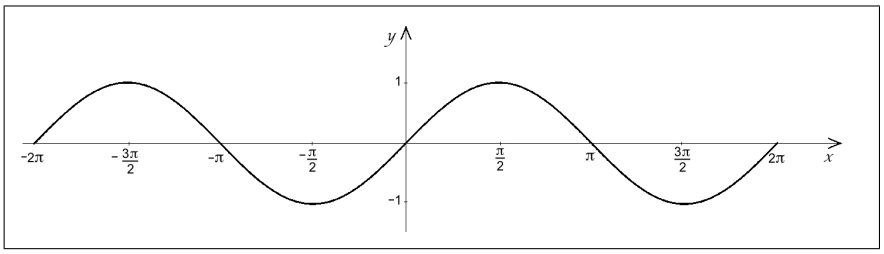
22.) a) $-\frac{336}{625}$ b) $-\frac{527}{625}$ c) $-\frac{24}{25}$ d) $\frac{7}{25}$ e) $\frac{117}{125}$ f) $\frac{4}{5}$ g) $\frac{117}{44}$ h) $\frac{3}{4}$ 23.) $\frac{20}{17}$ 24.) $\frac{10}{11}$

25.) a) $\pm\frac{3\sqrt{10}}{10}$ b) $-\frac{3}{5}$ c) $\frac{4}{5}$ 26.) $\frac{24}{25}$ 27.) $\frac{4\sqrt{6}}{25}$ 28.) $(-1,1)$ and $(0,-2)$

29.) a) $f(x) = \sin x$

domain: $\mathbb{R}$
range: $[-1,1]$

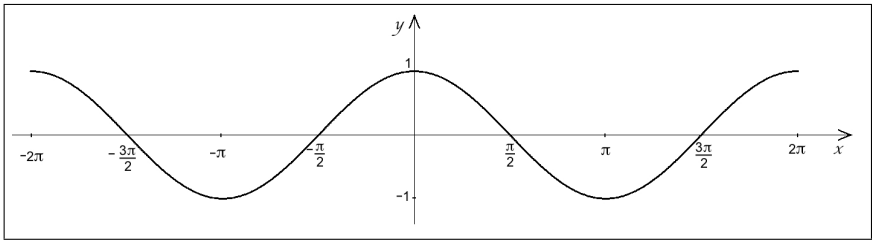
periodic with period 2π: for all x, $\sin(x+2\pi) = \sin x$
odd function: for all x, $\sin(-x) = -\sin x$



b) $f(x) = \cos x$

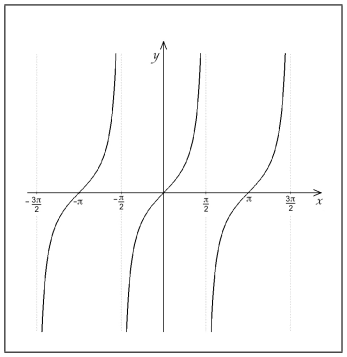
domain: $\mathbb{R}$
range: $[-1,1]$

periodic with period 2π: for all x, $\cos(x+2\pi) = \cos x$
even function: for all x, $\cos(-x) = \cos x$



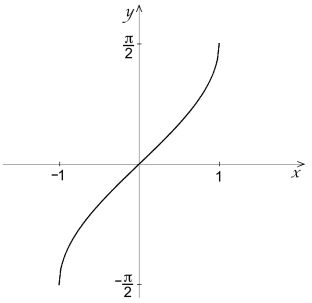
c) $f(x) = \tan x$

domain: $x \neq \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$
range: $\mathbb{R}$
periodic with period π: for all x, $\tan(x+\pi) = \tan x$
vertical asymptotes at $x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$
odd function: for all x, $\tan(-x) = -\tan x$



d) $f(x) = \sin^{-1} x$

domain: $[-1,1]$
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



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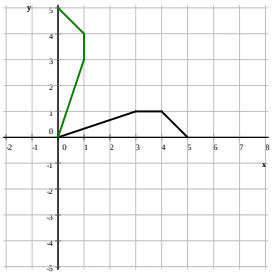
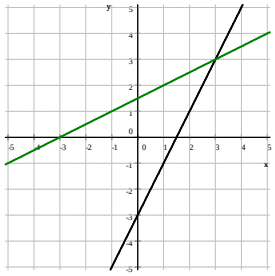
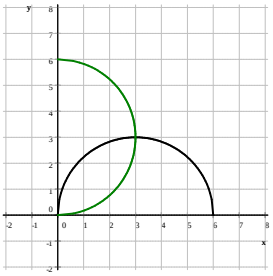
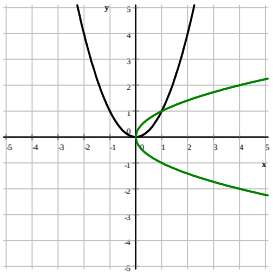
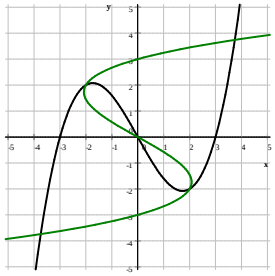
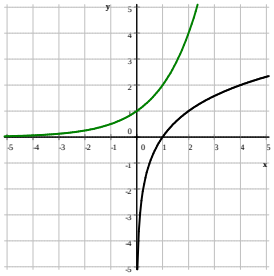
30.) a) $A = \frac{1}{2}ah$ and $h = b\sin\gamma$ b) $12\sqrt{2}\,\text{cm}$ 31.) $60°$

32.) a) $f^{-1}(x) = \frac{1}{3}x - \frac{1}{3}$ b) $f^{-1}(x) = \frac{1}{2}\left(x^3 + 5\right)$ c) $f^{-1}(x) = \frac{1}{3}(1 + \ln x)$ d) $f^{-1}(x) = \frac{1}{5^x + 1}$

e) $f^{-1}(x) = \frac{5x+3}{-x+2}$

33.)

34.) a) $c \approx 25.69345\,\text{cm}$ $\beta = 63°$ $b \approx 34.89477\,\text{cm}$ b) no such triangle

Solution for 8:

a) Given that $a = \log_5 4$ and $b = \log_2 6$. Express $\log_3 5$ in terms of a and b.

Solution: In the statement $a = \log_5 4$ we switch to base 2:

$a = \log_5 4 = \dfrac{\log_2 4}{\log_2 5} = \dfrac{2}{\log_2 5}$ Thus $a = \dfrac{2}{\log_2 5}$ we solve for $\log_2 5 = \dfrac{2}{a}$

$b = \log_2 6 = \log_2(2\cdot 3) = \log_2 3 + \log_2 2 = \log_2 3 + 1$ Thus $b = \log_2 3 + 1$ We solve for $\log_2 3 = b - 1$

Now we're ready: $\log_3 5 = \dfrac{\log_2 5}{\log_2 3} = \dfrac{\frac{2}{a}}{b-1} = \dfrac{2}{a(b-1)}$

b) Given that $\log_3 6 = x$ and $\log_5 9 = y$. Express $\log_{18} 100$ in terms of x and y.

Solution: We will switch to base 3.

$x = \log_3 6 = \log_3(2\cdot 3) = \log_3 3 + \log_3 2 = 1 + \log_3 2$ Thus $x = 1 + \log_3 2$, we solve for $\log_3 2$ and get $\log_3 2 = x - 1$

$y = \log_5 9 = \dfrac{\log_3 9}{\log_3 5} = \dfrac{2}{\log_3 5}$ Thus $y = \dfrac{2}{\log_3 5}$ We solve for $\log_3 5$ and get $\log_3 5 = \dfrac{2}{y}$

We are now ready to solve the problem:

$$\log_{18} 100 = \frac{\log_3 100}{\log_3 18} = \frac{\log_3\left(2^2\cdot 5^2\right)}{\log_3(2\cdot 3^2)} = \frac{\log_3\left(2^2\right) + \log_3\left(5^2\right)}{\log_3(2) + \log_3\left(3^2\right)} = \frac{2\log_3 2 + 2\log_3 5}{\log_3 2 + 2}$$

$$= \frac{2(x-1) + 2\frac{2}{y}}{x - 1 + 2} = \frac{2x - 2 + \frac{4}{y}}{x+1}\cdot\frac{y}{y} = \frac{2xy - 2y + 4}{xy + y}$$

Answer: $\dfrac{2x-2+\frac{4}{y}}{x+1} = \dfrac{2xy-2y+4}{xy+y} = \dfrac{2(xy-y+2)}{y(x+1)}$ All three forms are acceptable as final answer.

Last revised: April 7, 2016

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