Quiz 10 will also cover material from Quizzes 1-9 and Exam 1. Review those topics even if they do not appear in this review.

1. Simplify each of the following.

   a) $\dfrac{3a^2b^{-3}\left(2a^3b^7\right)^0}{-2b^0a^5\left(3b^{-1}\right)}$

   b) $\dfrac{a^{-1}b^{-2}}{a^{-2}b^{-1}}$

   c) $\dfrac{a^{-1}+b^{-2}}{a^{-2}-b^{-1}}$

   d) $\left(3^{-1}+1\right)^{-1}$

   e) $(-1)^{-1}$

   f) $\left(\dfrac{3ab^5}{-2a^3b^{-1}}\right)^{-2}$

   g) $\left(\dfrac{2x^{1/2}y^{-1/2}}{-x^{3/2}y^{-3/2}}\right)^4$

2. Simplify each of the following.

   a) $\sin 30^\circ \sin 45^\circ \sin 60^\circ + \cos 30^\circ \cos 45^\circ \cos 60^\circ$

   b) $\tan 30^\circ - \tan 45^\circ + \tan 60^\circ$

   c) $\cos^2 17^\circ + \sin^2 17^\circ$

   d) $\cos^2 30^\circ - \sin^2 30^\circ$

3. Simplify each of the following.

   a) -3^{-2}

   b) $(-3)^{-2}$

   c) $81^{-3/4}$

   d) $16^{7/4}$

   e) $\log_{81} 3$

   f) $\log_5 1$

   g) $\log_5 0$

   h) $\log_m\left(m^4\right)$

   i) $\log_{1/2}\left(\dfrac{1}{8}\right)$

   j) $\log_{49} 7$

   k) $\log_4\left(\dfrac{1}{32}\right)$

   l) $\log_8 16$

   m) $5^{\log_5 125}$

   n) $2^{\log_2 16}$

   o) $\log_8\left(\dfrac{1}{16}\right)$

   p) $\log_{27}\left(\dfrac{1}{9}\right)$

   q) $3^{\log_3(1/9)}$

   r) $\log_{100} 1000$

   s) $\log_2\left(\dfrac{1}{32}\right)$

   t) $2^{-\log_2 4}$

4. Place each of the following logarithms between two consecutive integers.

   a) $\log_2 20$

   b) $\log_5 100$

   c) $\log_3 10$

   d) $\log_{10} 2012$

   e) $\log_{10} 314$

   f) $\log_2 100$

   g) $\log_2 1000$

   h) $\log_3 100$

5. Re-write each of the following logarithmic statements as an exponential statement.

   a) $\log_2 x = 10$

   b) $\log_3 P = -2$

   c) $\log_7 21 = y$

   d) $\log_x y = 4$

   e) $\log_A B = C$

   f) $\log_2(x+1) = -2$

   g) $\log_x 5 = 2$

   h) $\log_{1.3}(x-5) = 1$

6. Solve each of the following basic logarithmic equations.

   a) $\dfrac{2\log_2(3x-1)-4}{3} = 2$

   b) $\dfrac{2}{3}\log_2(3x-1) - 4 = 2$

   c) $4\ln(x-2)+1 = -2$

   d) $4(\ln(x-2)+1) = -2$

   e) $\dfrac{2}{5}\log_3(8x+1) - 1 = 3$

   f) $\dfrac{2\log_3(8x+1)-1}{5} = 3$

   g) $\log_5(3x-1) = -2$

7. Re-write each of the following exponential statements as a logarithmic statement.

   a) $A^B = C$

   b) $4^x = 7$

   c) $3^{A-1} = 12$

   d) $e^x = 24$

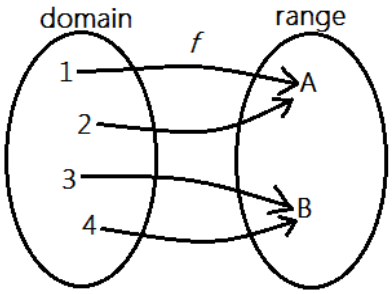
8. Solve each of the following basic exponential equations.

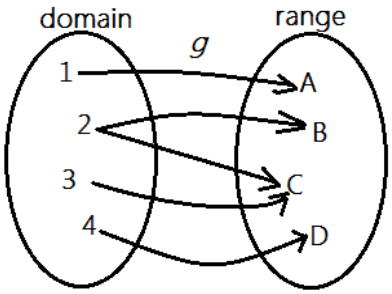
   a) $3^x = 243$
   b) $3^x = 38$
   c) $2^{4x-1} = 128$
   d) $2^{4x-1} = 42$
   e) $10^{3x-1} = 10^{41}$
   f) $10^{3x-1} = 19$
   g) $e^{2x-8} = e^{10}$
   h) $e^{3x-7} = 5$
   i) $3^{2x-5} = -27$

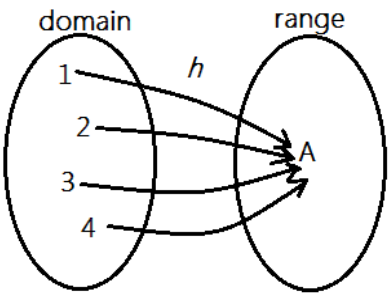
9. Let $f(x) = \dfrac{x-2}{3x+6}$. Simplify each of the following.

   a) $f(0)$
   b) $f(1)$
   c) $f(-1)$
   d) $f\left(\frac{2}{3}\right)$
   e) $f(-2)$
   f) $f(2) + f(3)$
   g) $f(2+3)$
   h) $f(a)$
   i) $2f(a)$
   j) $f(2a)$
   k) $f(f(2))$
   l) $f\left(f\left(\frac{1}{2}\right)\right)$
   m) Compute the value of x for which $f(x) = \dfrac{1}{7}$.

10. Which of the following relations are functions?







11. The sum of a number a and four times a number b is 30.
    a) Find the greatest value of ab.
    b) Find the greatest value of $6b^2 - a^2$

12. We are driving toward a tower. The angle of elevation is 35°. Then we drive 50 ft toward the tower. Now the angle of elevation is 40°. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.

13. A number is 3 greater than its own reciprocal. Find this number.

14. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

    a) $y = x^2 + 7x - 6$ and $y = 3x - 1$
    b) $y = -\frac{1}{2}x^2 + 5x - 3$ and $y = 3x - 1$
    c) $y = 3x^2 - x + 7$ and $y = x - 4$
    d) $x^2 + (y+4)^2 = 20$ and $x - 3y = 2$
    e) $(x-2)^2 + (y+5)^2 = 10$ and $3y - x = -7$
    f) $x^2 + (y+1)^2 = 16$ and $y + 2x = 15$

15. Find an equation for the line connecting the points of intersections of the circles $(x+1)^2 + (y-2)^2 = 10$ and $(x-1)^2 + (y-6)^2 = 50$.

16. Find an equation of the tangent line drawn
    a) to $2y - 6x + x^2 + y^2 = 40$ at the point $(10, -2)$.
    b) to $2y + x^2 + y^2 = 3(2x+5)$ at the point $(7, -4)$.

17. We are planning to build a fence around a rectangular a garden that is next to a river. Because of that, we only need to build a fence around three of its sides. What is the maximum area possible to fence in if we have 400 meters of fencing? What dimensions of a garden should we plan to achieve this maximum area?

18. Consider the right triangle ABC. Find all sides if $\gamma = 90°$, $\beta = 24°$ and $a = 17$. Present your answer as an approximation, accurate up to at least four decimal places.

19. True or false? For all angles α, $\sin 2\alpha = 2\sin\alpha$. (Hint: is it true for 30° and 60°?)

20. Find the exact value of each of the following expressions.

    a) $3\sin 30°\cos 60° - 2\sin 60°\cos 30°$

    b) $2\sin 60° + 2\sin 30° - 2\tan 45°$

    c) $\sin^2 16° + \cos^2 16°$

    d) $\dfrac{\cos 40°}{\sin 50°}$

    e) $\dfrac{\sec 45°\cos 30° - \csc 60°}{\sin 60°}$

    f) $\dfrac{\cot 30° - \tan^2 60°}{\cot 30° + \tan^2 60°}$

    g) $\sin 30°\cos 60° - \cos 30°\sin 60°$

    h) $\cot 30° + \cot 45° + \cot 60°$

21. Solve each of the following inequalities.

    a) $x^2 \geq x$ b) $8x - 2x^2 > 58$ c) $x^2 + 7 \leq 6x$ d) $4x^2 + 1 \leq 4x$

22. Solve each of the following equations.

    a) $\sqrt{x+6} + \sqrt{11-x} = 5$

    b) $\sqrt{3x+10} - \sqrt{x+4} = 2$

    c) $\sqrt{x-3} + 1 = \sqrt{x+2}$

    d) $\sqrt{x-1} - 1 = \sqrt{2x-9}$

23. A square-based pyramid has all of its edges measuring the same length, 10 meters.

    a) What is the height of the pyramid?

    b) Find the exact and approximate value of the angle that is formed between the square base and a triangulr face.

24. a) Find the exact and approximate value of the smaller angle that is formed between the line $y = 2x - 3$ and the positive part of the $x-$axis.

    b) Find the exact and approximate value of the smaller angle that is formed between the line $y = 4x - 3$ and the positive part of the $x-$axis.

    c) Based on the previous computations, determine whether doubling the slope of a line means doubling the angle or not.

25. Let ABC be a right triangle with hypotenuse AB. Let h denote the altitude drawn to the hypotenuse. Let D denote the point where h intersects the hypotenuse. Let x denote line segment AD and y denote line segment BD. Prove that $h = \sqrt{xy}$.

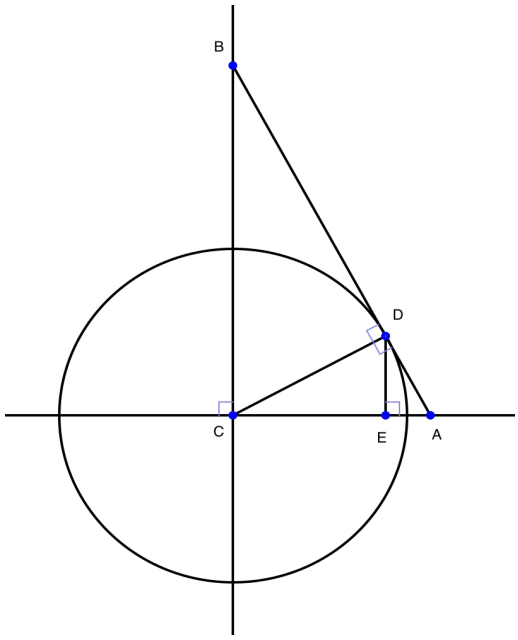
26. Find the area of a regular 9−sided polygon written into a circle of radius 10 cm.

27. Consider the triangle with sides 6 m, 6 m, and 8 m long.
    a) Compute the exact value of the area of the triangle.
    b) Compute the exact value of the smallest angle in the triangle.
    c) Compute an approximate value of the smallest angle in the triangle.

28. Suppose that circle C_1 has radius 4 and circle C_2 has radius 7. The two centers are 9 units apart. Find an approximate value of the angle formed by the two common tangent lines drawn to the circles.

29. Consider the picture shown below. Line AB is tangent to the unit circle, where D is the point of tangency. Let α denote angle DCE. Match each of the six trigonometric functions with the length of each of the line segments given.

$\sin\alpha,\ \cos\alpha,\ \tan\alpha,\ \csc\alpha, \sec\alpha, \cot\alpha$ and $AC,\ AD,\ BC,\ BD,\ CE,\ DE$



30. We are driving away from a tower. First, the angle of elevation from the car to the top of the tower is 52°. After we drove 300 feet away from the tower, the angle of elevation from the car to the top of the tower is 32°. How tall is the tower?

31. Prove each of the following identities.

a) $1+\tan^2 x = \sec^2 x$

b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

e) $\cos x\,(\sec x - \cos x) = \sin^2 x$

f) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

g) $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

Answers

1.) a) $-\dfrac{1}{2a^3b^2}$ b) $\dfrac{a}{b}$ c) $\dfrac{a^2+ab^2}{b^2-a^2b}$ d) $\dfrac{3}{4}$ e) -1 f) $\dfrac{4a^4}{9b^{12}}$ g) $\dfrac{16y^4}{x^4}$

2.) a) $\dfrac{\sqrt{6}}{4}$ b) $\dfrac{4\sqrt{3}}{3} - 1$ c) 1 d) $\dfrac{1}{2}$

3.) a) $-\dfrac{1}{9}$ b) $\dfrac{1}{9}$ c) $\dfrac{1}{27}$ d) 128 e) $\dfrac{1}{4}$ f) 0 g) undefined h) 4 i) 3 j) $\dfrac{1}{2}$
k) $-\dfrac{5}{2}$ l) $\dfrac{4}{3}$ m) 125 n) 16 o) $-\dfrac{4}{3}$ p) $-\dfrac{2}{3}$ q) $\dfrac{1}{9}$ r) $\dfrac{3}{2}$ s) -5 t) $\dfrac{1}{4}$

4.) a) $4 < \log_2 20 < 5$ b) $2 < \log_5 100 < 3$ c) $2 < \log_3 10 < 3$ d) $3 < \log_{10} 2012 < 4$
e) $2 < \log_{10} 314 < 3$ f) $6 < \log_2 100 < 7$ g) $9 < \log_2 1000 < 10$ h) $4 < \log_3 100 < 5$

5.) a) $2^{10} = x$ b) $3^{-2} = P$ c) $7^y = 21$ d) $x^4 = y$ e) $A^C = B$ f) $\frac{1}{4} = x + 1$
g) $x^2 = 5$ h) $1.3 = x - 5$

6.) a) 11 b) 171 c) $e^{-3/4} + 2 = \frac{1}{(\sqrt[4]{e})^3} + 2$ d) $e^{-3/2} + 2 = \frac{1}{(\sqrt{e})^3} + 2$ e) 7381 f) 820 g) $\frac{26}{75}$

7.) a) $\log_A C = B$ b) $x = \log_4 7$ c) $A - 1 = \log_3 12$ d) $x = \ln 24$

8.) a) 5 b) $\log_3 38$ c) 2 d) $\frac{1}{4}(1 + \log_2 42)$ e) 14 f) $\frac{1}{3}(1 + \log_{10} 19)$ g) 9
h) $\frac{1}{3}(7 + \ln 5)$ i) no solution

9.) a) $-\frac{1}{3}$ b) $-\frac{1}{9}$ c) -1 d) $-\frac{1}{6}$ e) undefined f) $\frac{1}{15}$ g) $\frac{1}{7}$ h) $\frac{a-2}{3a+6}$ i) $\frac{2a-4}{3a+6}$ j) $\frac{a-1}{3a+3}$
k) $-\frac{1}{3}$ l) $-\frac{11}{27}$ m) 5

10.) f and h 11.) a) $\frac{225}{4}$ b) 540 12.) 211.510 92 ft 13.) $\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}$

14.) a) $(1, 2)$ and $(-5, -16)$ b) $(2, 5)$ c) no intersection point d) $(-4, -2)$ and $(2, 0)$
e) $(1, -2)$ f) no intersection point 15.) $y = -\frac{1}{2}x - 1$

16.) a) $y + 2 = 7(x - 10)$ or $y = 7x - 72$ b) $\frac{4}{3}(x - 7) = y + 4$ or $y = \frac{4}{3}x - \frac{40}{3}$

17.) 100 m by 200 m will give us an area of 20 000 m^2 18.) $c \approx 18.608\,816\,7$ $b \approx 7.568\,888$

19.) False. For example, consider $\alpha = 30°$.
Then $\sin 2\alpha = \sin 60° = \frac{\sqrt{3}}{2}$ and $2\sin\alpha = 2\sin 30° = 2\left(\frac{1}{2}\right) = 1$.

20.) a) $-\frac{3}{4}$ b) $\sqrt{3} - 1$ c) 1 d) 1 e) $\sqrt{2} - \frac{4}{3}$ f) $\sqrt{3} - 2$ g) $-\frac{1}{2}$ h) $\frac{4\sqrt{3}}{3} + 1$

21.) a) $(-\infty, 0] \cup [1, \infty)$ b) no solution c) $\left[-\sqrt{2} + 3, \sqrt{2} + 3\right]$ d) $x = \frac{1}{2}$

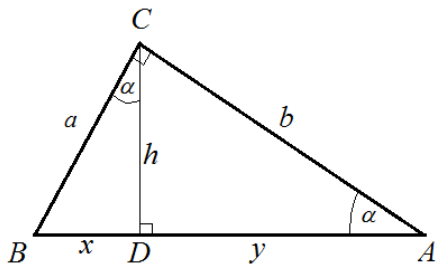
22.) 11.) a) $-5, 10$ b) 5 (-3 does not work) c) 7 d) 5 (17 doesn't work)

23.) a) $5\sqrt{2}$ m b) $\tan^{-1}\left(\sqrt{2}\right) \approx 54.735\,61°$

24.) a) $\tan^{-1} 2 \approx 63.434948823°$ b) $\tan^{-1}(4) \approx 75.963756\,532°$ c) not

25.) Consider the picture. Triangles ABC and ACD and BDC are all similar. Let us denote the angle at A by α. Then α also appears in triangle BDC, at point C. Consider now $\tan\alpha$ in triangles ACD and BDC.

$$\begin{aligned}\tan\alpha &= \frac{h}{y} = \frac{x}{h}\\ \frac{h}{y} &= \frac{x}{h}\\ h^2 &= xy\\ h &= \pm\sqrt{xy} \qquad \Longrightarrow \qquad h = \sqrt{xy}\end{aligned}$$



Note that we only used ratios and so this problem can be solved without any trignometry. (We just used it here because it made it so easy to explain which ratios will we use.)

26.) $900\cos 20^\circ \sin 20^\circ \,\text{cm}^2 \approx 289.254\,424\,36\,\text{cm}^2$ 27.) a) $8\sqrt{5}\,\text{m}^2$ b) $\cos^{-1}\left(\frac{4}{6}\right)$ c) 48.1897°

28.) $38.942\,44^\circ$ 29.) $\sin\alpha = DE$ $\cos\alpha = CE$ $\tan\alpha = AD$ $\csc\alpha = BC$ $\sec\alpha = AC$ $\cot\alpha = BD$

30.) 366.278 4958 feet 31.) a) $1 + \tan^2 x = \sec^2 x$

$$\text{LHS} = 1 + \tan^2 x = 1 + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

b) $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1 - \tan x}{1 + \tan x}$

$$\text{LHS} = \frac{\cot x - 1}{\cot x + 1} = \frac{\dfrac{1}{\tan x} - 1}{\dfrac{1}{\tan x} + 1} \cdot \frac{\tan x}{\tan x} = \frac{1 - \tan x}{1 + \tan x} = \text{RHS}$$

c) $\sin^2 x \cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right)\cos x = \sin^2 x\left(1 - \sin^2 x\right)\cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

d) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x = 1 + 2\sin x\cos x = \text{RHS}$$

e) $\cos x\,(\sec x - \cos x) = \sin^2 x$

$$\text{LHS} = \cos x\,(\sec x - \cos x) = \cos x\left(\frac{1}{\cos x} - \cos x\right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

f) $\dfrac{\sin x}{1 - \cos x} = \dfrac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x\,(1 + \cos x)}{1 - \cos^2 x} = \frac{\sin x\,(1 + \cos x)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

g) $\dfrac{1 + \sin x}{1 - \sin x} - \dfrac{1 - \sin x}{1 + \sin x} = 4\tan x \sec x$

$$\begin{aligned}\text{LHS} &= \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = \frac{(1 + \sin x)^2 - (1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)} = \frac{\left(1 + \sin^2 x + 2\sin x\right) - \left(1 + \sin^2 x - 2\sin x\right)}{1 - \sin^2 x}\\ &= \frac{1 + \sin^2 x + 2\sin x - 1 - \sin^2 x + 2\sin x}{\cos^2 x} = \frac{4\sin x}{\cos^2 x} = 4\frac{\sin x}{\cos x}\frac{1}{\cos x} = 4\tan x \sec x = \text{RHS}\end{aligned}$$

Last revised: February 25, 2015