1. Solve each of the following inequalities.

   a) $\dfrac{2}{x+3} < 1$ b) $\dfrac{3x+1}{x-1} \geq -2$ c) $\dfrac{-1}{x-5} \geq 2$ d) $\dfrac{x}{x+1} < \dfrac{3}{4}$ e) $\dfrac{3x-1}{x+4} \leq 3$

2. Solve each of the following equations.

   a) $\sqrt{2x-1} = 2 + \sqrt{x-4}$

   b) $\sqrt{x+1} + \sqrt{5x+1} = 6$

   c) $\sqrt{2x+1} = \sqrt{3x+4} - 1$

   d) $\sqrt{x+10} - \sqrt{x+3} = \sqrt{2x-11}$

3. Simplify each of the following.

   a) $2^{\log_2 4} + 2^{\log_2 8}$

   b) $2^{\log_2 4 + \log_2 8}$

   c) $\log_{25}\left(\dfrac{1}{\sqrt{125}}\right)$

   d) $\ln\left(e^{-7}\right)$

   e) $e^{-\ln 7}$

   f) $\log_{81} 3 \cdot \log_3 81$

   g) $5^{\log_5 M}$

   h) $25^{\log_5 M}$

   i) $5^{\log_{25} M}$

   j) $\log_{1/4}(8)$

   k) $3^{\log_3 9} + 3^{\log_3 27}$

   l) $3^{\log_3 9 + \log_3 27}$

   m) $\log_8(-64)$

   n) $e^{\ln A} + e^{\ln B}$

   o) $e^{\ln A + \ln B}$

   p) $\log_2(\sin 45°)$

   q) $\log_2\left(\sec^{10} 45°\right)$

   r) $\log_3(\tan 60°)$

   s) $\log_9\left(\dfrac{1}{\sqrt{27}}\right)$

   t) $\log_{\sqrt{2}} 32$

   u) $\log_A 1 - \log_A \sqrt{A} + \log_B \dfrac{1}{\sqrt[3]{B^2}}$

   v) $\ln\left(\dfrac{1}{\sqrt[3]{e}}\right) - 2\log_{10}(0.001) + \log_7 \dfrac{1}{49}$

   w) $2^{\log_2 x} + 2^{\log_2 y}$

   x) $2^{\log_2 x + \log_2 y}$

4. Simplify each of the following.

   a) $\log_3\left(\dfrac{1}{\sqrt{27}}\right)$ b) $2^{\log_2 8}$ c) $2^{\log_2 24}$ d) $\log_2(\cos 45°)$ e) $\log_{\sin 45°} 8$

5. Simplify each of the following.

   a) $\log_{10} 4 + \log_{10} 25$

   b) $\log_6 2 - \log_6 72$

   c) $\log_2 12 - \log_2 3$

   d) $\log_3 2 + \log_3 18 - 2\log_3 2$

   e) $\log_2 6 + \log_2 12 - 2\log_2 3$

   f) $\log_7 0.8 + \log_7 1.25$

6. Write each of the following as a single logarithm. Assume that A and B represent positive numbers.

   a) $\log_2 A + \log_2 B$

   b) $\log_2 A + 2\log_2 B$

   c) $\log_2 A - \log_2 B$

   d) $\dfrac{1}{2}\log_2 A + 2\log_2 B$

   e) $1 + \log_3 A$

   f) $2 - \log_{10} 5$

   g) $3 - \ln 8$

   h) $2 - \log_3 A$

   i) $2 - \dfrac{1}{4}\log_3 A$

7. a) Compute the area of the triangle determined by the points $A(5,2)$, $B(10,2)$, and $C(10,6)$.

   b) Compute the area of the triangle determined by the points $A(\log_2 x, \log_2 y)$, $B(\log_2 8x, \log_2 y)$, and $C(\log_2 x, \log_2 4y)$.

8. a) Suppose that α is NOT an acute angle. Find the exact value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \dfrac{1}{3}$. Rationalize the denominator in your answers.

   b) Suppose that β is an angle with $90° < \beta < 270°$. Find the value of $\sin\beta$ and $\cos\beta$ if $\tan\beta = 4$. Rationalize the denominator in your answer.

   c) Suppose that γ is an angle not in the third quadrant . Find the value of $\sin\gamma$, $\cos\gamma$, and $\tan\gamma$ if $\sec\gamma = -\dfrac{5}{2}$. Rationalize the denominator in your answer.

9. Suppose that $\tan x = -\frac{3}{2}$. Compute $\sin x \cos x$.

10. Simplify each of the following.

a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\cos\left(\frac{7\pi}{4}\right)$ c) $\tan(-3\pi)$ d) $\tan\left(\frac{\pi}{2}\right)$ e) $\sec\left(-\frac{7\pi}{3}\right)$ f) $\csc\left(\frac{3\pi}{4}\right)$

11. Simplify each of the following. (Write it in terms of $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$.

a) $\sin(\alpha + 360°)$ c) $\tan(\alpha - 180°)$ e) $\cos(-\alpha)$ g) $\sin(\alpha - 180°)$

b) $\cos(\alpha + 180°)$ d) $\sin(180° - \alpha)$ f) $\tan(-\alpha)$ h) $\tan(180° - \alpha)$

12. Consider the expression $\cos(180° - \alpha)$. All of the following expressions are equal to $\cos(180° - \alpha)$, except for one. Which one?

A) $-\cos\alpha$ B) $-\sin(90° - \alpha)$ C) $\sin(\alpha - 90°)$ D) $\cos\alpha$ E) $-\cos(-\alpha)$

13. Simplify each of the following.

a) $\log_3(\tan 60°) + \log_2(\sin 45°) - \ln(\tan 45°)$

b) $\sec 30° - \csc 45° + \cot 60°$

c) $\dfrac{\tan 120° - \tan 135°}{1 + \tan 120° \tan 135°}$

d) $\tan^2 120° - \tan^2 150°$

14. Prove that if α is any acute angle, then $\sin\alpha + \cos\alpha > 1$.

15. a) Compute the exact value of $\tan 30° + \tan 30°$

b) Compute the exact value of $\tan 60°$

c) Based on your findings, determine whether the following statement is true or false:

$$\tan(\alpha + \beta) = \tan\alpha + \tan\beta$$

16. Compute the exact value of each of the following. Rationalize denominators and simplify your answer.

a) $\cos 15°$ b) $\sin 15°$ c) $\tan 75°$

17. Prove each of the following co-function identities. For parts a) and b) , use an appropriate compund angle formula. Why is that not an option for part c)?

a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ c) $\tan\left(\frac{\pi}{2} - \alpha\right) = \cot\alpha$

18. Suppose that α and β are acute angles with $\sin\alpha = \frac{1}{3}$ and $\cos\beta = \frac{5}{13}$. Compute the exact value of each of the following.

a) $\cos\alpha$ d) $\tan\beta$ g) $\sin(\alpha + \beta)$ j) $\cos(\alpha - \beta)$

b) $\tan\alpha$ e) $\sin 2\alpha$ h) $\sin(\alpha - \beta)$ k) $\tan 2\alpha$

c) $\sin\beta$ f) $\cos 2\beta$ i) $\cos(\alpha + \beta)$ l) $\tan 2\beta$

19. Suppose that $\sin A = -\frac{3}{5}$. Compute the exact value of each of the following.

a) $\cos A$ b) $\tan A$ c) $\sin 2A$ d) $\cos 2A$

20. Suppose that $\sin B = \frac{1}{3}$ and B is not in the first quadrant. Compute the exact value of each of the following.

a) $\sec B$ b) $\tan 2B$ c) $\cos 2B$ d) $\tan\left(B - \frac{\pi}{4}\right)$

21. The expression $\dfrac{2\sin x}{\cos x - \sin x \tan x}$ is equivalent to which of the following?

A) $\tan 2x$ B) $\cot 2x$ C) $\tan x$ D) $\cot x$ E) $\sec x$

22. Compute the exact value of $\dfrac{\tan\left(\frac{4\pi}{3}\right) - \tan\left(\frac{\pi}{12}\right)}{1 + \tan\left(\frac{4\pi}{3}\right)\tan\left(\frac{\pi}{12}\right)}$. (Hint: there is an easy way and also a difficult way to do this.)

23. Simplify each of the following. Present exact values. Rationalize the denominator.

a) $\sin\left(\frac{5\pi}{3}\right) - 2\tan\left(-\frac{\pi}{6}\right) + \sin\left(\frac{7\pi}{6}\right)$ b) $\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 179^\circ + \cos 180^\circ$

24. Solve each of the following equations. You may present your answer in degrees.

a) $\sin x = -\frac{5}{2}$

b) $\cos x = -\frac{1}{\sqrt{2}}$

c) $\tan x = 1$

d) $\sin x = 0$

e) $\cos x = 1$

f) $\tan x = -\frac{1}{\sqrt{3}}$

g) $\cos\alpha = \frac{\sqrt{3}}{2}$

h) $\sin\beta = -\frac{1}{2}$

25. Solve each of the following equations.

a) $\tan^2 x + \tan x = 0$

b) $\sin^2 x = \frac{1}{2}$

c) $\sin x = 2\sin x \cos x$

d) $\sin x + 2\cos^2 x = 1$

e) $\cos x - 2\sin^2 x = 1$

26. Solve each of the following equations. Present exact values of all answers.

a) $\log_3(5x+1) = 4$ b) $3^{2x-1} = 18$ c) $\log_2(3x-2) = 4$ d) $5^{0.5x-3} = 10$

27. Solve each of the following equations. Make sure to check your solutions.

a) $\log_2(x-1) + \log_2(x-5) = 5$

b) $\log_6(3-x) + \log_6(-x-2) = 2$

c) $\log_2(x+1) + \log_2(x-1) = 3$

d) $\log_5(x-16) - \log_5(4x+20) = -2$

e) $\log_2(3x+6) - \log_2(x+7) = 3$

f*) $\log_{x-3}(x+20)\cdot\log_x(x-3) = 2$

28. Circle C_1 has a radius 5 unit long. Circle C_2 has a radius 11 unit long. The centers are at a distance of 12 units from each other. We draw the lines tangent to both circles.

a) Find an approximation of the angle formed by the two tangent lines.

b) Compute the distance between the two points of tangency on one of the common tangent lines.

29. Consider the parabola $y = 4x^2 - 3x - 8$.

a) Find both coordinates of the vertex. b) Find all $x-$intercepts. Present exact values.

30. Seattle, WA and San Francisco, CA are located approximately on the same longitude. The latitude of these cities are 47.5° N and 37.4° N. Find the distance between the two cities assuming that the Earth is a sphere with radius 3960 miles. Round your answer to the nearest mile.

31. The latitude of Seattle, WA is 47.5° N. A satellite above Seattle appears stationary in the sky. Find the speed of the satellite if it is at a height of 1200 miles above the surface of Earth. Assume that the Earth is a sphere with radius 3960 miles.

32. Find the domain for each of the following functions.

a) $f(x) = \dfrac{1}{x+2}$

b) $g(x) = \sqrt{x+2}$

c) $h(x) = \log_3(x+2)$

d) $f(x) = \dfrac{1}{x-x^2}$

e) $g(x) = \sqrt{x-x^2}$

f) $h(x) = \ln\left(x-x^2\right)$

g) $f(x) = \dfrac{2x-1}{x^2+9}$

h) $f(x) = \dfrac{3x-5}{8-\sqrt{x-1}}$

i) $f(x) = \ln\left(x^2-9\right) + \dfrac{1}{x^2-16}$

j) $f(x) = \tan x$

33. For each of the following pairs of graphs, find the coordinates of all points where they intersect.

a) $y = x + x^2 - 24$ and $y = 3x - 16$

b) $(x+3)^2 + (y+2)^2 = 10$ and $(x-1)^2 + (y-2)^2 = 10$

c) $(x+3)^2 + (y+2)^2 = 8$ and $x^2 + (y-1)^2 = 2$

34. Consider the pyramid $ABCDE$ if its base is a square $ABCD$ with sides 6 m and side $AE = BE = CE = DE = 10$ m. Compute an approximate value for the angle that is formed between a triangular face and the base.

35. Prove each of the following identities.

a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

c) $1 + \sin 2x = (\sin x + \cos x)^2$

36. Compute an approximate value for each of the angles in a triangle with sides 7 cm, 7 cm, and 10 cm.

37. One number a is 10 greater than twice another number b. Find each of the following.

a) The minimal value of $a^2 + b^2$.

b) The minimal value of ab

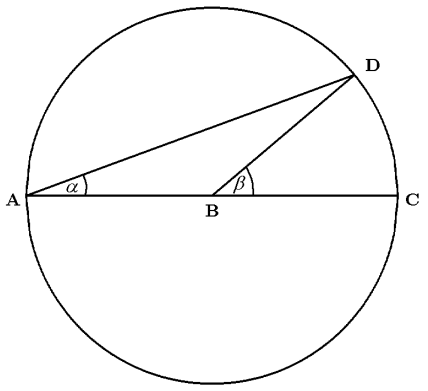
c) The maximal value of $b^2 - a^2$

38. We drew an n-sided regular polygon into a circle with radius R. In terms of R and n, express

a) the perimeter of the polygon

b) the area of the polygon.

39. Consider the picture shown. Given that B is the center of the circle, prove that $\beta = 2\alpha$.



Answers

1. a) $(-1,\infty)\cup(-\infty,-3)$ b) $\left(-\infty,\frac{1}{5}\right]\cup(1,\infty)$ c) $\left[\frac{9}{2},5\right)$ d) $(-1,3)$ e) $(-4,\infty)$

2. 5.) a) $13, 5$ b) 3 (24 doesn't work) c) $0, 4$ d) 6

3. a) 12 b) 32 c) $-\frac{3}{4}$ d) -7 e) $\frac{1}{7}$ f) 1 g) M h) M^2 i) $\sqrt{M}$ j) $-\frac{3}{2}$ k) 5
   l) 243 m) undefined n) $A+B$ o) AB p) $-\frac{1}{2}$ q) 5 r) $\frac{1}{2}$ s) $-\frac{3}{4}$ t) 10
   u) $-\frac{7}{6}$ v) $\frac{11}{3}$ w) $x+y$ x) xy

4. a) $-\frac{3}{2}$ b) 8 c) 24 d) $-\frac{1}{2}$ e) -6

5. a) 2 b) -2 c) 3 d) 2 e) 3 f) 0

6. a) $\log_2 AB$ b) $\log_2\left(AB^2\right)$ c) $\log_2\left(\frac{A}{B}\right)$ d) $\log_2\left(B^2\sqrt{A}\right)$ e) $\log_3(3A)$ f) $\log_{10} 20$
   g) $\ln\left(\frac{e^3}{8}\right)$ h) $\log_3\left(\frac{9}{A}\right)$ i) $\log_3\left(\frac{9}{\sqrt[3]{A}}\right)$

7. a) 10 unit^2 b) 3 unit^2

8. a) $\cos\alpha = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$ $\tan\alpha = -\frac{\sqrt{2}}{4}$ b) $\sin\beta = -\frac{4\sqrt{17}}{17}$ $\cos\beta = -\frac{\sqrt{17}}{17}$
   c) $\sin\gamma = \frac{\sqrt{21}}{5}$ $\cos\gamma = -\frac{2}{5}$ $\tan\gamma = -\frac{\sqrt{21}}{2}$

9. $-\frac{6}{13}$

10. a) -1 b) $\frac{\sqrt{2}}{2}$ c) 0 d) undefined e) 2 f) $\sqrt{2}$

11. a) $\sin\alpha$ b) $-\cos\alpha$ c) $\tan\alpha$ d) $\sin\alpha$ e) $\cos\alpha$ f) $-\tan\alpha$ g) $-\sin\alpha$ h) $-\tan\alpha$

12. D

13. a) 0 b) $\sqrt{3}-\sqrt{2}$ c) $2-\sqrt{3}$ d) $\frac{8}{3}$

14. Claim: if α is any acute angle, then $\sin\alpha+\cos\alpha>1$.

    Proof: Let a, b, and c be sides of a right triangle that contains α as an angle. If α is the angle opposite side a, then $\sin\alpha=\frac{a}{c}$ and $\cos\alpha=\frac{b}{c}$. To prove:

$$\begin{aligned}\sin\alpha+\cos\alpha &> 1\\ \frac{a}{c}+\frac{b}{c} &> 1\\ \frac{a+b}{c} &> \frac{c}{c}\end{aligned}$$

    Since $c>0$, we can multiply both sides by c.

$$a+b>c$$

    This is true for any triangle by the triangle inequality.

15. a) $\frac{2}{3}\sqrt{3}$ b) $\sqrt{3}$ c) false

16. a) $\frac{\sqrt{6}+\sqrt{2}}{4}$ b) $\frac{\sqrt{6}-\sqrt{2}}{4}$ c) $\sqrt{3}+2$

17. a) $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$$\sin\left(\frac{\pi}{2}-\alpha\right)=\sin\frac{\pi}{2}\cos\alpha-\cos\frac{\pi}{2}\sin\alpha=1\cdot\cos\alpha-0\cdot\sin\alpha=\cos\alpha$$

b) $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\cos\frac{\pi}{2}\cos\alpha+\sin\frac{\pi}{2}\sin\alpha=0\cdot\cos\alpha+1\cdot\sin\alpha=\sin\alpha$$

c) $\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha$ We can not use the difference formula for tangent here, because $\tan\frac{\pi}{2}$ is undefined. Instead, we need to use a compound angle formula separately for sine and cosine (see parts a and b).

$$\tan\left(\frac{\pi}{2}-\alpha\right)=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)}=\frac{\cos\alpha}{\sin\alpha}=\cot\alpha$$

18. a) $\frac{2\sqrt{2}}{3}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{12}{13}$ d) $\frac{12}{5}$ e) $\frac{4\sqrt{2}}{9}$ f) $-\frac{119}{169}$ g) $\frac{5+24\sqrt{2}}{39}$ h) $\frac{5+24\sqrt{2}}{39}$

i) $\frac{10\sqrt{2}-12}{39}$ j) $\frac{10\sqrt{2}+12}{39}$ k) $\frac{4\sqrt{2}}{7}$ l) $-\frac{120}{119}$

19. a) $\pm\frac{4}{5}$ b) $\pm\frac{3}{4}$ c) $\pm\frac{24}{25}$ d) $\frac{7}{25}$

20. a) $-\frac{3\sqrt{2}}{4}$ b) $-\frac{4\sqrt{2}}{7}$ c) $\frac{8}{9}$ d) $-\frac{1}{2}$

21. A

22. 1

23. a) $\frac{\sqrt{3}}{6}-\frac{1}{2}$ b) -1

24. a) no solution b) $\pm135^\circ+k\cdot360^\circ$ where $k\in\mathbb{Z}$ c) $45^\circ+k\cdot180^\circ$ where $k\in\mathbb{Z}$ d) $k\cdot180^\circ$ where $k\in\mathbb{Z}$
e) $k\cdot360^\circ$ where $k\in\mathbb{Z}$ f) $-30^\circ+k\cdot180^\circ$ where $k\in\mathbb{Z}$ g) $\pm30^\circ+k\cdot360^\circ$ where $k\in\mathbb{Z}$
h) $-30^\circ+k\cdot360^\circ$ and $210^\circ+k\cdot360^\circ$ where $k\in\mathbb{Z}$

25. a) $-\frac{\pi}{4}+k\pi$ and $k\pi$ where $k\in\mathbb{Z}$ b) $\frac{\pi}{4}+\frac{k\pi}{2}$ where $k\in\mathbb{Z}$ c) $k\pi$ or $\pm\frac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$
d) $x=\frac{\pi}{2}+2k\pi$ or $x=-\frac{\pi}{6}+2k\pi$ or $x=-\frac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$ e) $x=2k\pi$ where $k\in\mathbb{Z}$

26. a) 16 b) $\frac{1}{2}\left(1+\log_3 18\right)$ c) 6 d) $6+2\log_5 10$

27. a) 9 (-3 does not work) b) -6 (7 does not work) c) 3 (-3 does not work)
d) 20 e) no solution f) 5

28. a) $60°$ b) $\sqrt{108} = 6\sqrt{3}$

29. a) $\left(\frac{3}{8}, -\frac{137}{16}\right)$ b) $\left(\frac{3-\sqrt{137}}{8}, 0\right)$ and $\left(\frac{3+\sqrt{137}}{8}, 0\right)$

30. 698 miles

31. $912.644\,57\frac{\text{mi}}{\text{h}}$

32. a) $(-\infty,-2)\cup(-2,\infty)$ b) $[-2,\infty)$ c) $(-2,\infty)$ d) $(-\infty,0)\cup(0,1)\cup(1,\infty)$ e) $[0,1]$ f) $(0,1)$ g) $\mathbb{R}$ h) $[1,65)\cup(65,\infty)$ i) $(-\infty,-4)\cup(-4,-3)\cup(3,4)\cup(4,\infty)$ j) $x \neq \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

33. a) $(-2,-22)$ and $(4,-4)$ b) $(-2,1)$ and $(0,-1)$ c) $(-1,0)$

34. $71.670\,246\,2°$

35. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$

$$\begin{aligned} \text{RHS} &= \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \sec x = \text{LHS} \end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} \\ &= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS} \end{aligned}$$

c) $1 + \sin 2x = (\sin x + \cos x)^2$

$$\text{RHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x = 1 + \sin 2x = \text{LHS}$$

36. $44.415\,3°$, $44.415\,3°$, and $91.169\,4°$

37. a) 20 b) $-\frac{25}{2}$ c) $\frac{100}{3}$

38. a) $2nR\sin\left(\frac{180°}{n}\right)$ b) $nR\sin\left(\frac{180°}{n}\right)R\cos\left(\frac{180°}{n}\right) = nR^2\sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$

39. Line segments AB and BD are both radii in the circle, and so they are equal. So ABD triangle is isosceles and so the angles opposite AB and BD are also equal to each other. Thus $\measuredangle ADB = \alpha$. The third angle in triangle ADB is $180° - 2\alpha$. Angles ABD and DBC are supplementary beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ABD + \measuredangle DBC &= 180° \\ 180° - 2\alpha + \beta &= 180° && \text{subtract } 180° \\ -2\alpha + \beta &= 0 && \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$