1. Express $2^{2014} + 2^{2015} + 4^{2014}$ in terms of x if $x = 2^{2014}$.

2. Completely factor each of the following.

   a) $6m - 5n + 15mn - 2$ b) $54p^3 + 2$ c) $b^2 - a^2 + 3a^2x - 3b^2x$

3. Solve each of the following.

   a) $\dfrac{7}{2x-1} \leq -3$ b) $\dfrac{x-1}{3x+1} \leq \dfrac{1}{3}$ c) $\dfrac{x}{x+4} \geq \dfrac{2}{3}$ d) $\dfrac{x+3}{x-3} < 2$

4. We put \$2000 in a bank account with an annual compound interest rate of 7%. If we do not withdraw or add money to this account, how long do we need to wait until there is \$4000 in the account?

5. Graph each of the following functions.

   a) $f(x) = \log_2 x$
   b) $f(x) = 2^x$
   c) $f(x) = x^2 + 6x + 5$
   d) $f(x) = \dfrac{1}{x}$
   e) $f(x) = \sqrt[3]{x}$
   f) $f(x) = \sqrt{4 - x^2}$
   g) $f(x) = -|x|$
   h) $f(x) = \left(\dfrac{1}{2}\right)^x$

6. Graph the function $f(x) = \tan x$ on the interval $[-2\pi, 2\pi]$.

7. Find the exact value of each of the following expressions.

   a) $\sin\left(-\dfrac{5\pi}{2}\right) - \cos\left(\dfrac{7\pi}{3}\right) + \tan\left(\dfrac{3\pi}{4}\right) - \cos(7\pi)$
   b) $\log_{10}(0.01) + \log_8\left(\dfrac{1}{2}\right) - \ln\left(\dfrac{1}{e^5}\right)$
   c) $\cos 420^\circ - \tan 210^\circ + \sec 240^\circ + \cot 135^\circ$
   d) $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ$

8. Simplify each of the following.

   a) $\ln\left(e^{-5}\right)$
   b) $e^{-\ln 3}$
   c) $3^{\log_9 A}$
   d) $\log_m\left(m^4\right)$
   e) $\log_{27} 9$
   f) $\ln\left(-e^3\right)$
   g) $\ln\left(e^{-3}\right)$
   h) $9^{\log_3 7}$
   i) $e^{3\ln 2}$
   j) $\ln 1$
   k) $25^{\log_5 10}$
   l) $5^{\log_{25} 10}$
   m) $2^{\log_8 x}$

9. Simplify each of the following expressions.

   a) $\log_5\left(5^b\right)$
   b) $\log_5\left(125^m\right)$
   c) $\log_2\left[\left(\dfrac{1}{8}\right)^p\right]$
   d) $\log_{16}\left(2^x\right)$
   e) $3^{\log_3 4}$
   f) $9^{\log_3 5}$
   g) $3^{\log_9 2}$
   h) $2^{\log_8 10}$

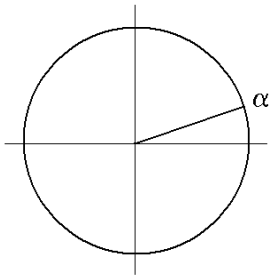
10. Simplify each of the following.

    a) $\log_{10} 2 + \log_{10} 5$
    b) $\log_6 180 - \log_6 5$
    c) $\log_2 24 - \log_2 3$
    d) $\log_{10} 40 - 2\log_{10} 2$
    e) $\log_3 18 + \log_3 24 - 4\log_3 2$
    f) $\log_5 0.4 + \log_5 2.5$

11. Write each of the following as a single logarithm. Assume that all variables represent positive numbers.

    a) $\log_{10} a + 2\log_{10} b$
    b) $\dfrac{1}{3}\log_2 a - 3\log_2 b$
    c) $1 + \log_3 2$
    d) $2 + \log_5 3$
    e) $\dfrac{1}{2} - \log_7 x$
    f) $2 + \ln 3 + \ln x - 2\ln y$

12. The picture shows an angle α on the unit circle. Draw each of the following angles in the same circle.



a) $-\alpha$ b) $\alpha - 180°$ c) $\alpha + 90°$

13. Suppose that x is an angle with $\sin x = -\frac{2}{5}$ and we also know that x is not in the third quadrant. Compute each of the following.

a) $\cos x$ c) $\sin 2x$ e) $\tan 2x$ g) $\cos 3x$

b) $\tan x$ d) $\cos 2x$ f) $\sin 3x$ h) $\cos 4x$

14. Suppose that β is an angle with $\tan 2\beta = \frac{3}{4}$. Find the exact value of $\tan \beta$.

15. Find the exact value of the tangent of the smaller angle that is formed where the lines $2x - 3y = -24$ and $x + 5y = -11$ intersect each other.

16. Simplify each of the following. (i.e. write it in terms of trigonometric functions of α.)

a) $\sin(90° - \alpha)$ d) $\sin(\alpha + 180°)$ g) $\cos(-\alpha)$ j) $\tan(180° - \alpha)$

b) $\sin(180° - \alpha)$ e) $\cos(90° - \alpha)$ h) $\cos(\alpha + 180°)$ k) $\tan(-\alpha)$

c) $\sin(-\alpha)$ f) $\cos(180° - \alpha)$ i) $\tan(90° - \alpha)$ l) $\tan(\alpha + 180°)$

17. Find all angles β so that twice β is coterminal with $120°$. Express your answer

a) in degrees b) in radians

c) Find all coterminal angles β such that $-500° < \beta < 500°$.

18. Solve each of the following equations.

a) $\cos 2x = \cos x - 1$ b) $\sin 4x = -\frac{1}{2}$ c) $\tan 5x = -1$

19. a) Solve the equation $\cos 8x = 1 - \sin 4x$ for $\sin 4x$.

b) Solve the equation $\cos 8x = 1 - \sin 4x$ for $4x$.

c) Solve the equation $\cos 8x = 1 - \sin 4x$ for x.

d) List all solutions of the equation that fall in the interval $[0, 360°)$. Present your answer in degrees.

20. Prove each of the following identities.

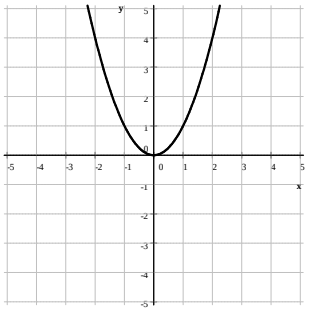
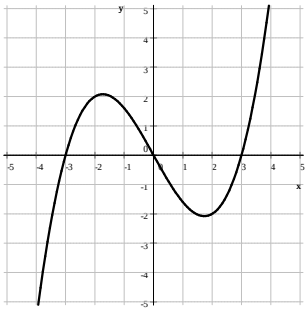
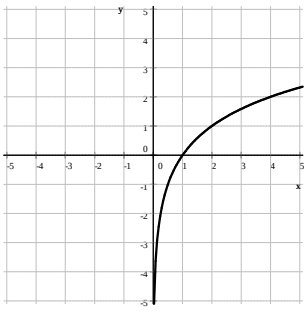
a) $\frac{2}{1 - \tan^2 x} = \frac{1 + \cos 2x}{\cos 2x}$ b) $\sin(30° + x) + \sin(30° - x) = \cos x$ c) $\frac{\tan(45° - x)}{\tan(45° + x)} = \frac{1 - \sin 2x}{1 + \sin 2x}$

21. For each of the following functions given, find an equation for the inverse.

a) $f(x) = 3x + 1$ c) $f(x) = e^{3x-1}$ e) $f(x) = \frac{2x - 3}{x + 5}$

b) $f(x) = \sqrt[3]{2x - 5}$ d) $f(x) = \log_5\left(\frac{1}{x} - 1\right)$

22. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.

23. Solve each of the following equations for x. Present the exact value of each solution. (Hint: re-write the logarithmic statements as exponential statements!)

a) $\log_5 x = 3$

b) $\log_x 16 = 2$

c) $\frac{1}{2}\log_2(3x+1) = 3$

d) $\ln(x-1) = -2$

e) $\log_3\left(2x^2-5\right) = 3$

f) $\log_{10}(x-2) = -1$

g) $\frac{2}{3}\ln(x-1) - 1 = 5$

h) $\frac{\log_2(5x+1) - 1}{3} = -1$

24. Solve each of the following equations.

a) $\log_6(x-3) + \log_6 3 + \log_6(x+1) = 2$

b) $\log_2(x-5) - \log_2(2x-14) = -2$

c) $\log_2(1-x) + \log_2(9-x) = 7$

25. Solve each of the following equations.

a) $5\cdot 2^x - 2\left(2^x - 1\right) = -2^x + 4$

b) $3^x - 3^{x+2} + 21 = 2\cdot 3^{x+1} - 7$

c) $5\cdot 2^x - 2 = 2^{x+1} - 8$

d) $4^x - 2^{x+3} = -7$

e) $25^x + 4\cdot 5^{x+1} + 64 = 0$

f) $7\cdot 2^{2x-1} = 5\cdot 3^{x+1}$

26. Find the domain of each of the following functions.

a) $f(x) = \log_3(x+7)$

b) $f(x) = \sqrt{x+7}$

c) $f(x) = \frac{1}{x+7}$

d) $f(x) = \log_5\left(x^2-7x-30\right)$

e) $f(x) = \sqrt{x^2-7x-30}$

f) $f(x) = \frac{1}{x^2-7x-30}$

g) $f(x) = \csc x$

h) $f(x) = \tan x$

i) $f(x) = \frac{x-2}{\sqrt{x-4}-3}$

27. Compare the domains of $f(x) = \log_3\left(x^2-4\right)$ and $g(x) = \log_3(x+2) + \log_3(x-2)$.

28. Graph each of the following pairs of functions together, in the same coordinate system.

a) $f(x) = x^2$ and $g(x) = (x+3)^2$

b) $f(x) = x^3$ and $g(x) = (x+3)^3$

c) $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$

d) $f(x) = 2^x$ and $g(x) = \log_2 x$

29. a) Samantha got a 4% raise. Now her monthly salary is \$2496. How much was her salary before the raise?

b) A TV set went on a 15% off sale. The sale price is \$1020. What was the original price of the TV set?

30. Write $\log_9 15 - \log_3 5$ as a single logarithm.

31*. Which is greater: $2^{\log_{10} 3}$ or $3^{\log_{10} 2}$?

Answers

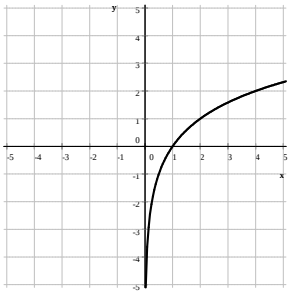
1. $x^2 + 3x$

2. a) $(3m-1)(5n+2)$ b) $2(3p+1)(9p^2-3p+1)$ c) $(a+b)(a-b)(3x-1)$

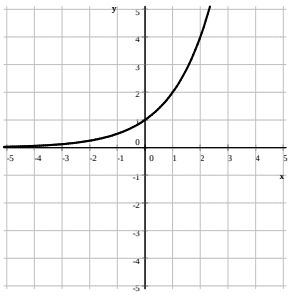
3. a) $\left[-\frac{2}{3}, \frac{1}{2}\right)$ b) $\left(-\frac{1}{3}, \infty\right)$ c) $(-\infty, -4) \cup [8, \infty)$ d) $(-\infty, 3) \cup (9, \infty)$

4. It will happen sometime during the 11th year. $\log_{1.07} 2 = \dfrac{\ln 2}{\ln 1.07} \approx 10.244\,768\,351\,058\,7$

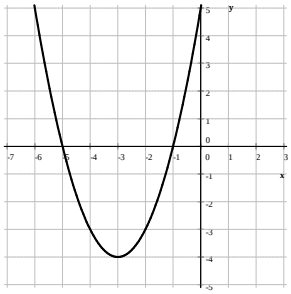
5. a) $f(x) = \log_2 x$



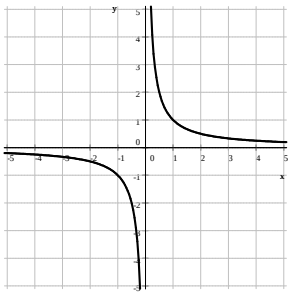
b) $f(x) = 2^x$



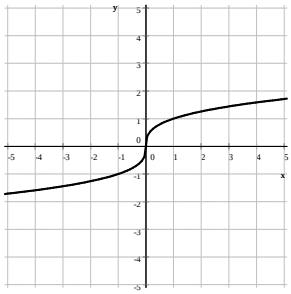
c) $f(x) = x^2 + 6x + 5$



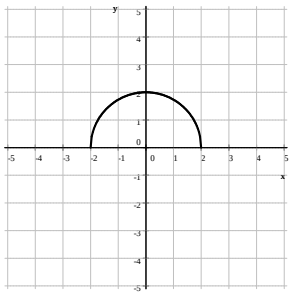
d) $f(x) = \dfrac{1}{x}$



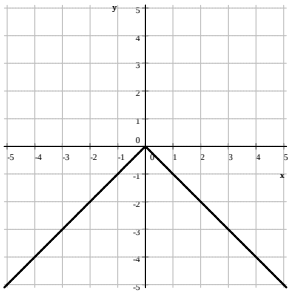
e) $f(x) = \sqrt[3]{x}$



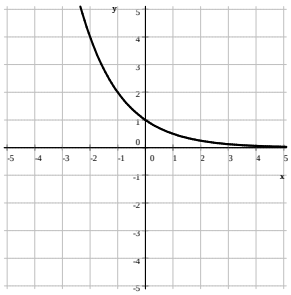
f) $f(x) = \sqrt{4 - x^2}$



g) $f(x) = -|x|$



h) $f(x) = \left(\dfrac{1}{2}\right)^x$



6. See handout

7. a) $-\frac{3}{2}$ b) $\frac{8}{3}$ c) $-\frac{\sqrt{3}}{3} - \frac{5}{2}$ d) $\frac{1}{2}$

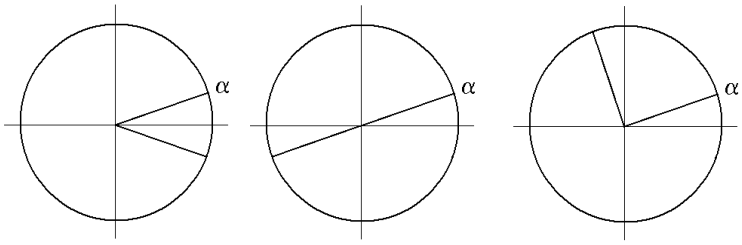
8. a) -5 b) $\frac{1}{3}$ c) $\sqrt{A}$ d) 4 e) $\frac{2}{3}$ f) undefined g) -3 h) 49 i) 8 j) 0 k) 100 l) $\sqrt{10}$ m) $\sqrt[3]{x}$

9. a) b b) $3m$ c) $-3p$ d) $\frac{1}{4}x$ e) 4 f) 25 g) $\sqrt{2}$ h) $\sqrt[3]{10}$

10. a) 1 b) 2 c) 3 d) 1 e) 3 f) 0

11. a) $\log_{10} ab^2$ b) $\log_2\left(\dfrac{\sqrt[3]{a}}{b^3}\right)$ c) $\log_3 6$ d) $\log_5 75$ e) $\log_7\left(\dfrac{\sqrt{7}}{x}\right)$ f) $\ln\left(\dfrac{3e^2x}{y^2}\right)$

12. a) b) c)



13. a) $\frac{\sqrt{21}}{5}$ b) $-\frac{2\sqrt{21}}{21}$ c) $-\frac{4\sqrt{21}}{25}$ d) $\frac{17}{25}$ e) $\frac{-4\sqrt{21}}{17}$ f) $-\frac{118}{125}$ g) $\frac{9\sqrt{21}}{125}$ h) $-\frac{47}{625}$

14. $\frac{1}{3}$ or -3

15. 1

16. a) $\cos\alpha$ b) $\sin\alpha$ c) $-\sin\alpha$ d) $-\sin\alpha$ e) $\sin\alpha$ f) $-\cos\alpha$ g) $\cos\alpha$ h) $-\cos\alpha$ i) $\cot\alpha$ j) $-\tan\alpha$ k) $-\tan\alpha$ l) $\tan\alpha$

17. a) $60° + k \cdot 180°$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$ b) $\frac{\pi}{3} + k\pi$ where $k = 0, 1, -1, 2, -2, 3, -3, ...$ c) $-480°, -300°, -120°, 60°, 240°, 420°$

18. a) $\frac{\pi}{2} + k\pi$ $\pm\frac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ b) $-\frac{\pi}{24} + \frac{k\pi}{2}$ $-\frac{5\pi}{24} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$ c) $-\frac{\pi}{20} + \frac{k\pi}{5}$ where $k \in \mathbb{Z}$

19. a) $0, \frac{1}{2}$ b) $k\pi$, $\frac{\pi}{6} + 2k\pi$, $\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ c) $\frac{k\pi}{4}$, $\frac{\pi}{24} + \frac{k\pi}{2}$, $\frac{5\pi}{24} + \frac{k\pi}{2}$ where $k \in \mathbb{Z}$ d) $0°, 7.5°, 37.5°, 45°, 52.5°, 82.5°, 90°, 97.5°, 127.5°, 135°, 142.5°, 172.5°, 180°, 187.5°, 217.5°, 225°, 232.5°, 262.5°, 270°, 277.5°, 307.5°, 315°, 322.5°, 352.5°$

20. a) see handout trig identites 4

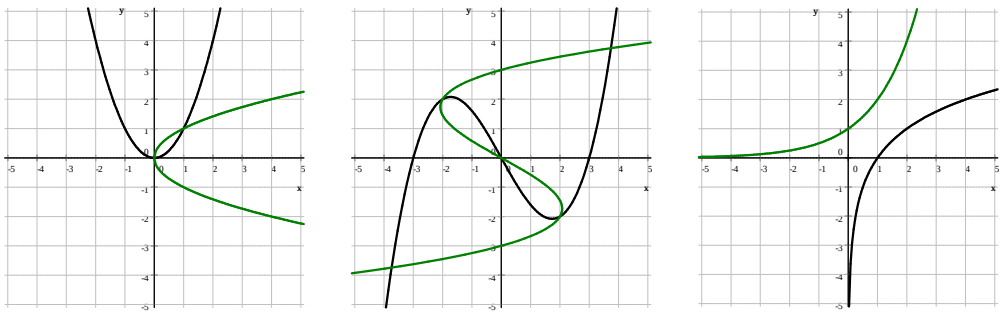
b)

$$\begin{aligned} \text{LHS} &= \sin(30° + x) + \sin(30° - x) = \sin 30° \cos x + \cos 30° \sin x + \sin 30° \cos x - \cos 30° \sin x \\ &= 2 \sin 30° \cos x = 2 \cdot \frac{1}{2} \cos x = \cos x = \text{RHS} \end{aligned}$$

c) see handout trig identites 4

21. a) $f^{-1}(x) = \frac{1}{3}x - \frac{1}{3}$ b) $f^{-1}(x) = \frac{1}{2}(x^3 + 5)$ c) $f^{-1}(x) = \frac{1}{3}(1 + \ln x)$ d) $f^{-1}(x) = \frac{1}{5^x + 1}$ e) $f^{-1}(x) = \frac{5x + 3}{-x + 2}$

22. .



23. a) 125 b) 4 c) 21 d) $\frac{1}{e^2}+1$ e) $4,-4$ f) 2.1 g) e^9+1 h) $-\frac{3}{20}$

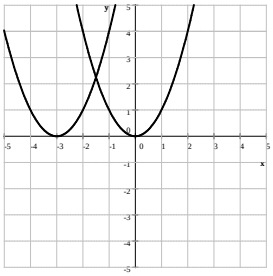
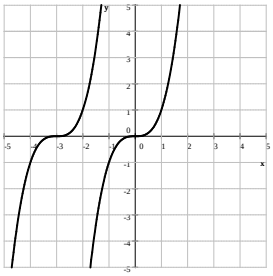
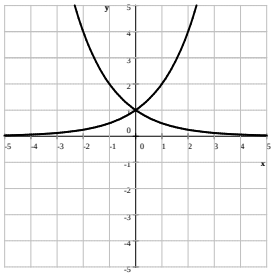
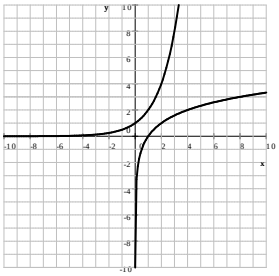
24. a) 5 (-3 does not work) b) no solution (3 does not work) c) -7 (16 does not work)

25. a) -1 b) $\log_3 2$ c) no solution d) $0, \log_2 7$ e) no solution f) $\log_{3/4}\left(\frac{7}{30}\right)$

26. a) $x > -7$ b) $x \geq -7$ c) $x \neq -7$ d) $x < -3$ or $x > 10$ e) $x \leq -3$ or $x \geq 10$
f) $x \neq -3, 10$ g) $x \neq k\pi, \quad k \in \mathbb{Z}$ h) $x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$ i) $x \geq 4$ but $x \neq 13$

27. The doomain of f is $(-\infty, -2) \cup (2, \infty)$ and the domain of g is $(2, \infty)$

28. a) b) c) d)

29. a) \$2400 b) \$1200

30. $\log_9 15 - \log_3 5 = \frac{\log_3 15}{\log_3 9} - \log_3 5 = \frac{\log_3 15}{2} - \log_3 5 = \frac{1}{2}\log_3 15 - \log_3 5 = \log_3 \sqrt{15} - \log_3 5 = \log_3\left(\frac{\sqrt{15}}{5}\right)$

31. not tellin'

Last revised: April 11, 2016