Please note that Quiz 20 will also cover topics covered on Quizzes 1-19 and Exams 1, 2, and 3. Please review those topics as well, even if they do not appear in this document.

1. Solve each of the following triangles:

   a) $b = 16\,\text{ft}$, $\alpha = 38°$, and $\beta = 83°$.

   b) $a = 4\,\text{cm}$, $b = 7\,\text{cm}$, and $\alpha = 58°$.

   c) $a = 6\,\text{in}$, $b = 4\sqrt{3}\,\text{in}$, and $\alpha = 60°$.

   d) $a = 12\,\text{m}$, $c = 15\,\text{m}$, and $\alpha = 32°$

   e) $a = 4\,\text{m}$, $c = 7\,\text{m}$ and $\gamma = 70°$

   f) $\beta = 78°$ $\gamma = 49°$ and $c = 15\,\text{ft}$

   g) $\alpha = 62°$ $a = 7\,\text{m}$ and $b = 12\,\text{m}$

   h) $\alpha = 31°$ $a = 4\,\text{cm}$ and $c = 5\,\text{cm}$

2. Find the exact value of $\log_2\left(\log_2\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}}\right)$.

3. Re-write $0.3\overline{90} = 0.390909090....$ as a ratio of two integers. You do not need to reduce the fraction.

4. Simplify each of the following.

   a) $2^{\log_2 A} - 2^{\log_2 B} + 2^{\frac{1}{3}\log_2 C}$ b) $2^{\log_2 A - \log_2 B + \frac{1}{3}\log_2 C}$ c) $\tan\left(\frac{5\pi}{12}\right)$ d) $\log_{\tan 60°}\left(2^3+1\right)$

5. Find $\tan x$ if we know that $\tan 2x = \frac{48}{55}$.

6. Solve each of the following inequalities.

   a) $\frac{2x-5}{3x+1} < -2$ b) $2x + \frac{1}{3}x^2 \le -3$ c) $\frac{3}{x-1} \le \frac{x}{2}$ d) $\frac{2}{x+1} \le \frac{3}{4}$ e) $\frac{1}{x} \ge 2$

7. Solve each of the following equations.

   a) $\sin 2x = \cos x$

   b) $\cos 2x = \sin x$

   c) $\sin 2x = \sin x$

   d) $\cos 2x = \cos x$

   e) $5 \cdot 2^{3x-2} = 7 \cdot 3^{2x-1}$

   f) $2^{x+2} - 5 = 2^{x+3} + 1$

   g) $3^{x+1} + 13 = 3^{x+2} - 17$

   h) $4^x - 2^{x+3} = -7$

   i) $\sin x - \sqrt{3}\cos x = -1$

   j) $\sin 5x - \sqrt{3}\cos 5x = -1$

8. a) Find all solutions of the equation $2 - 3\sin 4x = \cos 8x$. Present your answer in degrees.
   b) Present the final answer in radians.
   c) List all solutions that fall between 0° and 360°. Use degrees.

9. Compute the inverse for each of the following functions.

   a) $f(x) = \frac{2}{3}x - 6$

   b) $f(x) = \sqrt[3]{2x+1}$

   c) $f(x) = (2x-3)^3 + 8$

   d) $f(x) = \frac{3x-1}{7x-5}$

   e) $f(x) = \frac{7x+10}{3x-7}$

   f) $f(x) = e^{4x-1} - 3$

   g) $f(x) = \log_3(5x-4)$

10. Find the domain for each of the following functions.

    a) $f(x) = \frac{1}{\log_2 x + \log_2(x-4)}$ b) $g(x) = \sec x$ c) $f(x) = \frac{2x-3}{x^2+1}$ d) $g(x) = \sqrt{\frac{x}{x+1}}$

11. Find an equation for all tangent lines drawn to the given functions.

    a) to $f(x) = -\frac{1}{2}x^2 - 3x + 7$ from the point $(-1, 14)$

    b) to $f(x) = \frac{1}{2}x^2 + x - 1$ from the point $(-1, -6)$

12. Suppose that $f(x) = x^2 - 3x + 1$ and $g(x) = -2x + 1$. Compute each of the following.
    a) $f(g(-1))$ b) $g(f(-1))$ c) $f(2a)$ d) $g(2a)$ e) $f(g(x))$ f) $g(f(x))$
    g) Is there any value of x for which $f(g(x)) = g(f(x))$?

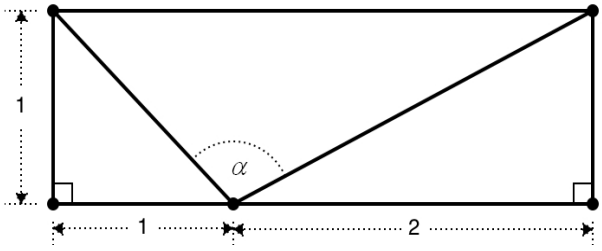
13. Graph each of the following functions on the intervals indicated.
    a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \tan^{-1} x$ c) $h(x) = \sqrt{9 - x^2}$ on its natural domain.

14. Perform the following divisions between the polynomials given.
    a) $x^4 + 5x^3 - x^2 + 10$ by $x + 2$
    b) $5x^4 - 7x^3 + 3x^2 - 9x + 8$ by $x - 1$
    c) $x^4 - 6x^3 + 3x^2 - 2x$ by $x - 5$
    e) $x^4 + x^2 - 3$ by $x^2 - x + 2$
    d) $x^4 - 3x^3 + 5x^2 - 6x + 1$ by $x^2 - 2$
    f) $x^6 - 1$ by $x^2 - 1$

15. a) Solve the equation $x^7 - 2x^6 - 7x^5 + 20x^4 - 12x^3 = 0$ if we know that $x = 2$ is a solution.
    b) Solve the equation $x^4 + 18x^3 + 85x^2 - 500 = 0$ if we know that $x = -5$ is a solution.

16. Graph each of the following.
    a) $f(x) = x^4 - 4x^2$ b) $g(x) = -x^3 + 6x$ c) $h(x) = -(x+2)^2 x^3 (x-2)^2$

17. Compute the exact value of $\tan\alpha$ if α is the angle shown on the picture.



18. Suppose that x is an angle belonging to the second quadrant and y is an angle belonging to the fourth quadrant. We also know that $\sin x = \frac{2}{3}$ and $\cos y = \frac{3}{5}$. Compute each of the following.
    a) $\cos x$ c) $\sin 2x$ e) $\sin(x - y)$ g) $\tan 2x$
    b) $\sin y$ d) $\cos 2x$ f) $\cos(x + y)$

19. Simplify each of the following.
    a) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$
    b) $\sin^{-1}(-1)$
    c) $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$
    d) $\cos^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$
    e) $\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$
    f) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$
    g) $\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$
    h) $\cos\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$
    i) $\sin\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$
    j) $\tan\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$
    k) $\sin\left(2\cos^{-1}\left(\frac{2}{5}\right)\right)$
    l) $\cos\left(2\tan^{-1}(-3)\right)$
    m) $\tan\left(2\tan^{-1}(-3)\right)$
    n) $\sin\left(\sin^{-1}\left(-\frac{3}{5}\right) + \cos^{-1}\left(-\frac{12}{13}\right)\right)$
    o) $\sin\left(\cos^{-1} x\right)$
    p) $\cos\left(\tan^{-1} x\right)$
    q) $\sin\left(\tan^{-1} x\right)$

20. Compute the exact value of each of the following.
    a) $\cos\frac{\alpha}{2}$ if we know that $\cos\alpha = -\frac{1}{3}$ b) $\sin\frac{\beta}{2}$ if we know that $\sin\beta = -\frac{4}{5}$ c) $\tan 22.5°$

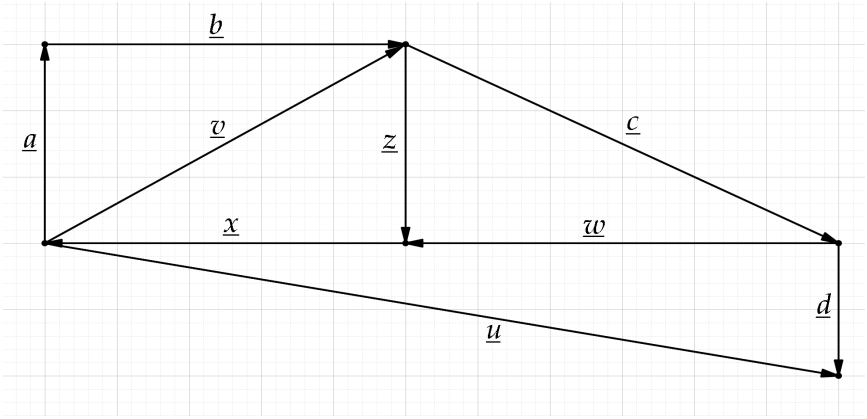
21. Point D is on side AB of triangle ABC, with $\angle ACD = \angle BCD = 60°$, $AC = 5$, and $BC = 15$. Find the length of line segment CD.

22. a) Compute $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that α is an acute angle and that $\tan\alpha = M$.

    b) Compute $\sin\beta$ and $\cos\beta$ in terms of T if we know that β is an angle in the fourth quadrant that $\tan\beta = T$.

23. Consider $f(x) = 8\sin x - 3\cos x$. What is the maximum value of f? Also, find the approximate value of the $x-$coordinate of each maximum.

24. Consider the picture given. Find each of the following in terms of vector(s) shown.



a) $\overline{a}+\overline{b}$ b) $\overline{z}-\overline{x}$ c) $\overline{d}-\overline{u}$ d) $\overline{a}+\overline{b}+\overline{c}+\overline{w}$

Answers

1. a) $\gamma = 59°$, $a \approx 9.924\,56\,\text{ft}$, $c \approx 13.817\,671\,64\,\text{ft}$ b) no solution
   c) $\beta = 90°$, $\gamma = 30°$, $c = 2\sqrt{3}\,\text{in}$
   d) $\gamma_1 \approx 41.483\,098°$ $\beta_1 \approx 106.516\,902°$ and $b_1 \approx 21.710\,535\,35\,\text{m}$
   $\gamma_2 \approx 138.516\,902°$ $\beta_2 \approx 9.483\,098°$ and $b_2 \approx 3.730\,907\,55\,\text{m}$
   e) $\alpha \approx 32.477\,421°$ $\beta \approx 77.522\,579°$ $b \approx 7.273\,302\,4\,\text{m}$
   f) $\alpha = 53°$ $b \approx 19.44087\,\text{ft}$ $a \approx 15.8730364\,\text{ft}$ g) no solution
   h) $\gamma_1 \approx 40.075\,583°$, $\beta_1 \approx 108.924\,417°$, $b_1 \approx 7.346\,62\,\text{cm}$ or
   $\gamma_2 \approx 139.924\,417°$, $\beta_2 \approx 9.075\,583°$, $b_2 \approx 1.225\,0532\,\text{cm}$ 2. -7 3. $\dfrac{387}{990}$

4. a) $A - B + \sqrt[3]{C}$ b) $\dfrac{A\sqrt[3]{C}}{B}$ c) $2+\sqrt{3}$ d) 4 5. $\dfrac{3}{8}$ or $-\dfrac{8}{3}$

6. a) $\left(-\dfrac{1}{3}, \dfrac{3}{8}\right)$ b) $\{-3\}$ c) $[-2,1)\cup[3,\infty)$ d) $(-\infty,-1)\cup\left[\dfrac{5}{3},\infty\right)$ e) $\left(0,\dfrac{1}{2}\right]$

7. a) $\dfrac{\pi}{2}+k\pi$, $\dfrac{\pi}{6}+2k\pi$, $\dfrac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$ b) $-\dfrac{\pi}{2}+2k\pi$, $\dfrac{\pi}{6}+2k\pi$, $\dfrac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$
   c) $k\pi$, $\pm\dfrac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$ d) $2k\pi$, $\pm\dfrac{2\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$ e) $\log_{9/8}\left(\dfrac{15}{28}\right)$
   f) no solution g) $\log_3 5$ h) $0, \log_2 7$ i) $-\dfrac{\pi}{2}+2k\pi$ or $\dfrac{\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$
   j) $-\dfrac{\pi}{10}+\dfrac{2k\pi}{5}$ or $\dfrac{\pi}{30}+\dfrac{2k\pi}{5}$ where $k\in\mathbb{Z}$

8. a) $x = 22.5° + k \cdot 90°$ or $x = 7.5° + k \cdot 90°$ or $x = 37.5° + k \cdot 90°$ where $k \in \mathbb{Z}$

   b) $x = \frac{\pi}{8} + \frac{k\pi}{4}$ or $x = \frac{\pi}{24} + \frac{k\pi}{4}$ or $x = \frac{5\pi}{24} + \frac{k\pi}{4}$ where $k \in \mathbb{Z}$

   c) $7.5°, 22.5°, 37.5°, 97.5°, 112.5°, 127.5°, 187.5°, 202.5°, 217.5°, 277.5°, 292.5°, 307.5°$

9. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$ c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$

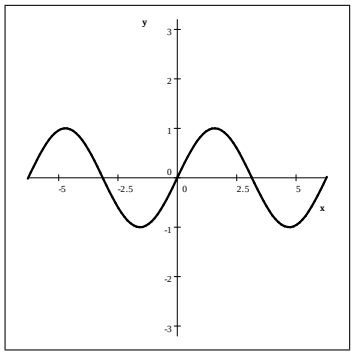
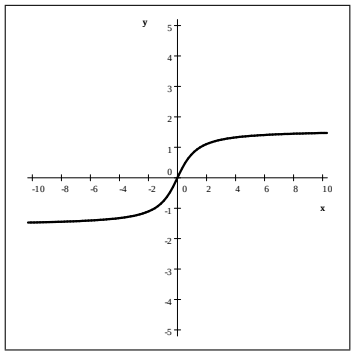
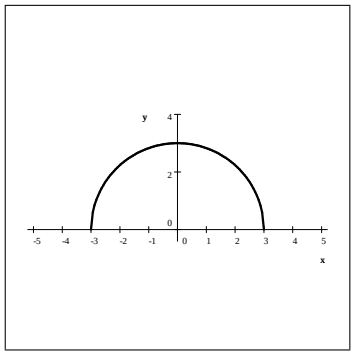
   e) $f^{-1}(x) = \frac{7x+10}{3x-7}$ f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3)+1)$ g) $f^{-1}(x) = \frac{1}{5}(3^x + 4)$

10. a) $\left\{x : x > 4 \text{ and } x \neq 2 + \sqrt{5}\right\}$ b) $\left\{x : x \neq \frac{\pi}{2} + 2k\pi \text{ where } k \in \mathbb{Z}\right\}$ c) $\mathbb{R}$ d) $(-\infty, -1) \cup [0, \infty)$

11. a) $y = -5x + 9$ and $y = x + 15$ b) $y = -3x - 9$ and $y = 3x - 3$

12. a) 1 b) -9 c) $4a^2 - 6a + 1$ d) $-4a + 1$ e) $4x^2 + 2x - 1$ f) $-2x^2 + 6x - 1$ g) 0 and $\frac{2}{3}$

13. a) $f(x) = \sin x$ on $[-2\pi, 2\pi]$ b) $g(x) = \tan^{-1} x$ c) $h(x) = \sqrt{9 - x^2}$ on $[-3, 3]$

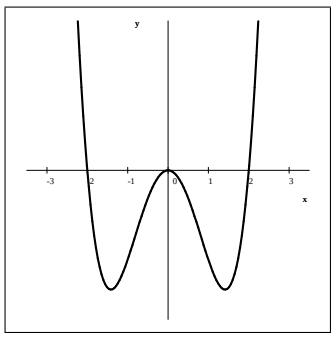
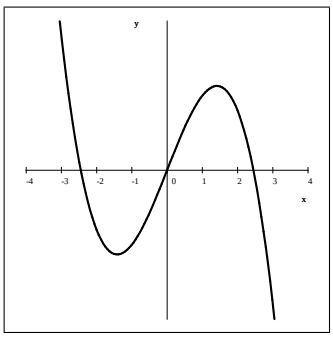
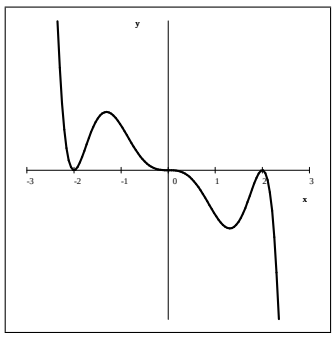




14. a) $x^3 + 3x^2 - 7x + 14$ R -18 b) $5x^3 - 2x^2 + x - 8$ c) $x^3 - x^2 - 2x - 12$ R -60

    d) $x^2 - 3x + 7$ R $-12x + 15$ e) $x^2 + x$ R $-2x - 3$ f) $x^4 + x^2 + 1$

15. a) $-3, 0, 1, 2$ b) $-10, -5, 2$

16. a) $f(x) = x^4 - 4x^2$ b) $g(x) = -x^3 + 6x$ c) $h(x) = -(x+2)^2 x^3 (x-2)^2$

17. -3 18. a) $-\frac{\sqrt{5}}{3}$ b) $-\frac{4}{5}$ c) $-\frac{4\sqrt{5}}{9}$ d) $\frac{1}{9}$ e) $\frac{6 - 4\sqrt{5}}{15}$ f) $-\frac{3\sqrt{5} - 8}{15}$ g) $4\sqrt{5}$

19. a) $-\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\sqrt{2}}{2}$ d) $\frac{\pi}{4}$ e) $\frac{1}{2}$ f) $\frac{\pi}{3}$ g) $\frac{\sqrt{5}}{3}$ h) $\frac{2\sqrt{5}}{5}$ i) $-\frac{\sqrt{5}}{5}$

    j) $-\frac{3}{4}$ k) $\frac{4\sqrt{21}}{25}$ l) $-\frac{4}{5}$ m) $\frac{3}{4}$ n) $\frac{56}{65}$ o) $\sqrt{1 - x^2}$ p) $\frac{1}{\sqrt{x^2 + 1}}$ q) $\frac{x}{\sqrt{x^2 + 1}}$

20. a) $\pm\frac{\sqrt{3}}{3}$ b) $\pm\frac{\sqrt{5}}{5}$ or $\pm\frac{2\sqrt{5}}{5}$ c) $\sqrt{2}-1$ 21. $\frac{15}{4}$

22. a) $\sin\alpha = \frac{M}{\sqrt{M^2+1}}$ and $\cos\alpha = \frac{1}{\sqrt{M^2+1}}$ b) T is negative. $\sin\beta = \frac{T}{\sqrt{T^2+1}}$ and $\cos\beta = \frac{1}{\sqrt{T^2+1}}$

23. $\sqrt{73}$ at $x = 110.556\,045\,22^\circ + k\cdot 360^\circ$ 24. a) $\overline{v}$ b) $\overline{c}$ c) $\overline{w}+\overline{x}$ d) $-\overline{x}$

Last revised: April 21, 2016