1. Graph each of the following. State the steps in graphing the functions.

   a) $f(x) = 3 - \dfrac{2}{x+4}$ b) $g(x) = -\sqrt{-x+3} + 1$ c) $h(x) = 2\log_2(x-1) - 3$

2. Graph each of the following functions by first applying a division and then transformations on the graph of $y = \dfrac{1}{x}$. List the steps in graphing each function.

   a) $f(x) = \dfrac{3x-10}{x-1}$ b) $f(x) = \dfrac{5x-1}{x+2}$ c) $f(x) = \dfrac{8x}{2x-4}$

3. Graph each of the pair of functions in the same coordinate system.

   a) $f(x) = \sin x$ and $g(x) = \csc x$ b) $f(x) = \sin x$ and $g(x) = 3\sin x$ c) $f(x) = \sin x$ and $g(x) = \sin(3x)$

   d) $f(x) = \cos x$ and $g(x) = \cos x + 1$ e) $f(x) = \cos x$ and $g(x) = \cos\left(x - \dfrac{\pi}{4}\right)$

4. Simplify each of the following.

   a) $\dfrac{\cos 35^\circ}{\sin 20^\circ \cos 35^\circ + \cos 20^\circ \sin 35^\circ}$

   b) $\log_{\sin 45^\circ} 2$

   c) $\log_{\sqrt{3}}(18x^2) - \log_{\sqrt{3}}(20x^3) + \dfrac{1}{2}\log_{\sqrt{3}}(900x^2)$

   d) $\sin\left(\dfrac{2015\pi}{6}\right)$

   e) $\dfrac{1 - \tan 75^\circ}{1 + \tan 75^\circ}$

   f) $\dfrac{\log_3 54 - \dfrac{1}{2}\log_3 108}{\log_2 12 - \dfrac{1}{2}\log_2 72}$

   g) $\cos 75^\circ \sin 75^\circ$

   h) $\left(\sin\left(\dfrac{11\pi}{4}\right) - \cos\left(\dfrac{11\pi}{4}\right)\right)^{10}$

5. If we know that $3\sin^2\alpha = 5\sin\alpha + 2$ and that $\cos\alpha < 0$, then find the exact value of $\sin 2\alpha$.

6. Find the exact value of the sum $\cos 1^\circ + \cos 2^\circ + ... + \cos 359^\circ$

7. Perform each of the following divisions.

   a) $(5x^5 - x^4 - 7x^3 + x^2 - 3x) \div (x^2 - 2)$

   b) $(x^4 - 5) \div (x + 2)$

   c) $(2x^3 - 5x^2 + 7x - 1) \div (x - 3)$

   d) $(x^7 - 1) \div (x^2 + 1)$

   e) $(x + 5) \div (x - 3)$

   f) $(2x - 1) \div (x + 1)$

   g) $(6x^4 - 15x^3 + x^2) \div (3x^2 - 1)$

   h) $x^6 \div (x - 2)$

8. a) Solve the equation $10x^5 - 426x^3 - 44x^4 - 693x^2 + x^6 = 0$ if we know that -3 is a solution of the equation.

   b) Solve the equation $48x - 44x^2 - 40x^3 + 21x^4 + 14x^5 + x^6 = 0$.

9. Solve each of the following ineqalities.

   a) $\dfrac{4x}{x+5} \le \dfrac{2}{3}$ b) $\dfrac{1}{x} \le \dfrac{6}{x-2}$ c) $x \le \dfrac{1}{x}$ d) $x^2 > 9$

10. Solve each of the following equations.

    a) $\log_2(3x+2) + \log_2(x-8) = 6$

    b) $\log_x(2x-1) = -1$

    c) $4^x - 2^{x+1} = 8$

    d) $\log_{x+1} 3 + \log_3(x+1) = \dfrac{5}{2}$

    e) $\sin 2x - \sqrt{3}\cos 2x = 1$

11. Find the smallest and greatest value of the function $f(x) = 3\sin x + 4\cos x$

12. Find the equation for all tangent lines drawn to the graph of $y = \dfrac{1}{2}x^2 - 3x - 7$ from the point $(1, -14)$.

13. Compute the exact value of each of the following.

a) $\sin\left(2\cos^{-1}\left(\frac{3}{4}\right)\right)$

b) $\sin\left(\frac{1}{2}\cos^{-1}\left(\frac{3}{4}\right)\right)$

c) $\cos\left(\tan^{-1}(-2)\right)$

d) $\tan\left(\tan^{-1}3+\tan^{-1}4\right)$

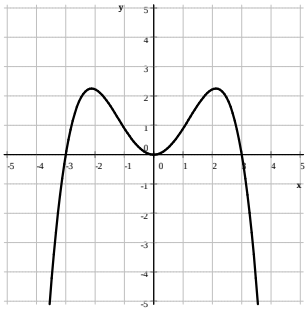
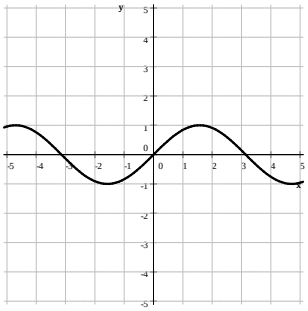
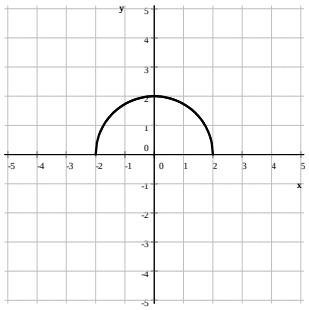
e) $\sin\left(\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)\right)$

f) $\cos\left(2\sin^{-1}x\right)$

14. Graph each of the following functions.

a) $f(x)=-2(x+2)^3x^2(x-2)(x-3)^2$ b) $f(x)=(x+2)x(x-3)$ c) $f(x)=(x+2)^2x(x-2)^3$

15. Given the same graphs as in the previous problem, graph the inverse relation in the same coordinate system.

16. Compute each of the following limits.

a) $\lim_{x\to-\infty}\left(-\frac{5}{3}x^7\right)$

b) $\lim_{x\to\infty}\left(-\frac{5}{3}x^7\right)$

c) $\lim_{x\to-\infty}\left(4.3x^6\right)$

d) $\lim_{x\to\infty}\left(4.3x^6\right)$

e) $\lim_{x\to-\infty}\left(-\frac{\sqrt{5}}{13}x^{12}\right)$

f) $\lim_{x\to\infty}\left(-\frac{\sqrt{5}}{13}x^{12}\right)$

g) $\lim_{x\to-\infty}\left(\log_3 x\right)$

h) $\lim_{x\to\infty}\left(\log_3 x\right)$

i) $\lim_{x\to-\infty}\sqrt{x-8}$

j) $\lim_{x\to\infty}\sqrt{x-8}$

k) $\lim_{x\to-\infty}\frac{5}{x^3}$

l) $\lim_{x\to\infty}\frac{5}{x^3}$

m) $\lim_{x\to-\infty}\frac{2^{3x-1}}{5^{2x+1}}$

n) $\lim_{x\to\infty}\frac{2^{3x-1}}{5^{2x+1}}$

o) $\lim_{x\to-\infty}\frac{2^{3x-1}}{5^{x-2}}$

p) $\lim_{x\to\infty}\frac{2^{3x-1}}{5^{x-2}}$

17. The number of cells in a sample at time t (measured in hours) is $N(t)=20\,000\left(1.2^{0.5t}\right)$.

a) How many cells are in the sample at $t=0$?

b) How long will it take for the sample to double from the amount that it had at $t=0$?

c) How many cells are in the sample at $t=6$?

d) How long will it take for the sample to double from the amount that it had at $t=6$?

e) What do you observe? Can we make (and perhaps prove) a general statement?

18. Classify the discontinuities of each of the following functions as a hole or a vertical asymptote.

a) $f(x)=\dfrac{x^3-x}{x^2(x-1)}$ b) $g(x)=\dfrac{(x+3)^2(x+1)^3x^4(x-2)^5(x-5)^8}{(x+3)^3(x+1)^2x^4(x-2)^8(x-5)^9}$

19. Prove the identity $\dfrac{\tan\left(\frac{\pi}{4}-x\right)}{\tan\left(\frac{\pi}{4}+x\right)}=\dfrac{1-\sin 2x}{1+\sin 2x}$

20. Find the domain for each of the following functions.

a) $f(x)=\dfrac{\ln(x+4)}{\ln(x-5)}$ b) $g(x)=\ln\left(\dfrac{x+4}{x-5}\right)$ c) $h(x)=\dfrac{1}{\sin x-\cos x}$ d) $h(x)=\dfrac{\cos^4x-\sin^4x}{\sin 2x}$

21. Compute the exact value of each of the following.

a) $\sin x$ if $\cos 2x = -\frac{2}{3}$ b) $\cos x$ if $\cos 2x = \frac{4}{5}$ c) $\cos x$ if $\sin 2x = \frac{5}{13}$ d) $\sin 2x$ if $\tan x = 2$

e) Compute the exact value of $\tan\alpha$ if we know that $90° \le \alpha \le 180°$ and $\cos 2\alpha = \frac{3}{5}$.

22. Express each of the following as a sum.

a) $\sin 5x \cos 17x$ b) $\sin 13A \sin A$ c) $\cos 3x \cos 5x$ d) $\sin 8M \sin 20M$

23. Express each of the following as a product.

a) $\sin 8x - \sin 6x$ b) $\cos 4x - \cos 12x$ c) $\sin 4x + \sin 12x$ d) $\sin 2x + \sin 4x$

24. Based on your answers for the previous problem, simplify the expression $\dfrac{\cos 4x - \cos 12x}{\sin 4x + \sin 12x}$

25. Solve each of the following triangles.

a) $a = 17$, $b = 13$, and $\beta = 20°$ b) $a = 17$, $b = 13$, and $\alpha = 20°$ c) $a = 17$, $b = 13$, and $\gamma = 20°$

26. Find the exact value of the cosine of the smallest angle in a triangle with sides 3, 5, and 6.

27. Find the **exact value** of the area of a triangle with sides 2, 3, and 4.

28. Triangle SML has sides of length 6, 7, and 8. Find the exact value of $\cos S + \cos M + \cos L$.

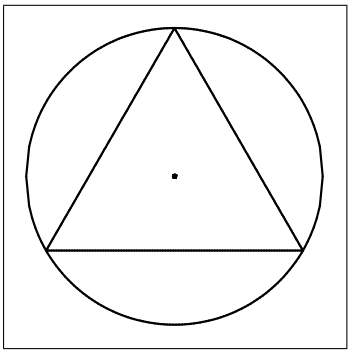
29. a) Find the exact value of $\cos\frac{\alpha}{2}$ if we know that $\cos\alpha = \frac{1}{3}$.

b) Find the exact value of $\sin\frac{\beta}{2}$ if we know that $\sin\beta = \frac{2}{3}$.

30. A triangle has sides of length a, b, and c, which are consecutive integers in increasing order, and $\cos\gamma = \frac{5}{16}$. Find $\cos\alpha$.

31. Consider an equilateral triangle with sides 1 unit long, inscribed in a circle. Let O be the center of the circle.

a) Find the radius of the circle.
b) Find the distance between O and the side of the triangle.



32. Suppose that $\underline{u} = 2\underline{i} - 7\underline{j}$ and $\underline{v} = 3\underline{i} + \underline{j}$. Compute each of the following.

a) $\underline{u}+\underline{v}$ b) $\underline{u}-\underline{v}$ c) $4\underline{u} - 5\underline{v}$ d) $\underline{u} \cdot \underline{v}$ e) $(2\underline{u} - \underline{v})(\underline{u} + 2\underline{v})$

Compute the approximate value of the angle formed between the given vectors.

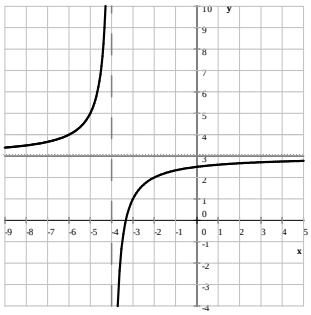
f) $\underline{u}$ and $\underline{v}$ g) $2\underline{u} - \underline{v}$ and $\underline{u} + 2\underline{v}$

33. The population of a town is growing exponentially. From 2007, it took A years for the population to double. From 2007, it took B years for the population to triple. Express B in terms of A.

Answers

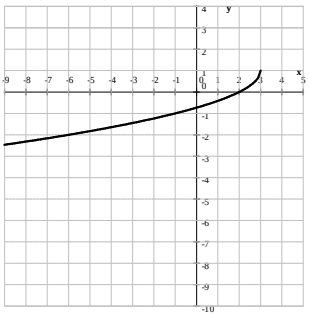
1. a) $f(x) = 3 - \dfrac{2}{x+4}$

Start with $y = \dfrac{1}{x}$
shift to the left by 4 units
stretch along by the $y-$axis by 2
reflect to the $x-$axis
shift up by 3 units



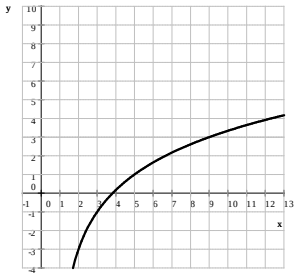
b) $g(x) = -\sqrt{-x+3} + 1$

Start with $y = \sqrt{x}$
reflect to the $y-$axis
shift to the right by 3 units
reflect to the $x-$axis
shift up by 1 unit



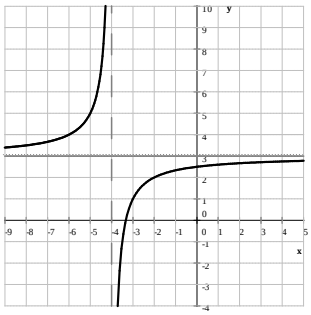
c) $h(x) = 2\log_2(x-1) - 3$

Start with $y = \log_2 x$
shift to the right by 1 unit
stretch along by the $y-$axis by 2
reflect to the $x-$axis
shift down by 3 units



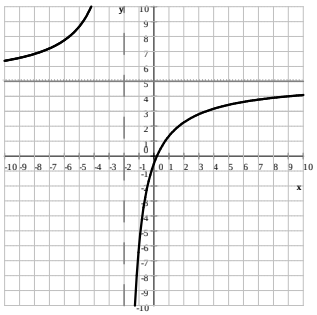
2. a) $f(x) = \dfrac{3x-10}{x-1} = -7 \cdot \dfrac{1}{x-1} + 3$

Start with $y = \dfrac{1}{x}$
shift to the right by 1 unit
stretch along by the $y-$axis by 7
reflect to the $x-$axis
shift up by 3 units



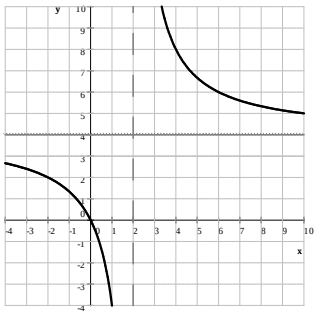
b) $f(x) = \dfrac{5x-1}{x+2} = -11 \cdot \dfrac{1}{x+2} + 5$

Start with $y = \dfrac{1}{x}$
shift to the left by 2 units
stretch along by the $y-$axis by 11
reflect to the $x-$axis
shift up by 5 units

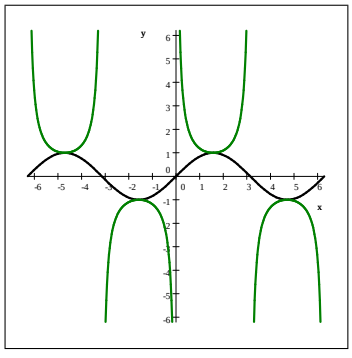


c) $f(x) = \dfrac{8x}{2x-4} = 8 \cdot \dfrac{1}{x-2} + 4$

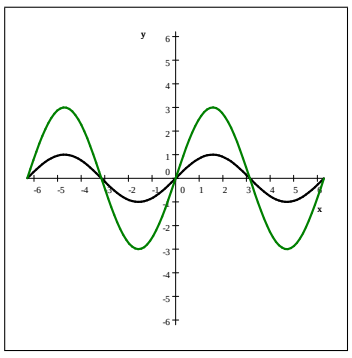
Start with $y = \dfrac{1}{x}$
shift to the right by 2 units
stretch along by the $y-$axis by 8
shift up by 4 units



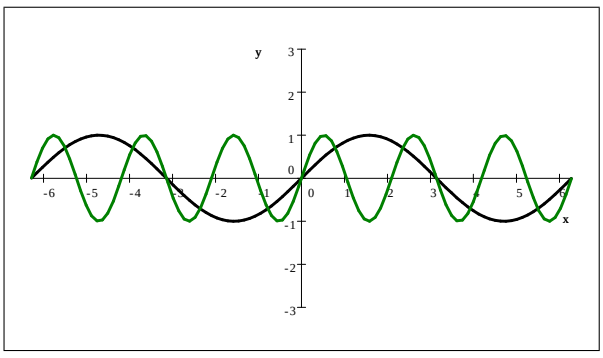
3. a) $f(x) = \sin x$ and $g(x) = \csc x$



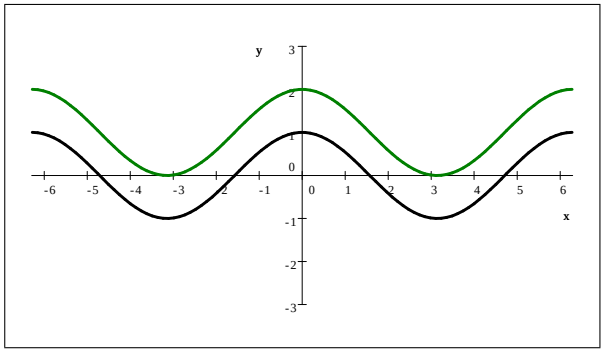
b) $f(x) = \sin x$ and $g(x) = 3\sin x$



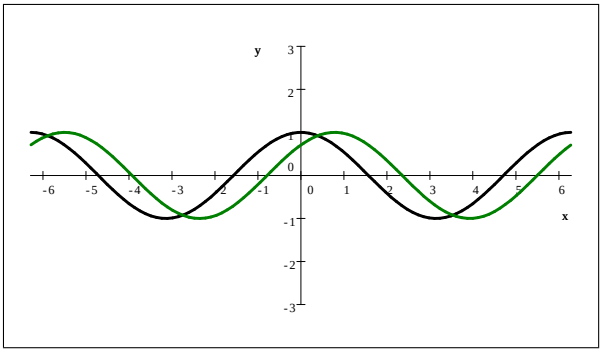
c) $f(x) = \sin x$ and $g(x) = \sin(3x)$



d) $f(x) = \cos x$ and $g(x) = \cos x + 1$



e) $f(x) = \cos x$ and $g(x) = \cos\left(x - \frac{\pi}{4}\right)$



4. a) 1 b) -2 c) 6 d) $-\frac{1}{2}$ e) $-\frac{\sqrt{3}}{3}$ f) 3 g) $\frac{1}{4}$ h) 32

5. $\frac{4}{9}\sqrt{2}$

6. -1

7. a) $5x^3 - x^2 + 3x - 1$ R $3x - 2$ b) $x^3 - 2x^2 + 4x - 8$ R 11 c) $2x^2 + x + 10$ R 29
   d) $x^5 - x^3 + x$ R $-x - 1$ e) 1 R 8 f) 2 R -3 g) $2x^2 - 5x + 1$ R $-5x + 1$
   h) $x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32$ R 64

8. a) $-11, -3, 0, 7$ b) $-12, -2, 0, 1$

9. a) $(-5, 1]$ b) $\left[-\frac{2}{5}, 0\right) \cup (2, \infty)$ c) $(0, 1] \cup (-\infty, -1]$ d) $(-\infty, -3) \cup (3, \infty)$

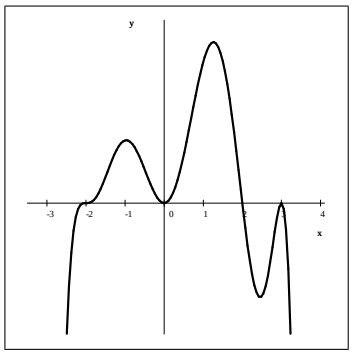
10. a) 10 b) no solution c) 2 d) $10, \sqrt{3} - 1$ e) $\frac{\pi}{4} + k\pi$, $\frac{7\pi}{12} + k\pi$ where $k \in \mathbb{Z}$

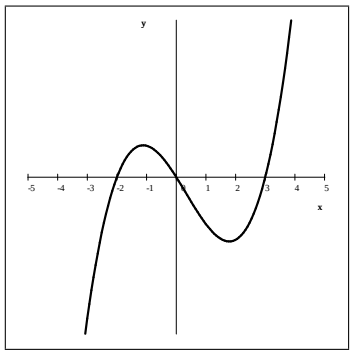
11. smallest value: -5 greatest value: 5

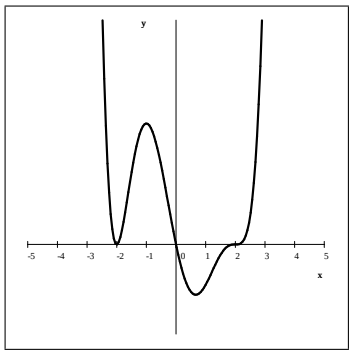
12. $y = -5x - 9$ and $y = x - 15$

13. a) $\frac{3\sqrt{7}}{8}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{\sqrt{5}}{5}$ d) $-\frac{7}{11}$ e) 1 f) $1 - 2x^2$

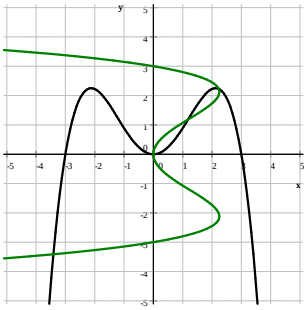
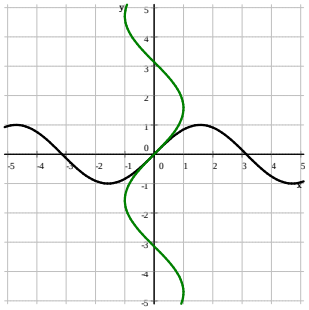
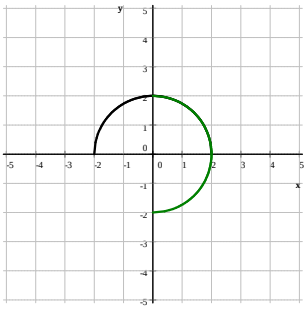
14. a) $f(x) = -2(x+2)^3 x^2 (x-2)(x-3)^2$ b) $f(x) = (x+2)x(x-3)$ c) $f(x) = (x+2)^2 x (x-2)^3$







15.

16. a) ∞ b) $-\infty$ c) ∞ d) ∞ e) $-\infty$ f) $-\infty$ g) undefined h) ∞ i) undefined
j) ∞ k) 0 l) 0 m) ∞ n) 0 o) 0 p) ∞

17. a) 20 000 b) 7. 603 568 hours c) 34 560 d) $\dfrac{\ln 2\left(1.2^3\right)}{0.5\ln 1.2} - 6 \approx 7.603\,568$ hours

e) For any time t, we will need to wait until $t + \dfrac{2\ln 2}{\ln 1.2}$ for the sample to double. So, the doubling time is independent of t and is constant. Let t_1 be any time and t_2 is the time when the amount is doubled from $A(t_1)$. In other words, $A(t_2) = 2A(t_1)$.

$$\begin{aligned}
A(t_2) &= 2A(t_1) \\
20\,000\left(1.2^{0.5t_2}\right) &= 2 \cdot 20\,000\left(1.2^{0.5t_1}\right) \\
1.2^{0.5t_2} &= 2\left(1.2^{0.5t_1}\right) \\
\frac{1.2^{0.5t_2}}{1.2^{0.5t_1}} &= 2 \\
1.2^{0.5(t_2-t_1)} &= 2 \\
0.5(t_2-t_1)\ln 1.2 &= \ln 2 \\
t_2 - t_1 &= \frac{\ln 2}{0.5\ln 1.2} = \frac{2\ln 2}{\ln 1.2} \approx 7.603\,568 \text{ hours}
\end{aligned}$$

18. a) vertical asymptote at $x = 0$ hole at $x = 1$ b) vertical asymptote at $x = -3, 2, 5$ hole at $x = -1, 0$

19. see handout Trigonometric Identities 4.

20. a) $x > 5$ and $x \neq 6$ b) $x < -4$ or $x > 5$ c) $x \neq \dfrac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$ d) $x \neq \dfrac{k\pi}{2}$ where $k \in \mathbb{Z}$

21. a) $\pm\dfrac{\sqrt{30}}{6}$ b) $\pm\dfrac{3\sqrt{10}}{10}$ c) $\pm\dfrac{\sqrt{26}}{26}, \pm\dfrac{5\sqrt{26}}{26}$ d) $\dfrac{4}{5}$ e) $-\dfrac{1}{2}$

22. a) $\dfrac{1}{2}\sin 22x - \dfrac{1}{2}\sin 12x$ b) $\dfrac{1}{2}\cos 12A - \dfrac{1}{2}\cos 14A$ c) $\dfrac{1}{2}\cos 8x + \dfrac{1}{2}\cos 2x$ d) $\dfrac{1}{2}\cos 12M - \dfrac{1}{2}\cos 28M$

23. a) $2\sin x\cos 7x$ b) $2\sin 4x\sin 8x$ c) $2\cos 4x\sin 8x$ d) $2\cos x\sin 3x$

24. $\tan 4x$

25. a) $\alpha_1 \approx 26.567\,838°$ $\gamma_1 \approx 133.432\,162°$ $c_1 \approx 27.602\,045$
$\alpha_2 \approx 153.432\,162°$ $\gamma_2 \approx 6.567\,838°$ $c_2 \approx 4.347\,503\,459$
b) $\beta \approx 15.161\,745\,79°$ $\gamma \approx 144.838\,254\,21°$ $c \approx 28.624\,256\,733\,067\,1$
c) $c = 6.531\,146$ $\alpha = 117.095\,52°$ $\beta = 42.904\,48°$

26. $\dfrac{13}{15}$

27. $\dfrac{3\sqrt{15}}{4}$

28. $\dfrac{47}{32}$

29. a) $\pm\dfrac{\sqrt{6}}{3}$ b) $\pm\dfrac{\sqrt{6}}{6}$ or $\pm\dfrac{\sqrt{30}}{6}$

30. $\dfrac{13}{20}$

31. a) $\dfrac{\sqrt{3}}{3}$ b) $\dfrac{\sqrt{3}}{6}$

32. a) $5\underline{i}-2\underline{j}$ b) $-\underline{i}-4\underline{j}$ c) $-7\underline{i}-17\underline{j}$ d) -1 e) 15 f) $92.489\,553°$ g) $74.744\,881\,3°$

33. $B=\dfrac{\ln 3}{\ln 2}A$ or $B=A\log_2 3$

Solution: Let $f(x)=c\cdot d^x$ $(d>1)$ be the exponential function expressing the population. Let x express the years passed, starting at 2007. For A, we write the equation

$$\begin{aligned} 2c &= c\cdot d^A \\ 2 &= d^A \\ \ln 2 &= A\ln d \\ \frac{\ln 2}{\ln d} &= A \end{aligned}$$

For B, we write

$$\begin{aligned} 3c &= c\cdot d^B \\ 3 &= d^B \\ \ln 3 &= B\ln d \\ \frac{\ln 3}{\ln d} &= B \end{aligned}$$

So $A=\dfrac{\ln 2}{\ln d}$ and $B=\dfrac{\ln 3}{\ln d}$. Then

$$\begin{aligned} \frac{B}{A} &= \frac{\frac{\ln 3}{\ln d}}{\frac{\ln 2}{\ln d}} = \frac{\ln 3}{\ln d}\cdot\frac{\ln d}{\ln 2} = \frac{\ln 3}{\ln 2} \\ \frac{B}{A} &= \frac{\ln 3}{\ln 2} \\ B &= A\frac{\ln 3}{\ln 2} = A\log_2 3 \end{aligned}$$