

Review Problems

1. Simplify each of the following.

a) 6^{-2} b) $32^{3/5}$ c) $64^{-2/3}$ d) $\log_9 \sqrt{27}$ e) $8^{\log_2 5}$

2. Factor $3x^2 - 4x - 319$ by completing the square.

3. In case of each polynomial given, determine (by completing the square) whether it factors or not. (You do not have to actually factor.)

a) $20x + 2x^2 + 44$ b) $20x - 5x^2 - 25$

4. A citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. Find the total number of trees the grower should plant to maximize yield.

5. The cost of manufacturing q units of a product is given by $C(q) = 6q^2 + 10q$. Suppose we can sell all q units for a total of $142q + 1674$ dollars. Find the maximum profit we can achieve.

6. Graph each of the following equations in the same coordinate system.

a) $f(x) = x + 3$, $g(x) = x - 3$, $h(x) = (x + 3)(x - 3)$, $k(x) = \frac{1}{(x + 3)(x - 3)}$

b) $f(x) = 2^x$ and $g(x) = \log_2 x$

7. Consider the parametric equation $x^2 + 2mx + 46 = 10x + 6m$. Find all values of the parameter m for which the equation has exactly one real solution for x .

8. Find each of the following limits.

a) $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^2)$ e) $\lim_{x \rightarrow -\infty} \frac{3x^2 - 1}{5x^2 - 3x + 2}$ i) $\lim_{x \rightarrow \infty} \frac{-x^3 + 2x + 1}{x - 3}$

b) $\lim_{x \rightarrow \infty} (-2x^5 + 8x^2)$ f) $\lim_{x \rightarrow -\infty} \frac{100x - 1}{5x^2 - 3x + 2}$ j) $\lim_{x \rightarrow -\infty} 2^x$

c) $\lim_{x \rightarrow -\infty} (-2x^5 + 8x^6)$ g) $\lim_{x \rightarrow -\infty} \log_2 x$ k) $\lim_{x \rightarrow \infty} 2^x$

d) $\lim_{x \rightarrow \infty} \frac{3^{x-1}}{2^{2x-3}}$ h) $\lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{3x^2 - 5x + 2}$ l) $\lim_{x \rightarrow \infty} \frac{12 + \log_7 3x}{15 + \log_7 x}$

9. Let $f(x) = \frac{x^2 - 2x - 8}{x^2 - 4}$. Find each of the following limits.

a) $\lim_{x \rightarrow -\infty} f(x)$ c) $\lim_{x \rightarrow -2^-} f(x)$ f) $\lim_{x \rightarrow 0^-} f(x)$ i) $\lim_{x \rightarrow 2^-} f(x)$

b) $\lim_{x \rightarrow \infty} f(x)$ d) $\lim_{x \rightarrow -2^+} f(x)$ g) $\lim_{x \rightarrow 0^+} f(x)$ j) $\lim_{x \rightarrow 2^+} f(x)$

e) $\lim_{x \rightarrow -2} f(x)$ h) $\lim_{x \rightarrow 0} f(x)$ k) $\lim_{x \rightarrow 2} f(x)$

10. Find each of the following limits.

a) $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$ b) $\lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h}$ c) $\lim_{x \rightarrow 0} \frac{|x|}{x}$ d) $\lim_{x \rightarrow 5} \frac{-3x}{(x - 5)^2}$

11. Define $f(x)$ as follows:

$$f(x) = \begin{cases} 3x + m & \text{if } x < 5 \\ -x^2 + 4x & \text{if } x \geq 5 \end{cases}$$

Find the value of m so that f is continuous on \mathbb{R} .

12. Find the derivative of each function given, and prove your result using the definition of the derivative as a limit.

a) $f(x) = \frac{1}{x}$ b) $f(x) = \sqrt{x}$ c) $f(x) = x^3$

13. Differentiate each of the following.

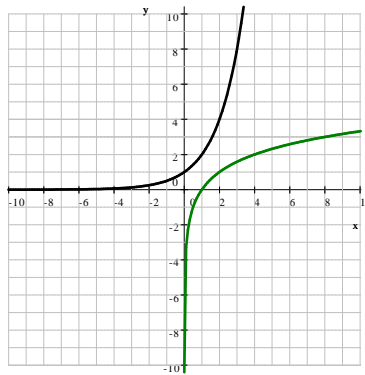
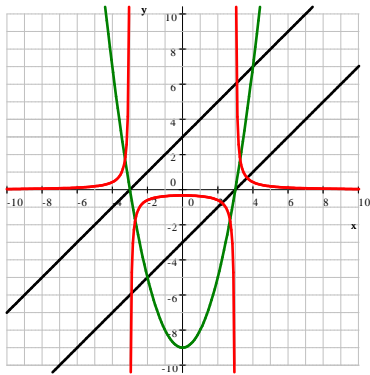
a) $f(x) = x^2 - 5x + 2 - \frac{1}{x}$ b) $f(x) = 3x - \cos 30^\circ$ c) $f(x) = \sqrt[3]{x^5} - e^2$

14. An object is moving along a vertical line. (Upward is positive.) The location function of the object is given by $L(t) = -3t^3 + 5t^2 - 8t + 100$. In what direction is the object moving at $t = 4$?

15. An object is moving along a vertical line. (Upward is positive.) The location function of the object is given by $L(t) = t^3 - 18t^2 + 60t - 120$. Find all values of t for which the object is moving upward.

Review Problems - Answers

1. a) $\frac{1}{36}$ b) 8 c) $\frac{1}{16}$ d) $\frac{3}{4}$ e) 125
2. $3\left(x + \frac{29}{3}\right)(x - 11)$ or $(3x + 29)(x - 11)$ see Completing the square - Part 3
3. a) factors b) does not factor
4. If we plant 80 trees, then we will obtain a maximal yield of 25600 oranges. (see Optimization 1)
5. \$2400 (Optimization 1)
6. a) $f(x)$ and $g(x)$ are in black,
 $h(x)$ is the green graph and
 $k(x)$ is the red graph



7. 7, -3
8. a) ∞ b) $-\infty$ c) ∞ d) 0 e) $\frac{3}{5}$ f) 0
 g) undefined since the function is not defined on negative numbers
 h) $\frac{2}{3}$ i) $-\infty$ j) 0 k) ∞ l) 1
9. a) 1 b) 1 c) $\frac{3}{2}$ d) $\frac{3}{2}$ e) $\frac{3}{2}$ f) 2 g) 2 h) 2 i) ∞ j) $-\infty$ k) undefined
10. a) $-\frac{1}{4}$ b) $\frac{1}{6}$ c) undefined d) $-\infty$
11. -20
12. a) $f'(x) = -\frac{1}{x^2}$ b) $f'(x) = \frac{1}{2\sqrt{x}}$ c) $f'(x) = 3x^2$ (see handout)
13. a) $f'(x) = 2x - 5x + \frac{1}{x^2}$ b) $f'(x) = 3$ c) $f'(x) = \frac{5}{3}x^{2/3}$
14. downward, $v(4) = L'(4) = -112$
15. $t < 2$ or $t > 10$