Exam 2 will cover the following topics. All topics covered by Quizzes 1-4 and Exam 1. These topics include quadratic inequalities, functions and their graphs, inverse functions, exponents and logarithms, limits, differentiation by using the definition, the sum rule, the constant multiplier rule, the generalized power rule, the product rule, chain rule and quotient rule, derivative of trigonometric functions, and applications of the derivative: increasing and decreasing functions, relative and absolute extrema, optimization problems, tangent lines, and antiderivatives. These include all handouts and the following sections from the book: Chapter R: all, Chapter 1: all except 1.6, Chapter 2: 2.1, 2.2, 2.3 (fractional powers only from 2.3), 2.5 Chapter 3: 3.1, 3.2, 3.5, Chapter 5: 5.1, 5.2, Chapter 6: 6.1, 6.2

Review Problems

1. Simplify each of the following.

   a) $\log_8\left(\frac{1}{16}\right)$ b) $\log_{27}\left(\frac{1}{9^a}\right)$ c) $4^{\log_2 7}$ d) $2^{-\log_8 27}$

2. Find the domain of each of the following functions.

   a) $f(x) = \log_3\left(12x - x^2 - 32\right)$ b) $f(x) = \frac{1}{x^2-2} - \log_2\left(x^2-9\right)$ c) $f(x) = \frac{1}{\sqrt{x^2-6x+13}}$

3. Graph the functions f and g in the same coordinate system if

   a) $f(x) = 2^x$, $g(x) = \left(\frac{1}{2}\right)^x$ b) $f(x) = \log_3 x$, $g(x) = \log_{(1/3)} x$ c) $f(x) = 2^x$, $g(x) = \log_2 x$

4. Solve the equation $x^2 + 7 = 6x$. Present exact values for solution(s) and check your solution(s).

5. Find the inverse for each of the following functions.

   a) $f(x) = x^3 - 5$ b) $f(x) = \frac{2x+8}{x-9}$ c) $f(x) = \log_3(x+2)$

6. Let $f(x) = \frac{6x + x^2 - 7}{36x + 3x^2 + 105}$. Find each of the following limits.

   a) $\lim_{x\to-\infty} f(x)$ c) $\lim_{x\to-10^-} f(x)$ f) $\lim_{x\to-7^-} f(x)$ i) $\lim_{x\to-5^-} f(x)$

   b) $\lim_{x\to\infty} f(x)$ d) $\lim_{x\to-10^+} f(x)$ g) $\lim_{x\to-7^+} f(x)$ j) $\lim_{x\to-5^+} f(x)$

   e) $\lim_{x\to-10} f(x)$ h) $\lim_{x\to-7} f(x)$ k) $\lim_{x\to-5} f(x)$

7. Based on your answers to the previous problem, graph $f(x) = \frac{6x + x^2 - 7}{36x + 3x^2 + 105}$.

8. Graph each of the following functions.

   a) $f(x) = \frac{x-3}{x-3}$ c) $f(x) = \frac{(x-3)^7}{(x-3)^5}$ e) $f(x) = \frac{(x-3)^2}{(x-3)^4}$

   b) $f(x) = \frac{(x-3)^4}{(x-3)}$ d) $f(x) = \frac{(x-3)}{(x-3)^4}$

9. Graph each of the following.

   a) $y=(x+4)(x+1)^2(x-3)$ c) $y=\dfrac{(x-3)(x+1)}{(x+4)(x+1)}$ e) $y=\dfrac{1}{(x+4)(x+1)^2(x-3)}$

   b) $y=\dfrac{(x+4)(x+1)(x-3)}{(x+1)}$ d) $y=\dfrac{(x+4)(x-3)}{(x+1)^2}$

10. Differentiate each of the following, using the definition of the derivative.

    a) $f(x)=\dfrac{1}{x-2}$ b) $f(x)=\sqrt{2x}$

11. Differentiate each of the following functions.

    a) $f(x)=\dfrac{5x^3-1}{x^3}$ d) $f(x)=17x-24\sqrt{x}$ g) $f(x)=(2x-7)^{15}$

    b) $f(x)=\sin\left(2x^5\right)$ e) $f(x)=\tan x$ h) $f(x)=\dfrac{1}{\cos(2x-\pi)}$

    c) $f(x)=\sqrt[3]{x^5}-\dfrac{2}{x}+e^5$ f) $f(x)=\dfrac{2x^2-5x+8}{x^3}$ i) $f(x)=\sqrt{1-x^2}$

12. Evaluate each of the following indefinite integrals.

    a) $\displaystyle\int 10x-3\,dx$ c) $\displaystyle\int \sqrt[3]{x^5}-\sqrt[5]{x^3}\,dx$ e) $\displaystyle\int 3abm-2a+1\,da$

    b) $\displaystyle\int 3\sin x-\cos x\,dx$ d) $\displaystyle\int mx+b\,dx$ f) $\displaystyle\int 3abm-2a+1\,dm$

13. Find an equation of the tangent line drawn to the graph of $f(x)=x^3-7x+6$ at $x=-1$.

14. Let $f(x)=16x+\dfrac{1}{x}$. Find the equation of all tangent lines drawn to the graph of f that are perpendicular to the line $x+12y=-5$.

15. We know the following things about a function f. $f'(x)=20x^3-3$ and $f(-1)=16$. Find f.

16. Prove that the function $f(x)=3x^5-50x^3+390x-1200$ is one-to-one.

17. Consider the function $f(x)=(138-6x)\left(x^2-7x-8\right)$ on the interval $[-5,24]$. Discuss the following: domain, range, intercepts, relative and absolute minimums and maximums. Sketch f, f', and f'' in the same coordinate system.

18. A closed box with a square base is to have a volume of 250 cubic meters. The material for the top and bottom of the box costs \$ 2 per square meter, and the material for the sides costs \$ 1 per square meter. Can the box be constructed for less than \$ 300?

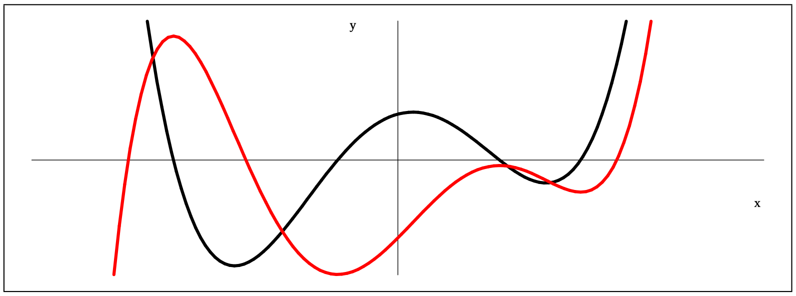
19. A rectangular box, open at the top, is to be constructed from a rectangular sheet of cardboard 50 centimeters by 80 centimeters by cutting out equal squares in the corners and folding up the sides. What sides squares should be cut out for the container to have maximal volume?

20. Find a third degree polynomial $P(x)$ such that $P(0)=-5$, $P'(0)=3$, $P''(0)=-6$ and $P'''(0)=60$.

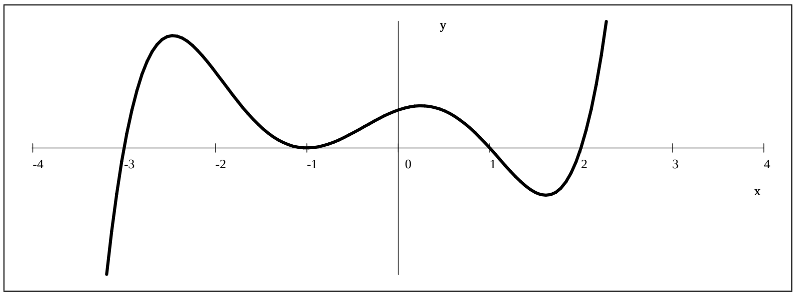
21. Find the $x-$coordinate of all relative maximums and minimums of each of the functions given below.

    a) $f(x)=(1-x)^3(x-5)^8$ b) $f(x)=\dfrac{(2x-7)^5}{x^2}$ c) $f(x)=x-\sin 2x$

22. The graph below shows a function f and its first derivative, f'. Which is which?



23. The graph below shows f', the first derivative of a function f.



a) Find all values of x for which the function f has a local maximum at x.

b) Find all values of x for which the function f has a local minimum at x.

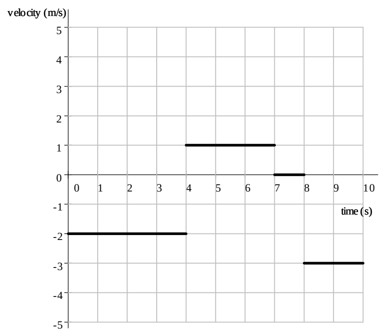
24. We shoot a small object upward, from the top of a tower. The acceleration function of the object is $a(t) = -10$. (Location is measured in meters, velocity in $\frac{\text{m}}{\text{s}}$, acceleration in $\frac{\text{m}}{\text{s}^2}$.)

a) Given that $v(0) = 160$, find $v(t)$, the velocity function of the object.

b) Given that $h(0) = 525$, find $h(t)$, the location function of the object.

c) Find the maximum height that the object reaches.

25. The picture below shows the velocity function, $v(t)$ of an object. (Time is measured in seconds, distance in meters, velocity in $\frac{\text{m}}{\text{s}}$. Positive direction is upward.).



a) Suppose that the object starts at a height of 5 m. Graph its location function.

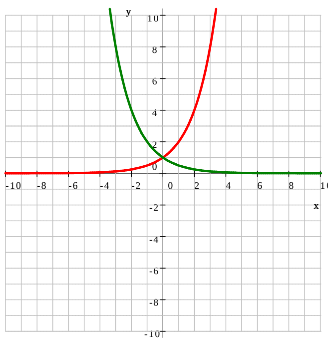
b) Suppose that the object starts at a height of 9 m. Graph its location function.

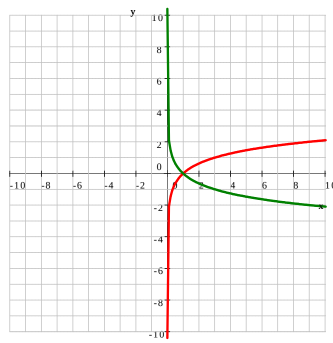
Review Problems - Answers

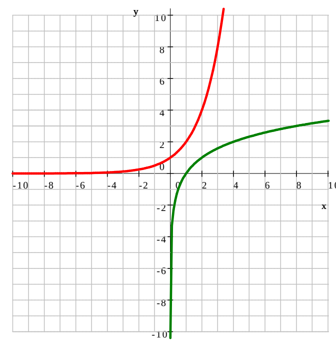
1.) a) $-\frac{4}{3}$ b) $\frac{-2a}{3}$ c) 49 d) $\frac{1}{3}$

2.) a) $(4,8)$ b) $(-\infty,-3)\cup(3,\infty)$ c) $\mathbb{R}$

3.) a) b) c)





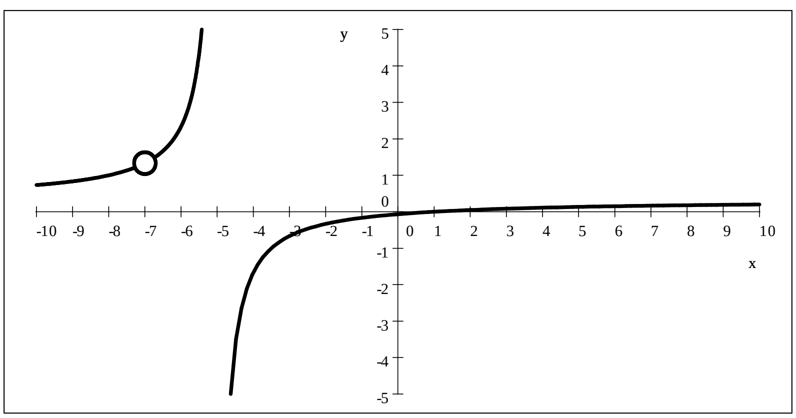


4.) $3+\sqrt{2}$ and $3-\sqrt{2}$

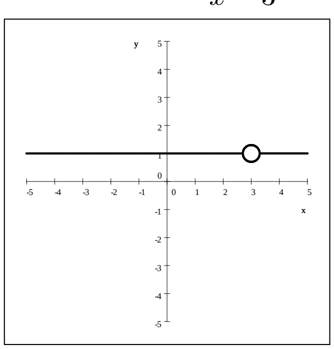
5.) a) $f^{-1}(x)=\sqrt[3]{x+5}$ b) $f^{-1}(x)=\frac{9x+8}{x-2}$ c) $f^{-1}(x)=3^{x}-2$

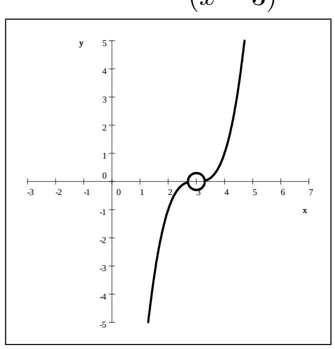
6.) a) $\frac{1}{3}$ b) $\frac{1}{3}$ c) $\frac{11}{15}$ d) $\frac{11}{15}$ e) $\frac{11}{15}$ f) $\frac{4}{3}$ g) $\frac{4}{3}$ h) $\frac{4}{3}$ i) ∞ j) $-\infty$ k) undefined

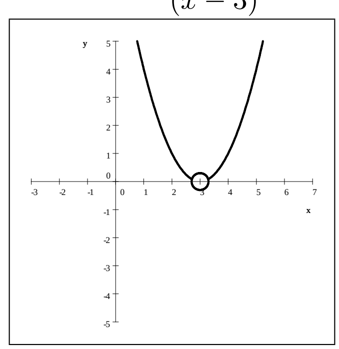
7.) $$f(x)=\frac{6x+x^2-7}{36x+3x^2+105}=\frac{(x+7)(x-1)}{3(x+5)(x+7)}=\begin{cases}\frac{x-1}{3(x+5)} & \text{if } x\neq -7\\ \text{undefined} & \text{if } x=-7\end{cases}$$

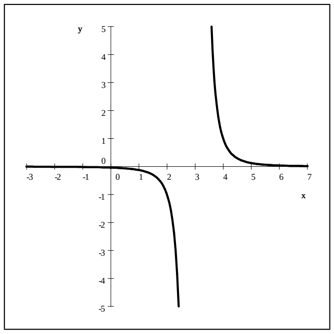


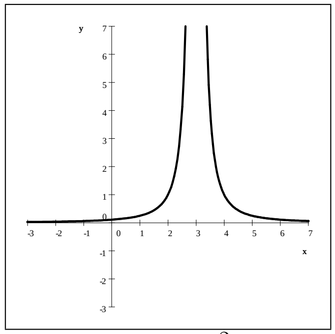
8.) a) $f(x)=\frac{x-3}{x-3}$ b) $f(x)=\frac{(x-3)^4}{(x-3)}$ c) $f(x)=\frac{(x-3)^7}{(x-3)^5}$





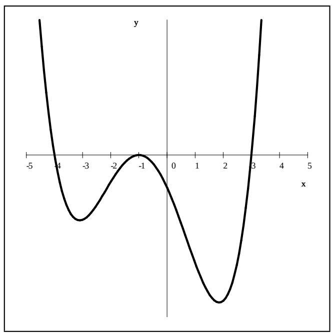


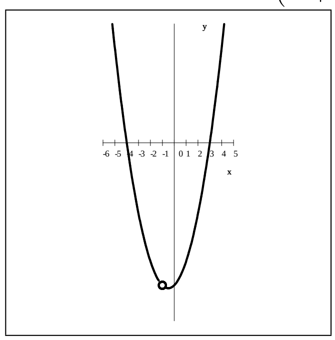


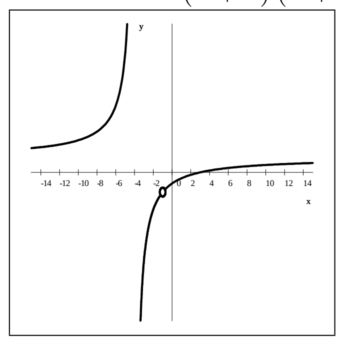


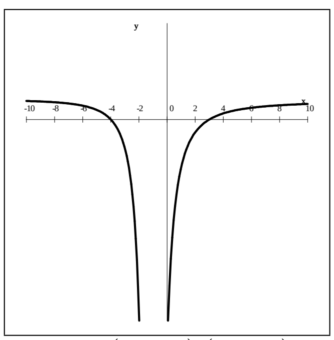
d) $f(x) = \dfrac{(x-3)}{(x-3)^4}$ e) $f(x) = \dfrac{(x-3)^2}{(x-3)^4}$

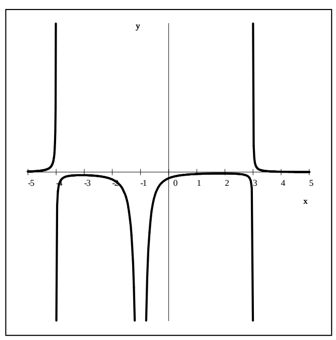
9.) a) $y = (x+4)(x+1)^2(x-3)$ b) $y = \dfrac{(x+4)(x+1)(x-3)}{(x+1)}$ c) $y = \dfrac{(x-3)(x+1)}{(x+4)(x+1)}$











d) $y = \dfrac{(x+4)(x-3)}{(x+1)^2}$ e) $y = \dfrac{1}{(x+4)(x+1)^2(x-3)}$

10.) a) $f(x) = \dfrac{1}{x-2}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\dfrac{1}{x+h-2} - \dfrac{1}{x-2}}{h} = \lim_{h\to 0} \frac{\dfrac{x-2-(x+h-2)}{(x+h-2)(x-2)}}{h} = \lim_{h\to 0} \frac{1}{h} \cdot \frac{x-2-x-h+2}{(x+h-2)(x-2)} \\ &= \lim_{h\to 0} \frac{-h}{h(x+h-2)(x-2)} = \lim_{h\to 0} \frac{-1}{(x+h-2)(x-2)} = -\frac{1}{(x-2)^2} \end{aligned}$$

b) $f(x) = \sqrt{2x}$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} = \lim_{h\to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}} \\ &= \lim_{h\to 0} \frac{\left(\sqrt{2x+2h}\right)^2 - \left(\sqrt{2x}\right)^2}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)} = \lim_{h\to 0} \frac{2x+2h-2x}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)} = \lim_{h\to 0} \frac{2\not h}{\not h\left(\sqrt{2x+2h} + \sqrt{2x}\right)} \\ &= \frac{2}{\sqrt{2x} + \sqrt{2x}} = \frac{1}{\sqrt{2x}} \end{aligned}$$

11.) a) $f'(x) = \dfrac{3}{x^4}$ b) $f'(x) = 10x^4 \cos 2x^5$ c) $f'(x) = \dfrac{5}{3}\sqrt[3]{x^2} + \dfrac{2}{x^2} = \dfrac{5}{3}x^{2/3} + 2x^{-2}$

d) $f'(x) = 17 - \dfrac{12}{\sqrt{x}}$ e) $f'(x) = \dfrac{1}{\cos^2 x} = \tan^2 x + 1$ f) $f'(x) = -\dfrac{2}{x^2} + \dfrac{10}{x^3} - \dfrac{24}{x^4}$

g) $f'(x) = 30(2x-7)^{14}$ h) $f'(x) = -\dfrac{2\sin 2x}{\cos^2 2x}$ i) $f'(x) = -\dfrac{x}{\sqrt{1-x^2}}$

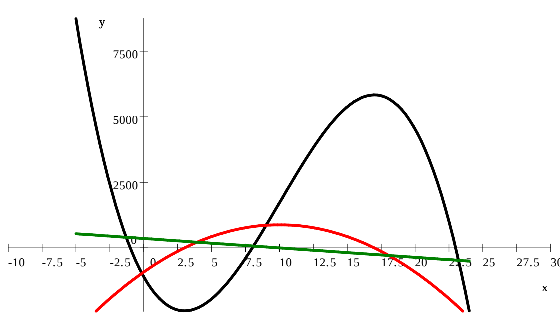
12.) a) $5x^2 - 3x + C$ b) $-3\cos x - \sin x + C$ c) $\dfrac{3}{8}x^{8/3} - \dfrac{5}{8}x^{8/5} + C$

d) $\dfrac{1}{2}mx^2 + bx + C$ e) $\dfrac{3}{2}a^2bm - a^2 + a + C$ f) $\dfrac{3}{2}abm^2 - 2am + m + C$

13.) $y = -4x + 8$ 14.) $y = 12x + 4$ and $y = 12x - 4$ 15.) $f(x) = 5x^4 - 3x + 8$

16.) $f'(x) = 15\left(\left(x^2 - 5\right)^2 + 1\right)$ is always positive, hence f is always increasing. Thus one-to-one.

17.) domain: $[-5, 24]$, range: $[-2400, 5832]$
$y-$intercept: $(0, -1104)$, $x-$intercepts: $(8, 0)$, $(-1, 0)$, $(23, 0)$
increasing on $[3, 17]$, decreasing on $[-5, 3]$ and $[17, 24]$
relative minimum: $(3, -2400)$ absolute minimums: $(3, -2400)$ and $(24, -2400)$
relative maximum: $(17, 5832)$ absolute maximum: $(-5, 8736)$



18.) no, 5 by 5 by 10 costs exactly \$300 19.) 10 cm by 10 cm

20.) $P(x) = 10x^3 - 3x^2 + 3x - 5$

21.) a) relative maximum at $x = 5$ relative minimum at $x = \dfrac{23}{11}$

b) relative maximum at $x = -\dfrac{7}{3}$ no relative minimum

c) relative maximum at $x = -\dfrac{\pi}{6} + k\pi$, relative minimum at $x = \dfrac{\pi}{6} + k\pi$ where $k \in \mathbb{Z}$

22.) the red graph is f, the black graph is f' 23.) a) 1 b) $-3, 2$

24.) a) $v(t) = -10t + 160$ b) $h(t) = -5t^2 + 160t + 525$ c) $h_{\max} = 1805\,\mathrm{m}$

25.) a) red graph b) green graph