Exam 3 will cover the following topics. All topics covered by Quizzes 1-7, Exams 1,2, and Problem Sets 1, 2, 3, and 4. These topics include quadratic inequalities, functions and their graphs, inverse functions, exponents and logarithms, limits, differentiation by using the definition, the sum rule, the constant multiplier rule, the generalized power rule, the product rule, chain rule and quotient rule, derivative of trigonometric functions, and applications of the derivative: increasing and decreasing functions, relative and absolute extrema, optimization problems, tangent lines, and antiderivatives. These include all handouts and the following sections from the book: Chapter R: all, Chapter 1: all except 1.6, Chapter 2: all, Chapter 3: 3.1, 3.2, 3.3, 3.4, 3.5, Chapter 4: 4.1, 4.2, 4.3, 4.4, 4.5 Chapter 5: 5.1, 5.2, 5.3, 5.4, Chapter 6: all

Review Problems

1. Consider $f(x) = \dfrac{x+4}{3x-5}$. Find the equation for the inverse of f, $f^{-1}(x)$.

2. Prove that $f(x) = x^8 + 3x^4 - 1$ has at least two distinct zeros.

3. Find each of the following limits.

   a) $\displaystyle\lim_{x\to 1+} \frac{x^2-1}{x^2-3x+2}$ c) $\displaystyle\lim_{x\to\infty} \frac{2x^2+3x+1}{3x^2-5x+2}$ e) $\displaystyle\lim_{x\to 3} \frac{x^2-3x}{x^2-9}$

   b) $\displaystyle\lim_{x\to 2+} \frac{x^2-1}{x^2-3x+2}$ d) $\displaystyle\lim_{x\to\infty} \frac{-x^3+2x+1}{x-3}$ f) $\displaystyle\lim_{x\to 3+} \frac{x^2+3x}{x^2-9}$

4. Consider $g(x) = \dfrac{-(x+2)(x+1)^2(x-2)}{(x+1)x^2(x-1)(x-2)}$ and $f(x) = -(x+2)(x+1)^3 x^2 (x-1)(x-2)^2$.

   (a) Graph $y = f(x)$. (Notice that f was created by multiplying all factors of numerator and denominator in $g(x)$.)

   (b) Use your graph from part a) to solve the inequality $f(x) < 0$

   (c) Use your solution for part b) to solve the inequality $g(x) \le 0$.

   (d) Graph $y = g(x)$.

5. Find the derivative of each of the following functions.

   a) $f(x) = 7^x - x^7 + \sqrt{7x} - e^7$ d) $f(x) = \dfrac{8x^2-20x+1}{4x-10}$ g) $f(x) = \arctan(3x)$

   b) $f(x) = \dfrac{e^x - e^{-x}}{2}$ e) $f(x) = \log_x(x^2+1)$ h) $f(x) = \tan(3x)$

   c) $f(x) = \dfrac{1}{4}xe^{4x} - \dfrac{1}{16}e^{4x}$ f) $f(x) = \ln(\ln(x^5))$

6. Find the second derivative for the function $f(x) = \dfrac{5x-1}{5x+1}$.

7. Compute each of the following indefinite integrals.

   a) $\displaystyle\int 12x^2 - 2x + 1\ dx$ d) $\displaystyle\int x^5 - 2ax - a^2\ da$ g) $\displaystyle\int \sin 5x\ dx$

   b) $\displaystyle\int x^2 + x + 1 + \frac{1}{x} + \frac{1}{x^2}\ dx$ e) $\displaystyle\int e^{2x} dx$ h) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\ dx$

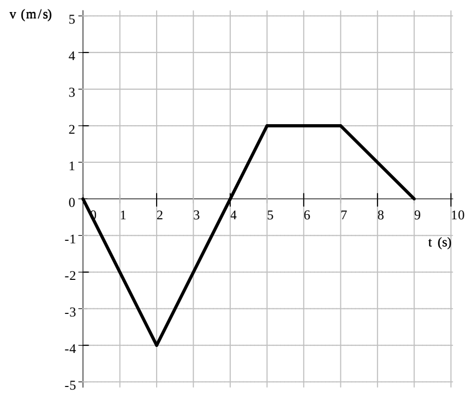
   c) $\displaystyle\int x^5 - 2ax - a^2\ dx$ f) $\displaystyle\int 5^x - x^5\ dx$ i) $\displaystyle\int 5^{2x-3} dx$

8. Find the equation of $F(x)$ if $F'(x) = 10x^4 - 6x^2 + 4x - 5$ and $F(-1) = 8$.

9. An object is moving along a vertical line. Its acceleration, as a function of time, is $a(t) = 24t - 10$. After 5 seconds, the velocity of the object is $v(5) = -20$ and its height is $h(5) = 25$.

   a) Find the velocity function $v(t)$. b) Find the location function $h(t)$.

10. Graph the location function $h(t)$ of an object if $h(0) = 0$ and the velocity function, $v(t)$ is given on the graph given below. Compute $h(2)$, $h(4)$, $h(5)$, $h(7)$, and $h(9)$.



11. Two vertices of a triangle ABC are $A(7, -5)$ and $B(10, 4)$. The third vertex, C is located on the graph of $f(x) = x^2 - x + 5$. Find the coordinates of C that minimizes the area of the triangle ABC.

12. Two sides of a triangle are 12 m long. Find the third side that maximizes the area of the triangle.

13. Two vertices of a triangle are given by $A(-4, 2)$ and $B(-1, -2)$. The third vertex of the triangle is a point on the circle $x^2 + y^2 - 16x - 4y + 43 = 0$. Find the coordinates of the third vertex so that the area of the triangle is a) minimal b) maximal.

14. Let $f(x) = \dfrac{x+1}{x^2+1}$

   a) Find all relative maximums and minimums of f. b) Find all points of inflections of f.

15. Find an equation of the tangent line drawn to the graph of $f(x) = \dfrac{1}{(3x-1)^2}$ at $x = 1$.

16. Find the $x-$coordinate of all relative maximums and minimums of the function $f(x) = \dfrac{x^2}{e^{2x}}$. State whether the extrema is a maximum or minimum.

17. Find an equation(s) of the tangent line(s) drawn to the graph of $2x^2 + y^2 = 5y - x$ at $x = -2$.

18. Find the exact values of a and b so that the curve $y = x^2 + ax + b$ will be tangent to the line $y = 2x + 1$ at the point $(1, 3)$.

19. Find a polynomial function $P(x)$ such that P is of degree four (or less) and $f(0) = P(0)$, $f'(0) = P'(0)$, $f''(0) = P''(0)$ $f'''(0) = P'''(0)$, and $f^{(4)}(0) = P^{(4)}(0)$ if $f(x)$ is defined as

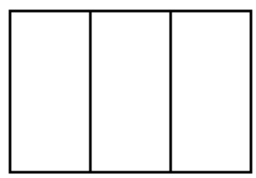
   a) $f(x) = \sin x$ b) $f(x) = e^{-x/2}$

20. Suppose that f is a function with derivative $f'(x) = \left(16 - x^2\right)^3 (1-x)^2 x^5 \left(9 - x^2\right)^3 (x+2)(2-x)^3 (x+3)$
   a) Graph f'
   b) Find all x for which f has a relative maximum.
   c) Find all x for which f has a relative minimum.
   d) How many points of inflection does f have?
   e) Plot the graph of f in the same coordinate system with f'.
   f) Is it possible that f does not have any $x-$intercept?
   g) Plot the graph of f'' in the same coordinate system with f'.

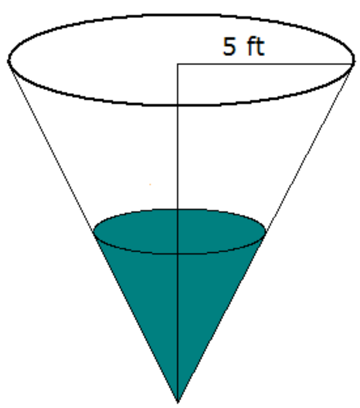
21. Consider the function $f(x) = (x-1)^{10} (4-x)^5$.
   a) Find all values of x for which $f(x)$ has a relative maximum.
   b) Find all values of x for which $f(x)$ has a relative minimum.
   c) Find all values of $f(x)$ for which $f(x)$ has a point of inflection.

22. Find two non-negative numbers x and y for which $2x + y = 30$, such that xy^2 is maximized.

23. We have P meters of fencing and want to create three adjacent rectangular enclosings as shown on the figure below. What is the maximal area we can enclose this way?



24. A tank, shaped like a cone shown on the picture below, is being filled up with water. The top of the tank is a circle with radius 5 ft, its height is 15 ft. Water is added to the tank at the rate of $V'(t) = 2\pi \dfrac{\text{ft}^3}{\text{min}}$. How fast is the water level rising when the water level is 6 ft high? (The volume of a cone with height h and base radius r is $V = \dfrac{\pi r^2 h}{3}$.)



25. A city is of a circular shape. The area of the city is growing at a constant rate of $2 \dfrac{\text{mi}^2}{\text{y}}$ (square miles per year). How fast is the radius growing when it is exactly 15 mi?

26. Consider the function $f(x) = \sqrt{1-x^2}$ on the interval $[-1, 1]$.
    a) Compute the left Riemann sum with a uniform partition of 5 subintervals.
    b) Compute the right Riemann sum with a uniform partition of 5 subintervals.

27. Consider the function $f(x) = \ln x$ on the interval $[1, 6]$.
    a) Compute the left Riemann sum with a uniform partition of 5 subintervals.
    b) Compute the right Riemann sum with a uniform partition of 5 subintervals.

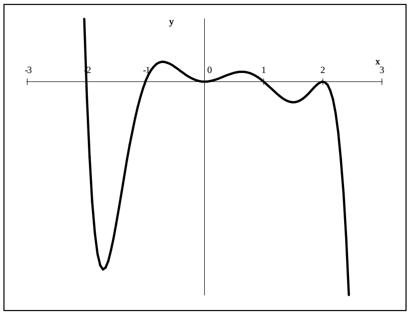
Answers

1. $f^{-1}(x) = \dfrac{5x+4}{3x-1}$

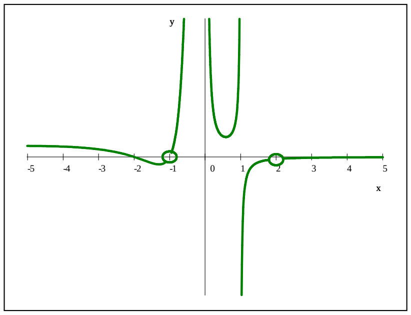
2. see solutions

3. a) -2 b) ∞ c) $\dfrac{2}{3}$ d) $-\infty$ e) $\dfrac{1}{2}$ f) ∞

4. b) $(-2,-1) \cup (1,2) \cup (2,\infty)$ c) $[-2,-1) \cup (1,2) \cup (2,\infty)$



a)



d)

5. a) $7^x \ln 7 - 7x^6 + \dfrac{7}{2\sqrt{7x}}$ b) $\dfrac{e^x + e^{-x}}{2}$ c) xe^{4x} d) $\dfrac{8x^2-40x+49}{4x^2-20x+25} = 2 - \dfrac{1}{(2x-5)^2}$

   e) $\dfrac{\dfrac{2x\ln x}{x^2+1} - \ln(x^2+1)\left(\dfrac{1}{x}\right)}{(\ln x)^2} = \dfrac{2x}{\ln x\,(x^2+1)} - \dfrac{\ln(x^2+1)}{x(\ln x)^2}$ f) $\dfrac{1}{x\ln x}$ g) $\dfrac{3}{1+9x^2}$

   h) $3\tan^2 3x + 3 = \dfrac{3}{1+\cos^2(3x)}$

6. $-\dfrac{100}{(5x+1)^3}$

7. a) $4x^3 - x^2 + x + C$ b) $\dfrac{x^3}{3} + \dfrac{x^2}{2} + x + \ln x - \dfrac{1}{x} + C$ c) $\dfrac{x^6}{6} - ax^2 - a^2x + C$

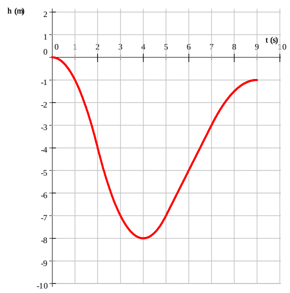
   d) $x^5a - xa^2 - \dfrac{a^3}{3} + C$ e) $\dfrac{1}{2}e^{2x} + C$ f) $\dfrac{5^x}{\ln 5} - \dfrac{x^6}{6} + C$ g) $-\dfrac{1}{5}\cos 5x + C$

   h) $\arcsin x + C$ i) $\dfrac{5^{2x-3}}{2\ln 5} + C$

8. $F(x) = 2x^2 - 5x - 2x^3 + 2x^5 + 1$

9. a) $v(t) = 12t^2 - 10t - 270$ b) $h(t) = 4t^3 - 5t^2 - 270t + 1000$

10. $h(2) = -4, \quad h(4) = -8, \quad h(5) = -7, \quad h(7) = -3, \quad h(9) = -1$



11. $(2, 7)$

12. $12\sqrt{2}\,\mathrm{m}$

13. a) $(4, -1)$ b) $(12, 5)$

14. a) maximum: $\left(-1+\sqrt{2}, \dfrac{1+\sqrt{2}}{2}\right)$ minimum: $\left(-1-\sqrt{2}, \dfrac{1-\sqrt{2}}{2}\right)$

b) $\left(-2-\sqrt{3}, \dfrac{1-\sqrt{3}}{4}\right)$, $\left(-2+\sqrt{3}, \dfrac{1+\sqrt{3}}{4}\right)$ and $(1, 1)$

15. $y - \dfrac{1}{4} = -\dfrac{3}{4}(x-1)$ or $y = -\dfrac{3}{4}x + 1$

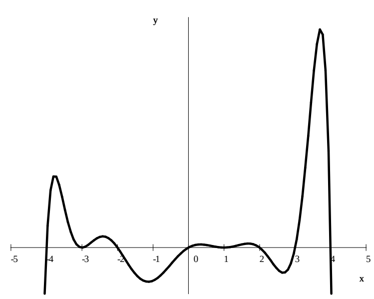
16. a relative minimum at $x = 0$, a relative maximum at $x = 1$

17. $y = -7x - 12$ and $y = 7x + 17$

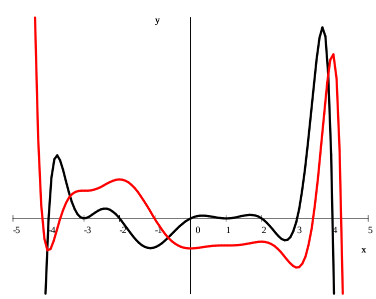
18. $a = 0, b = 2$

19. a) $x - \dfrac{1}{6}x^3$ b) $1 - \dfrac{1}{2}x + \dfrac{1}{8}x^2 - \dfrac{1}{48}x^3 + \dfrac{1}{384}x^4$

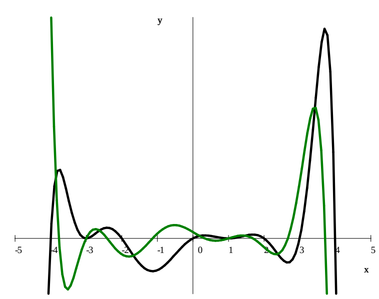
20. b) $-2, 2, 4$ c) $-4, 0, 3$ d) 9 f) no



a)



e)



g)

21. a) 3 b) 1 c) $4, 3 - \dfrac{1}{\sqrt{7}}, 3 + \dfrac{1}{\sqrt{7}}$ 22.) $x = 5, y = 20$

23.) $\dfrac{P^2}{32}$ 24.) $\dfrac{1}{2}\dfrac{\mathrm{ft}}{\mathrm{min}}$ 25.) $\dfrac{1}{15\pi}\dfrac{\mathrm{mi}}{\mathrm{y}}$ 26.) a) 1.42384 b) 1.42384

27.) a) 4.78749 b) 6.57925