1. Solve each of the following equations over $\mathbb{R}$.

   a) $\sqrt{x^2-4}+\sqrt{4-x^2}=\dfrac{x-2}{5x+1}$ b) $5^x+5^{-x}=1+\cos x$

2. Integrate.

   a) $\displaystyle\int \sin 5x \; dx$ b) $\displaystyle\int 2^x dx$ c) $\displaystyle\int (3x-8)^{100} \; dx$ d) $\displaystyle\int \frac{1}{2x-5} \, dx$ e) $\displaystyle\int \frac{x-3}{x+5} \, dx$

3. The demand for life insurance, L, and the demand for health insurance, H, can be modeled as functions of time, t:

$$\begin{aligned} L(t) &= t^3+9t+100 \quad \text{for} \quad 0 \le t \le 4 \\ H(t) &= 6t^2+102 \quad \text{for} \quad 0 \le t \le 4 \end{aligned}$$

   During the time period for $0 \le t \le 4$, the greatest difference between the two demands occurs n times. Determine n.

4. Let $f(x)=x^3+ax^2+bx-7$. Find values of a, and b such that

   a) f has a relative maximum at $x=1$ and a relative minimum at $x=2$.

   b) f has a point of inflection at $x=1$ and a relative minimum at $x=2$.

5. Find all relative extrema and points of inflection of the function $f(x)=\sin x+\cos x$.

6. Find an equation of the line tangent to the graph of $f(x)=\dfrac{x^4-1}{6x^2+1}$ at the point $(1,0)$.

7. Find an equation for the tangent line to the graph of the relation $y^3+y^2-x^2+30=5y$ at the point $(6,-2)$.

8. Find the equation(s) of the tangent line(s) drawn to $2x^2+2xy+y^2=8$ at $x=2$.

9. Find each of the following sums. (Recall that $1^2+2^2+3^2+...+n^2=\dfrac{n(n+1)(2n+1)}{6}$ for all natural number n.)

   a) $\displaystyle\sum_{k=1}^{207}(-2)$ b) $\displaystyle\sum_{k=1}^{207}(-2k)$ c) $\displaystyle\sum_{k=0}^{100}(3k-2)$ d) $\displaystyle\sum_{k=0}^{30}\left(k^2+k+2\right)$ e) $\displaystyle\sum_{k=0}^{100}\left((k+1)^3-k^3\right)$

10. Suppose that f is a differentiable odd function. Prove that then f' is an even function.

11. The area of a rectangle is a constant $100\,\text{in}^2$. One side is decreasing by a constant rate of $0.5\dfrac{\text{in}}{\text{s}}$. Find the rate of change of the length of the other side at the moment when

    a) the decreasing side is $20\,\text{in}$ long.

    b) the rectangle is a square.

12. Consider the function $f(x)=\dfrac{x^3-1}{x^2-1}$.

    a) Describe the points of discontinuity of f.

    b) Find all relative extrema for f.

    c) On what intervals is f concave up?

13. Find $g(x)$, where $g(x)$ is a polynomial of degree 3, and satisfies

$$f(0) = g(0), \quad f'(0) = g'(0), \quad f''(0) = g''(0), \text{ and } f'''(0) = g'''(0)$$

where $f(x)$ is given as

a) $f(x) = \sin x$ b) $f(x) = \dfrac{1}{x+1}$ c) $f(x) = e^x$

14. The volume of a sphere is increasing by a constant rate of $\dfrac{1}{4} \dfrac{\mathrm{m}^3}{\mathrm{s}}$. How fast is the radius of the sphere increasing when it is exactly 10 m long?

15. If $y = f(x)$ is a function, we define the curvature as

$$C(x) = \frac{|y''|}{\left(1 + (y')^2\right)^{3/2}}$$

Prove that if $f(x) = \sqrt{r^2 - x^2}$ where $r > 0$, then the curvature is constant on the interval $(-r, r)$.