

1. Simplify each of the following expressions.

a) $\frac{2x-1}{1-2x}$

b) $\frac{x^2-2x-8}{x^2-4}$

c) $\frac{1-x^2}{x^2-2x-3} \div \frac{3x-3}{3-x}$

d) $\frac{a^2-7}{a^2-8a+7} - \frac{a}{a-7}$

2. Simplify each of the following.

a) $\frac{x^3(-2x^3y^{-2})^{-4}2yx^{-1}}{x^{-3}yx^3y^0(-3x^5y^{-6})^{-2}y^{-3}}$

e) $\left(\frac{3q}{2p}\right)^{-2}$

k) $(-16)^{-3/4}$

b) $\frac{a^{-3}(b^{-4})^2}{(ab^2)^{-3}}$

f) -2^{-2}

l) $\frac{5^{-1}\left(-\frac{1}{2}\right)^2}{5^{-1}2^{-1}} - 2^{-1}$

c) $\frac{ab^{-2}}{a^2b^{-5}}$

g) $(-2)^{-2}$

h) $8^{2/3}$

d) $\frac{a-b^{-2}}{a^2+b^{-5}}$

i) $-2^{1/2}$

j) $(\sqrt{2})^5$

m) $\frac{5^{-1} + \left(-\frac{1}{2}\right)^2}{5^{-1} + 2^{-1}} - 2^{-1}$

3. Simplify each of the following.

a) $(3-\sqrt{5})^2$

c) $\frac{2}{(\sqrt{3}-1)^2}$

e) $\frac{3}{\sqrt{8}-\sqrt{5}}$

b) $5\sqrt{18} - 7\sqrt{32} + 4\sqrt{50}$

d) $(3x-1)(3x+9x^2+1)$

f) $\frac{\sqrt{20}-6}{2}$

4. Simplify each of the following expressions.

a) $\log_6 12 + \log_6 24 - 3 \log_6 2$

d) $\frac{1}{2} \ln(A^{10})$

g) $\log_5(2x^3) - 2 \log_5(3x) - \frac{1}{\log_x 5}$

b) $\log_5(x^2-x) - \log_5(x-1)$

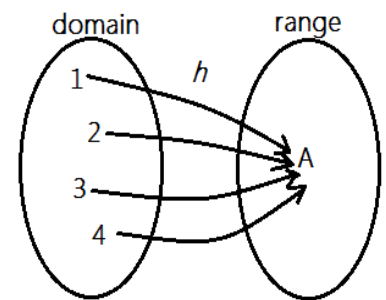
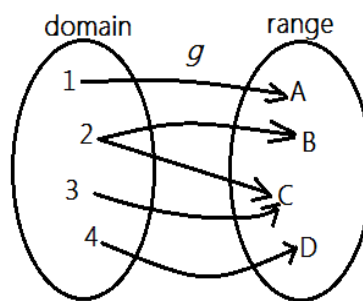
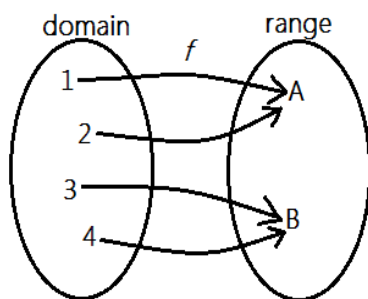
e) $\log_2(\log_3(3^{64}))$

c) $\log_6(12a^3) + \log_6(3a^5)$

f) $e^{\ln 2} - e^{2 \ln 5} + e^{-\ln 3}$

h) $\log_2(\sqrt{32})$

5. The following figure depicts three relations, f , g , and h . Which of them (if any) are also functions?



6. Let f be the function defined by $f(x) = x^2 - 4x + 9$.

- a) Compute $f(3)$. b) Is it true that $f(2+3) = f(2) + f(3)$? c) Simplify $f(2a)$

7. Let f be the function defined by $f(x) = \sqrt{x-5}$.

- a) Compute $f(3)$. b) Compute $f(9) + f(21)$. c) Compute $f(9+21)$.

8. Given that $f(x) = 2x - 3$ and $g(x) = x^2 + 5$ compute each of the following.

- a) $f(5) + f(5)$ b) $f(5+5)$ c) $f(g(2))$ d) $f(g(0))$ e) $g(f(2))$ f) $g(f(0))$

9. Simplify each of the following.

a) $\left(\frac{x^{-3}}{x^{-5}}\right)^4$ b) $-(x-2)^2 - (x+1)(2x-3)$

10. Solve each of the following equations over the real numbers.

a) $\frac{3x-1}{4} - \frac{2x+1}{3} = 2x+9$ c) $\frac{1}{x} + \frac{6}{x^2} = 1$ e) $6x - 3x^2 = 15$

b) $\frac{2}{3}(x+1) - \frac{3}{5}(2x-1) = \frac{x}{4} - 5$ d) $x^2 + 1 = 3x$

11. Solve

a) $2\pi r^2 + 2\pi r h = A$ for h b) $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$ for q c) $P = \frac{3x}{\pi} - \frac{2x-1}{4}$ for x

12. Completely factor each of the following.

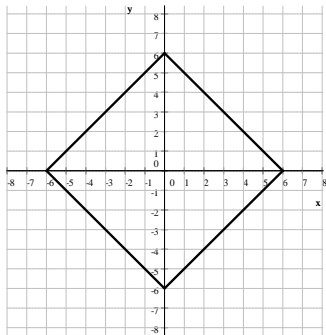
a) $12x^3 - 3x$ b) $49x^6 - 100$ c) $5x^2 + 80$ d) $x^4 - 16$

13. Factor by completing the square.

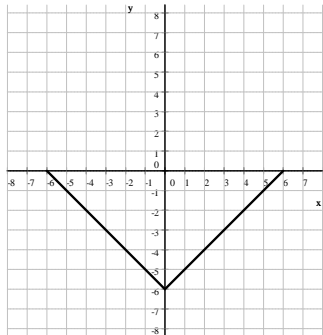
a) $x^2 - 10x + 16$ b) $x^2 + 20x + 36$ c) $x^2 - 10x + 29$ d) $x^2 - 8x + 15$ e) $x^2 - 4x + 4$

14. Consider the graphs shown on the pictures below.

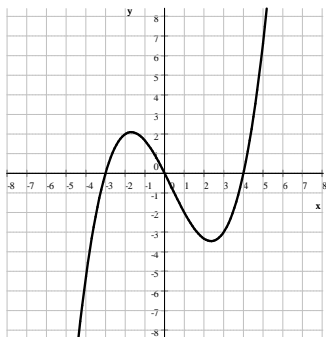
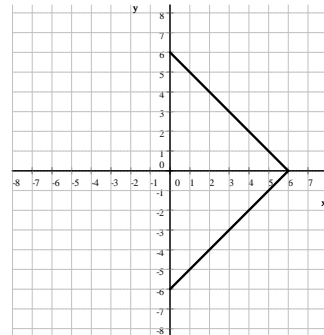
Graph A



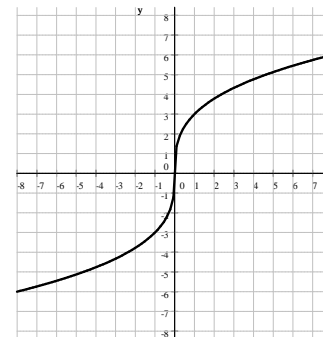
Graph B



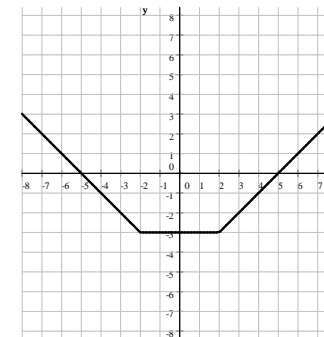
Graph C



Graph D



Graph E



Graph F

- a) Which of the graphs belong to a function?
 b) Which of the graphs belong to a one-to-one function?

15. Give a complete analysis for each of the following functions given below. (Discuss domain, range, intercepts, maximum, minimum, one-to-one-ness, and graph.)

a) $f(x) = \log_2 x$

d) $t(x) = 2 + \sqrt{x}$ on $(0, 5]$

f) $m(x) = \left(\frac{1}{3}\right)^x$

b) $g(x) = x^2 - 6x$

c) $h(x) = x^2 - 6x$ on $[2, 8]$

e) $p(x) = |x| - 3$ on $[-2, 4]$

g) $p(x) = -\frac{1}{x}$

Answers

1. a) -1 b) $\frac{x-4}{x-2}$ c) $\frac{1}{3}$ d) $\frac{1}{a-1}$

2. a) $\frac{9}{8y}$ b) $\frac{1}{b^2}$ c) $\frac{b^3}{a}$ d) $\frac{ab^5 - b^3}{a^2b^5 + 1}$ e) $\frac{4p^2}{9q^2}$ f) $-\frac{1}{4}$ g) $\frac{1}{4}$ h) 4 i) $-\sqrt{2}$

j) $4\sqrt{2}$ k) undefined l) 0 m) $\frac{1}{7}$

3. a) $14 - 6\sqrt{5}$ b) $7\sqrt{2}$ c) $\sqrt{3} + 2$ d) $27x^3 - 1$ e) $2\sqrt{2} + \sqrt{5}$ f) $\sqrt{5} - 3$

4. a) 2 b) $\log_5 x$ c) $2 + 8\log_6 a$ d) $5 \ln A$ e) 6 f) $-\frac{68}{3}$ g) $\log_5 \left(\frac{2}{9}\right) = \log_5 2 - \log_5 9$

h) $\frac{5}{2}$ i) 1

5. f and h are functions. g isn't

6. a) $f(3) = 6$ b) $f(5) = 14$ and $f(2) + f(3) = 11$ c) $f(2a) = 4a^2 - 8a + 9$

7. a) $f(3) = \text{undefined}$ b) $f(9) + f(21) = 6$ c) $f(30) = 5$

8. a) 14 b) 17 c) 15 d) 7 e) 6 f) 14

9. a) x^8 b) $-3x^2 + 5x - 1$

10. a) $x = -5$ b) $x = 8$ c) $-2, 3$ d) $\frac{3 \pm \sqrt{5}}{2}$ e) no real solutions

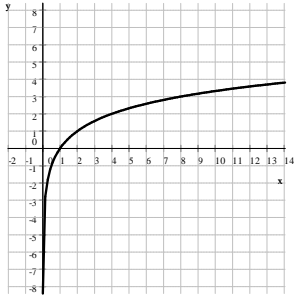
11. a) $h = \frac{A - 2\pi r^2}{2\pi r}$ or $h = \frac{A}{2\pi r} - r$ b) $q = \frac{pr}{p-r}$ c) $x = \frac{\pi - 4\pi P}{2\pi - 12}$

12. a) $3x(2x-1)(2x+1)$ b) $(7x^3-10)(7x^3+10)$ c) $5(x^2+16)$ d) $(x^2+4)(x+2)(x-2)$

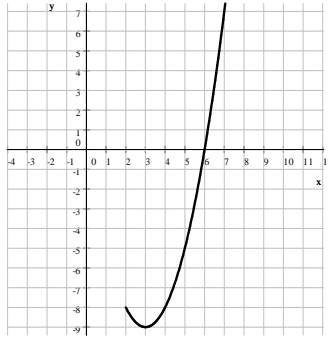
13. a) $(x-2)(x-8)$ b) $(x+18)(x+2)$ c) $x^2 - 10x + 29$ d) $(x-3)(x-5)$ e) $(x-2)^2$

14. a) B, D, E, F b) E

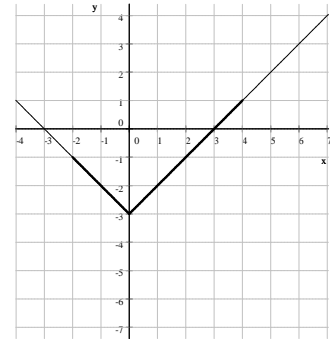
15. a) $f(x) = \log_2 x$
 domain: $(0, \infty)$
 range: \mathbb{R}
 x -intercept: $(1, 0)$
 y -intercept: none
 maximum: none
 minimum: none
 one-to-one



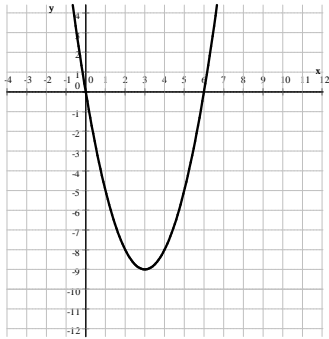
- domain: $[2, 8]$
 range: $[-9, 16]$
 x -intercepts: $(6, 0)$
 y -intercept: none
 maximum: $(8, 16)$
 minimum: $(3, -9)$
 not one-to-one



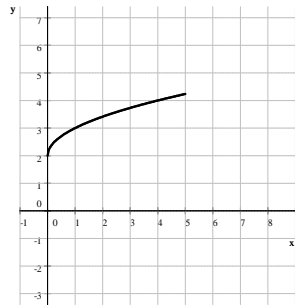
- e) $p(x) = |x| - 3$ on $[-2, 4]$
 domain: $[-2, 4]$
 range: $[-3, 1]$
 x -intercepts: $(3, 0)$
 y -intercept: $(0, -3)$
 maximum: $(4, 1)$
 minimum: $(0, -3)$
 not one-to-one



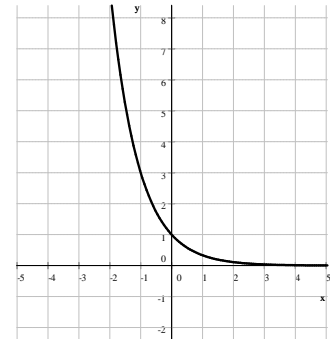
- b) $g(x) = x^2 - 6x$
 domain: \mathbb{R}
 range: $[-9, \infty)$
 x -intercepts: $(0, 0)$ and $(6, 0)$
 y -intercept: $(0, 0)$
 maximum: none
 minimum: $(3, -9)$
 not one-to-one



- d) $t(x) = 2 + \sqrt{x}$ on $(0, 5]$
 domain: $(0, 5]$
 range: $(2, 2 + \sqrt{5}]$
 x -intercepts: none
 y -intercept: none
 maximum: $(5, 2 + \sqrt{5})$
 minimum: $(3, -9)$
 one-to-one



- f) $m(x) = \left(\frac{1}{3}\right)^x$
 domain: \mathbb{R}
 range: $(0, \infty)$
 x -intercept: none
 y -intercept: $(0, 1)$
 maximum: none
 minimum: none
 one-to-one



- c) $h(x) = x^2 - 6x$ on $[2, 8]$

- g) $p(x) = -\frac{1}{x}$
 domain: $\{x : x \neq 0\}$
 range: $\{y : y \neq 0\}$
 x -intercepts: none
 y -intercept: none
 maximum: none
 minimum: none
 one-to-one

