1. Simplify each of the following.

   a) $\dfrac{x^3y^{-2}}{x^{-1}y^8}$

   b) $\dfrac{x^3+y^{-2}}{x^{-1}-y^8}$

   c) $\left(\dfrac{2c}{c+2}+\dfrac{2c}{6-3c}+\dfrac{8c}{c^2-4}\right)\div\dfrac{c-4}{c-2}$

   d) $\left(2-\sqrt{3}\right)\left(5\sqrt{3}+1\right)$

   e) $\left(\sqrt{2}-1\right)^3$

   f) $\dfrac{2}{1-\sqrt{5}}$

   g) $e^{\ln A}+e^{-\ln B}+e^{3\ln C}$

   h) $e^{\ln A-\ln B+3\ln C}$

   i) $\log\left(33x^5\right)-\dfrac{1}{2}\log\left(44x^6\right)-\log\left(15x^2\right)-\log\sqrt{1100}$

2. Factor completely each of the following.

   a) x^4-9x^3 b) x^4-4x^2 c) x^4-81

3. Factor each of the following by completing the square.

   a) $12x-3x^2+288$ b) $x^2-12x+45$ c) $6x^2-7x-3$ d) x^2+x-1

4. Solve each of the following inequalities.

   a) $x^2\geq 25$ b) $x^2-6x\leq -9$ c) $x^2+4x<-6$ d) $x^2-4x\leq 3$

5. Express the solution sets from the previous problem as sets.

6. Solve each of the following inequalities.

   a) $(x+4)(x-7)<0$ b) $\dfrac{x+4}{x-7}<0$ c) $\dfrac{1}{(x+4)(x-7)}<0$

7. Find the equation of the straight line passing through the points $(-6,6)$ and $(2,2)$.

8. A straight line passes through points $(a,0)$ and $(0,b)$, where $a,b\neq 0$. Prove that the line has the following equation:

$$bx+ay=ab$$

9. Banks X and Y offer slightly different business checking accounts. Bank X charges \$10 per month for the account and then 12 cents for every check cashed. Bank Y charges \$14 per month for the account and 10 cents for every check cashed.

   a) Which deal is better if we cash 85 checks per month? Explain your answer.
   b) Which deal is better if we cash 300 checks per month? Explain your answer.
   c) If we cash n checks in a month, the two offers are identical. Find the value of n.

10. Suppose that a small object is moving up and down along a vertical line. We monitor the location of the object as a function of time. We set ground level to represent a height (or vertical location) to be zero. In each case, graph the location function given the data on the height.

    t is time, measured by seconds and h is the height, measured in meters.

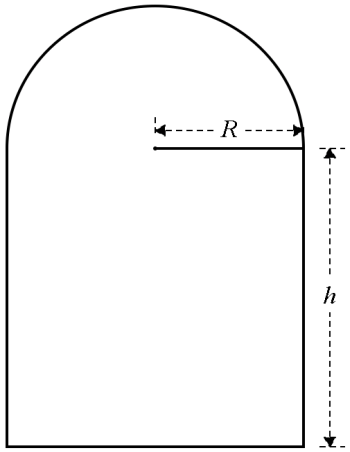
t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
h	38.3	52.8	63.9	71.9	76.8	78.4	76.75	72.1	64.15	52.7	38.38	20.75	0	0	0

    a) Create a coordinate system with to graph this data. Label both axis and set up a consistent scale on both of them. Then graph the data given. (x axis: time, measured in seconds from 0 to 15 and y axis: vertical position, measured in feet from 0 to 80)

    b) When is the object moving upward?

11. Find the equation for the set of all points $P(x, y)$ that are three times as far from $A(7, 6)$ as from $B(-1, -2)$. What kind of a curve is this?

12. A norman window is designed by placing a semicircle on a rectangle as shown on the picture.
    a) Express the area A of the window in terms of R and h.
    b) Solve for h in terms of A and R
    c) Solve for R in terms of h and A
    d) If in a word problem you have a choice between solving for h and solving for R, which one would you choose? Why?



13. Compute each of the limits. Show all steps using correct notation.

    a) $\lim_{x\to-\infty}\left(-5x^3-2x^2+10\right)$
    b) $\lim_{x\to\infty}\left(-5x^3-2x^2+10\right)$
    c) $\lim_{x\to-\infty}(\ln x)$
    d) $\lim_{x\to\infty}(\ln x)$
    e) $\lim_{x\to-\infty}\left(\frac{5^{x+2}}{2^{2x-1}}\right)$
    f) $\lim_{x\to\infty}\left(\frac{5^{x+2}}{2^{2x-1}}\right)$
    g) $\lim_{x\to-\infty}\left(\frac{5^{x+2}}{2^{3x-1}}\right)$
    h) $\lim_{x\to\infty}\left(\frac{5^{x+2}}{2^{3x-1}}\right)$
    i) $\lim_{x\to-\infty}\frac{3x^2-5x+1}{-2x^2+8}$
    j) $\lim_{x\to\infty}\frac{3x^2-5x+1}{-2x^2+8}$
    k) $\lim_{x\to-\infty}\frac{8x^5-3x^4-x^2+6}{-3x^2+1}$
    l) $\lim_{x\to\infty}\frac{8x^5-3x^4-x^2+6}{-3x^2+1}$
    m) $\lim_{x\to-\infty}\frac{-2x^4+7x^2-x+1}{-3x^5-9x^2+x-3}$
    n) $\lim_{x\to\infty}\frac{-2x^4+7x^2-x+1}{-3x^5-9x^2+x-3}$
    p) $\lim_{x\to-\infty}x\left(\frac{2}{2-\frac{1}{x}}-1\right)$
    q) $\lim_{x\to\infty}\cos x$
    r) $\lim_{x\to\infty}\frac{\frac{1}{x}}{5-\sqrt{25-\frac{1}{x}}}$
    s) $\lim_{x\to-\infty}\sqrt{2x-1}$
    t) $\lim_{x\to\infty}\left(3^{x+2}-3^x\right)$
    u) $\lim_{x\to\infty}\left(\log_2 10x-\log_2 x\right)$
    v) $\lim_{x\to\infty}\left(\log_4 x-\log_2 x\right)$
    x) $\lim_{x\to\infty}\tan x$
    y) $\lim_{x\to\infty}\left(\frac{\sin x}{x}\right)$
    z) $\lim_{x\to\infty}\left(\frac{2^{x-1}\cdot 3^{x+1}}{6^{x-1}}\right)$

14. A company is introducing a new product. The marketing manager determines that t weeks after an advertising campaign begins, $P(t)$ percent of the potential market is aware of the new product, where

$$P(t)=\frac{68t^2+2t+10}{2t^2+t+1}-2$$

    a) What percent of the potential market knows about the product when it is launched?
    b) What percent of the potential market knows about the product after 3 weeks?
    c) What happens to the percentage $P(t)$ in the long run?

15. Sketch the graph of and give a complete analysis for each of the following functions.

a) $f(x) = -\frac{1}{2}x + 3$

b) $g(x) = -\frac{1}{2}x + 3$ on $[-2, 4]$

c) $h(x) = -x^2 + 2x + 3$ on $(-2, 3]$

d) $t(x) = \frac{1}{x}$

e) $d(x) = \sqrt{x}$

f) $f(x) = \sqrt{16 - x^2}$

g) $f(x) = 2^x$ on $[-5, 3]$

h) $f(x) = \left(\frac{1}{2}\right)^x$ on $(-2, 3)$

i) $f(x) = \log_{1/2} x$

16. Let f be the function defined by $f(x) = x^2 - 4x + 1$ and g be defined by $g(x) = 2x - 1$. Simplify each of the following.

a) $f(g(x))$ b) $g(f(x))$

17*. Find the last digit of the number 7^{207}. (Hint: if you do not know it for a large number, try $1, 2, 3, 4, \ldots$ and the idea will come!)

18*. The perimeter of a right triangle is 60 m, its area is 120 m^2. How long are the sides?

19*. Suppose that a is a number such that $\frac{1}{a^3} + a^3 = 5$. Find the exact value of $\frac{1}{a^6} + a^6$. (Hint: you do not need to find the value of a.)

Answers

1. a) $\frac{x^4}{y^{10}}$ b) $\frac{x^4y^2+x}{y^2-xy^{10}}$ c) $\frac{4c}{3(c-4)}$ d) $9\sqrt{3}-13$ e) $5\sqrt{2}-7$ f) $-\frac{\sqrt{5}+1}{2}$ g) $A+\frac{1}{B}+C^3$ h) $\frac{AC^3}{B}$ i) -2

2. a) $x^3(x-9)$ b) $x^2(x+2)(x-2)$ c) $\left(x^2+9\right)(x+3)(x-3)$

3. a) $-3(x+8)(x-12)$ b) can not be factored c) $6\left(x-\frac{3}{2}\right)\left(x+\frac{1}{3}\right)=(2x-3)(3x+1)$ d) $\left(x+\frac{1+\sqrt{5}}{2}\right)\left(x+\frac{1-\sqrt{5}}{2}\right)$

4. a) $x\le -5$ or $x\ge 5$ b) $x=3$ c) there is no solution d) $2-\sqrt{7}\le x\le 2+\sqrt{7}$

5. a) $(-\infty,-5]\cup[5,\infty)$ b) $\{3\}$ c) $\varnothing$ d) $\left[2-\sqrt{7},2+\sqrt{7}\right]$

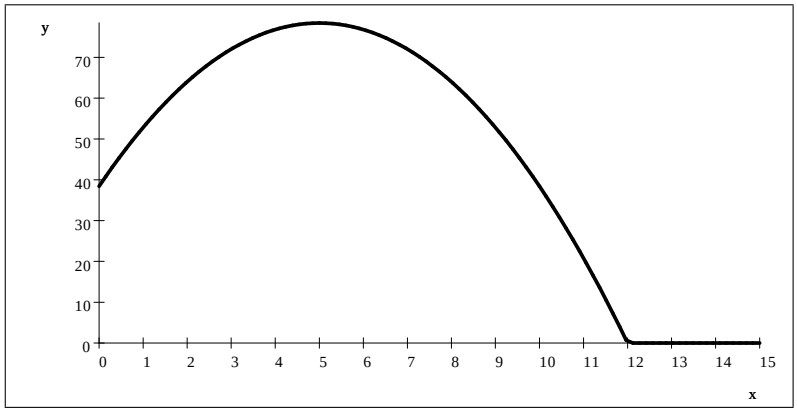
6. a) $-4<x<7$ b) $-4<x<7$ c) $-4<x<7$

7. $y=-\frac{1}{2}x+3$

8. Solution 1: verify that both points are on the line. Two points uniquely determine a line.
Solution 2: Use the two points to find the slope between them and then write the equation of the ;line.

9. a) Bank X is better since $X(85)=20.2$, $Y(85)=22.5$ b) Bank Y is better since $X(300)=46$, $Y(300)=44$
c) 200

10. a) b) between $t=0$ and $t=5$



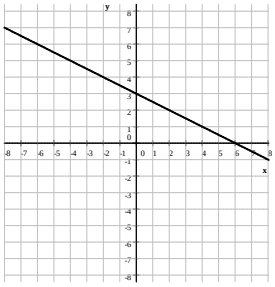
11. a circle with equation $(x+2)^2+(y+3)^2=18$ center: $(-2,-3)$ radius: $2\sqrt{3}$

12. a) $A=2Rh+\frac{1}{2}\pi R^2$ b) $h=\frac{2A-\pi R^2}{4R}$ c) $R=\frac{-2h+\sqrt{4h^2+2A\pi}}{\pi}$

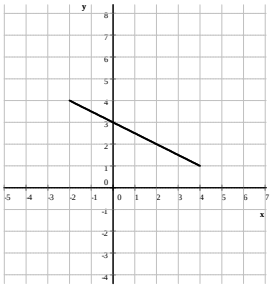
13. a) ∞ b) $-\infty$ c) undefined d) ∞ e) 0 f) ∞ g) ∞ h) 0 i) $-\frac{3}{2}$ j) $-\frac{3}{2}$ k) ∞ l) $-\infty$ m) 0 n) 0 p) $\frac{1}{2}$ q) undefined r) 10 s) undefined t) ∞ u) $\log_2 10$ v) $-\infty$ x) undefined y) 0 z) 9

14. a) 8% b) $26.\overline{54}\%$ c) 32%

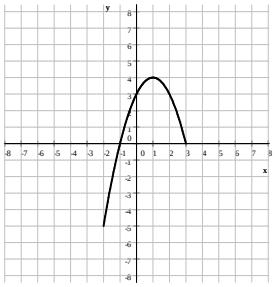
15. a) $f(x) = -\frac{1}{2}x + 3$
domain: $\mathbb{R}$
range: $\mathbb{R}$
$y-$intercept: $(0, 3)$
$x-$intercept: $(6, 0)$
maximum: none
minimum: none
one-to-one: yes



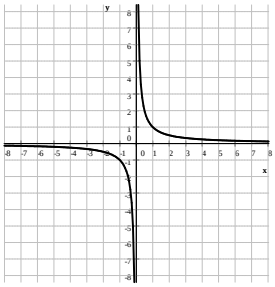
b) $g(x) = -\frac{1}{2}x + 3$ on $[-2, 4]$
domain: $\mathbb{R}$
range: $\mathbb{R}$
$y-$intercept: $(0, 3)$
$x-$intercept: none
maximum: $(-2, 4)$
minimum: $(4, 1)$
one-to-one: yes



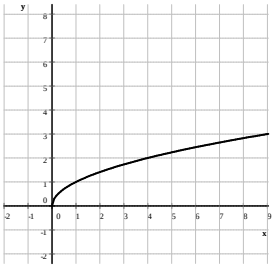
c) $h(x) = -x^2 + 2x + 3$ on $(-2, 3]$
domain: $(-2, 3]$
range: $(-5, 4]$
$y-$intercept: $(0, 3)$
$x-$intercept: $(-1, 0)$
maximum: $(1, 4)$
minimum: none
one-to-one: no



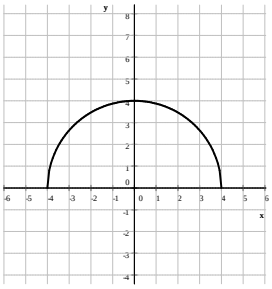
d) $t(x) = \frac{1}{x}$
domain: $\{x : x \neq 0\}$
range: $\{y : y \neq 0\}$
$y-$intercept: none
$x-$intercept: none
maximum: none
minimum: none
one-to-one: yes



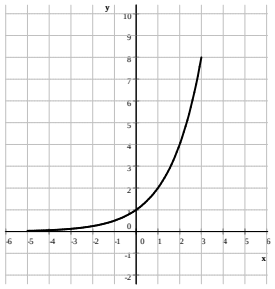
e) $d(x) = \sqrt{x}$
domain: $[0, \infty)$
range: $[0, \infty)$
$y-$intercept: $(0, 0)$
$x-$intercept: $(0, 0)$
maximum: none
minimum: $(0, 0)$
one-to-one: yes



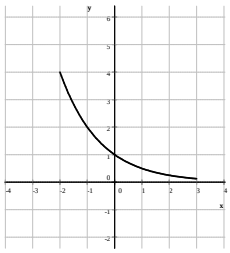
f) $f(x) = \sqrt{16 - x^2}$
domain: $[-4, 4]$
range: $[0, 4]$
$y-$intercept: $(0, 4)$
$x-$intercept: $(-4, 0)$ and $(4, 0)$
maximum: $(0, 4)$
minimum: $(-4, 0)$ and $(4, 0)$
one-to-one: no



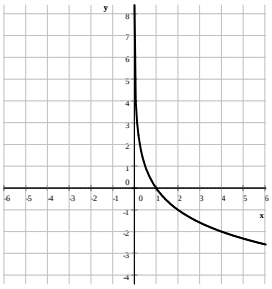
g) $f(x) = 2^x$ on $[-5, 3]$
domain: $[-5, 3]$
range: $\left[\frac{1}{32}, 8\right]$
$y-$intercept: $(0, 1)$
$x-$intercept: none
maximum: $(0, 4)$
minimum: $\left(-5, \frac{1}{32}\right)$
one-to-one: yes



h) $f(x) = \left(\frac{1}{2}\right)^x$ on $(-2, 3)$
domain: $(-2, 3)$
range: $\left(\frac{1}{8}, 4\right)$
$y-$intercept: $(0, 1)$
$x-$intercept: none
maximum: none
minimum: none
one-to-one: yes



i) $f(x) = \log_{1/2} x$
domain: $(0, \infty)$
range: $\mathbb{R}$
$y-$intercept: none
$x-$intercept: $(1, 0)$
maximum: none
minimum: none
one-to-one: yes



16. a) $f(g(x)) = 4x^2 - 12x + 6$ b) $g(f(x)) = 2x^2 - 8x + 1$

17. 3

18. 10 m, 24 m, 26 m

19. 23

