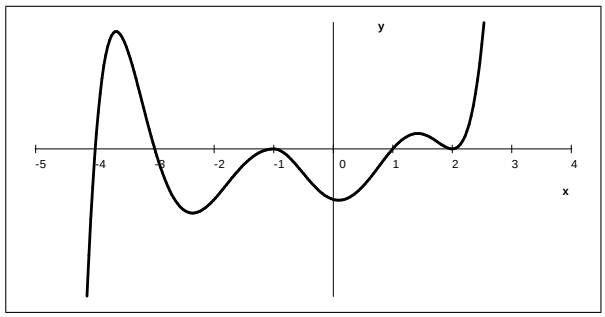
Review Problems

1. The picture below shows the graph of a function f. Sketch the graph of each of the following.



a) $g(x) = \frac{1}{2} f(x)$

b) $g(x) = f(2x)$

c) $g(x) = f(-x)$

d) $g(x) = \frac{1}{f(x)}$

e) $g(x) = f'(x)$

f) $g(x)$ where $g'(x) = f(x)$

g) $g(x) = |f(x)|$

h) $g(x) = f(|x|)$

i) $g(x) = \frac{f(x) - |f(x)|}{2}$

2. Sketch the graph of each of the following rational functions.

a) $f(x) = \frac{x(x+2)}{x^2-1}$

b) $f(x) = \frac{(x+4)(x+3)^3(x+2)^2(x+1)^3 x^2 (x-1)^3}{(x+3)^2(x+2)^8(x+1)^3 x^7 (x-1)(x-4)}$

3. Let f be the function that is given by $f(x) = \frac{ax+b}{x^2-c}$ and that has the following properties.

i) The graph of f is symmetric with respect to the $y-$axis. ii) $\lim_{x \to 2+} f(x) = \infty$ iii) $f'(1) = -2$

a) Determine the values of a, b, and c.

b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

c) Sketch the graph of $f(x)$.

4. Find each of the following limits.

a) $\lim_{x \to 5} \frac{x^2-25}{x^2-2x-15}$

b) $\lim_{x \to -3-} \frac{x^2-25}{x^2-2x-15}$

c) $\lim_{x \to -3} \frac{x^2-25}{x^2-2x-15}$

d) $\lim_{x \to \infty} \frac{x^2-1}{2^x+1}$

e) $\lim_{x \to \infty} \left(\log_2(4x^2-1) - \log_2(x^2+1)\right)$

f) $\lim_{x \to 3+} \frac{2(x+1)(x-3)(x+5)}{(x-1)(x-3)}$

g) $\lim_{x \to \infty} \frac{\ln x}{x}$

h) $\lim_{x \to 0+} \frac{1-e^x}{(x-1)^3}$

i) $\lim_{x \to 9+} \frac{\sqrt{x}-3}{9-x}$

5. Differentiate each of the following.

a) $f(x) = \ln \sqrt[3]{x^5 - 2x + 4}$

b) $f(x) = 2^{\frac{x+1}{x-1}}$

c) $f(x) = -3e^{-4x^2}$

d) $f(x) = x\left(\ln 5 + \ln x^2\right) - 2x$

e) $f(x) = \ln\left(\sin^2(\pi x - 1) + 1\right)$

f) $10 = x^2 y - x y^2 + y^3 - x^3$

g) $f(x) = e^{\sin x} + e^{\cos x}$

h) $f(x) = 3^x + x^3$

6. Compute each of the following integrals.

a) $\int_0^1 \frac{6x^2-4x+5}{x+1}dx$

b) $\int 5^x dx$

c) $\int_0^5 e^{-x}dx$

d) $\int \frac{e^{-x}}{e^{-x}+1}dx$

e) $\int_0^5 xe^{-x}dx$

f) $\int_0^\infty xe^{-x}dx$

g) $\int_0^{\pi/4} \tan x\ dx$

h) $\int_2^{28} (x-1)^{2/3}\,dx$

i) $\int_0^{\pi/2} \sin x \cos x\ dx$

7. Let f be a function given by $f(x) = \ln\left(\frac{x}{x-1}\right)$

a) Find the domain of f.

b) Find the value of the derivative of f at $x = -1$.

c) Find the formula for the inverse of f.

8. We are on the surface of the Moon. The gravitational acceleration there is $g = -1.6\frac{\text{m}}{\text{s}^2}$. A rock is thrown vertically upward, from an initial height of 19.2 m, with an initial velocity of $8\frac{\text{m}}{\text{s}}$.

a) Find the velocity function $v(t)$ of the object.
b) Find the location function $s(t)$ of the object.
c) Find the maximal height that the rock will reach.
d) How long until the rock hits the ground?
e) What is the velocity of the rock when it hits the ground?

9. A particle starts at $t = 0$ and moves along the $x-$axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

a) Find the velocity function $v(t)$ of the particle.
b) For what values of t is the object moving to the left?
c) Find all values of t for which the object is moving and but its acceleration is zero.

10. A function f has derivative $f'(x) = -18(x+5)^3(x+4)^2(x+2)x^6(2-x)^5(4-x)^2$.

a) Plot the graph of f'.
b) Find all critical points of f and classify each of them as a maximum, minimum, or a point of inflection.
c) How many points of inflection does f have?

11. Write the equation of the degree 3 polynomial $P(x)$, given that

a) the points $(-2, 69)$, $(-1, 14)$, $(2, -7)$, and $(1, 6)$ on the graph.
b) $P(0) = -2$, $P'(0) = 3$, $P''(0) = 12$, and $P'''(0) = -240$.
c) P has a relative minimum at $x = -3$ and a relative minimum at $x = 1$, $P(0) = 0$, and $P(1) = 1$.

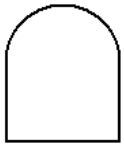
12. Find an equation for all tangent lines drawn to the graph of $2x^2 + y^2 = 5y - x$ at $x = -2$.

13. A company estimates that the total cost of producing q units is $C(q) = q^3 - 155q^2 + 6375q + 3000$

a) What is the fixed cost?
b) At what level of production will the total cost be minimized? What is the minimal cost?
c) At what level of production will the profit be maximized, provided that we can sell every item we produce, for \$775?

14. Let $f(x) = 2x^7 - 7x^4 + 70x + 4$. Let $g(x)$ be the inverse of $f(x)$. Find $g'(4)$.

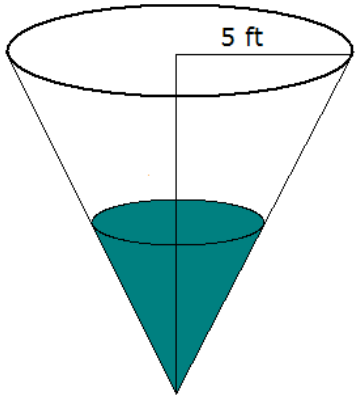
15. A Norman window has the outline of a semicircle on top of a rectangle, as shown on the picture below. Find the dimensions of the window that can be built using 8 meters of wood and has the maximal area.



16. Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$. The tangent line to the graph of f at the point $(k, f(k))$ intercepts the $x-$axis at $x = 4$. What is the value of k?

17. A town is of a circular shape. The area of the town is growing with a constant $3\pi \frac{\text{mi}^2}{\text{y}}$ (square mile per year). How fast is its radius changing when the radius is exactly 5 miles long?

18. A tank, shaped like a cone shown on the picture below, is being filled up with water. The top of the tank is a circle with radius 5 ft, its height is 15 ft. Water is added to the tank at the rate of $V'(t) = 2\pi \frac{\text{ft}^3}{\text{min}}$. How fast is the water level rising when the water level is 6 ft high? (Hint: the volume of a cone with height h and base radius r is $V = \frac{\pi r^2 h}{3}$.)



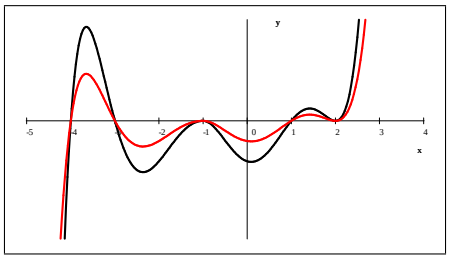
19. A tank, shaped like a cone shown on the picture above, is filled up with liquid. that weighs $300 \frac{\text{N}}{\text{ft}^3}$ (Newtons per cubic feet). How much work is needed to pump all of the liquid out of the tank through its top?

20. We have to lift a 300 N bucket to the top of a 120 m tall building using a rope that weighs 6 N per meter. How much work is needed to lift the bucket from the ground to the roof?

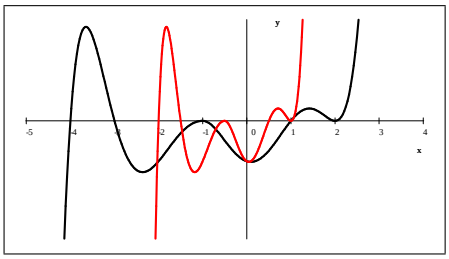
21. Find the area of the region bounded by the graphs of $f(x) = x^2 + 6x - 7$ and $g(x) = 5x + 5$

Answers

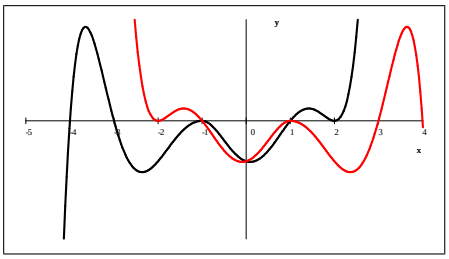
1. a) $g(x) = \frac{1}{2} f(x)$



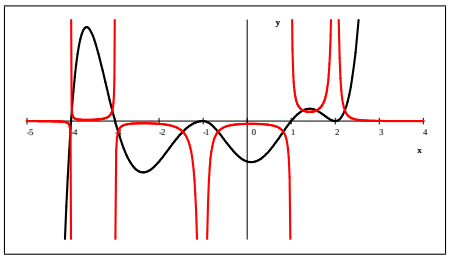
b) $g(x) = f(2x)$



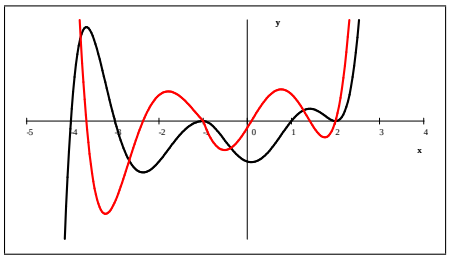
c) $g(x) = f(-x)$



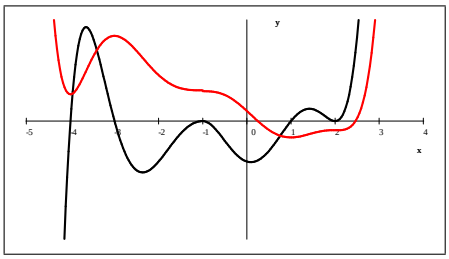
d) $g(x) = \frac{1}{f(x)}$



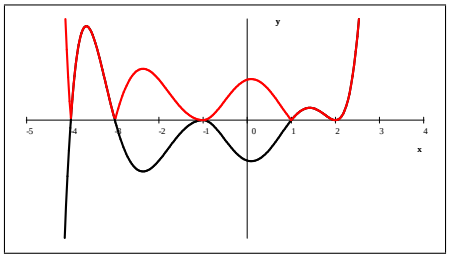
e) $g(x) = f'(x)$



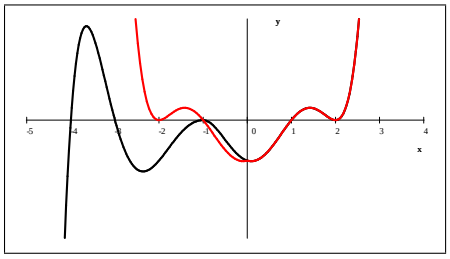
f) $g(x)$ where $g'(x) = f(x)$



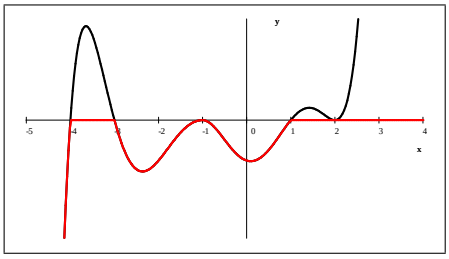
g) $g(x) = |f(x)|$



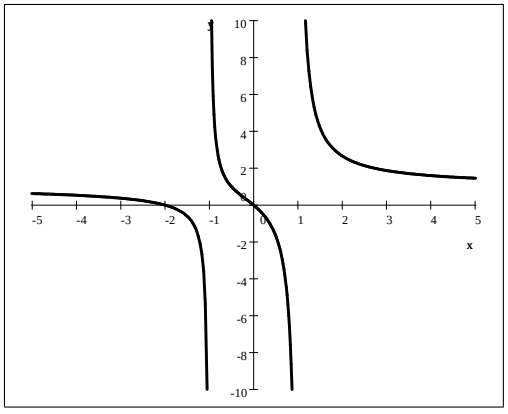
h) $g(x) = f(|x|)$



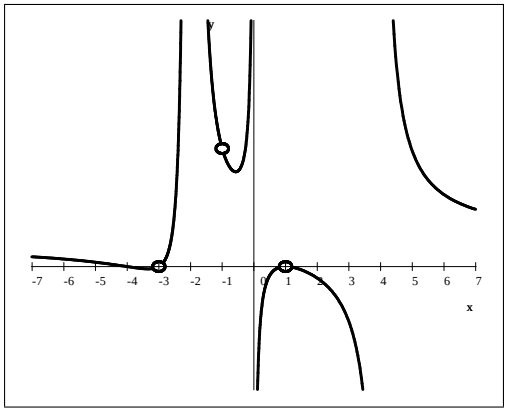
i) $g(x) = \frac{f(x) - |f(x)|}{2}$



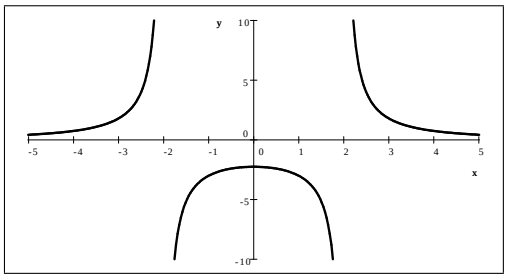
2. a) $f(x) = \frac{x(x+2)}{x^2 - 1}$



b) $f(x) = \frac{(x+4)(x+3)^3(x+2)^2(x+1)^3 x^2 (x-1)^3}{(x+3)^2(x+2)^8(x+1)^3 x^7 (x-1)(x-4)}$



3. a) $a=0,\quad b=9,\quad c=4$ b) vertical: $x=2,\quad x=-2,$ horizontal: $y=0$
c)



4. a) $\frac{5}{4}$ b) $-\infty$ c) undefined d) 0 e) 2 f) 32 g) 0 h) 0 i) $-\frac{1}{6}$

5. a) $\frac{5x^4-2}{3\left(x^5-2x+4\right)}$ b) $\ln 2\cdot 2^{\frac{x+1}{x-1}}\cdot\frac{-2}{(x-1)^2}$ c) $24xe^{-4x^2}$ d) $2\ln x+\ln 5$
e) $\frac{2\pi\sin(\pi x-1)\cos(\pi x-1)}{\sin^2(\pi x-1)+1}$ f) $y'=\frac{2xy-3x^2-y^2}{2xy-x^2-3y^2}$ g) $\cos x e^{\sin x}-\sin x e^{\cos x}$
h) $3x^2+(\ln 3)\,3^x$

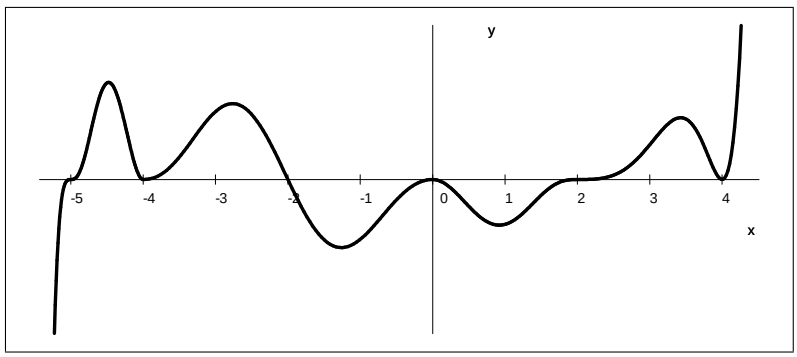
6. a) $15\ln 2-7$ b) $\frac{5^x}{\ln 5}+C$ c) $1-\frac{1}{e^5}$ d) $-\ln\left(e^{-x}+1\right)+C$ e) $1-6e^{-5}$
f) 1 g) $\frac{\ln 2}{2}$ h) $\frac{726}{5}$ i) $\frac{1}{2}$

7. a) $(-\infty,0)\cup(1,\infty)$ b) $-\frac{1}{2}$ c) $f^{-1}(x)=\frac{e^x}{e^x-1}$

8. a) $v(t)=-1.6t+8=-1.6(t-5)$ b) $s(t)=-0.8t^2+8.0t+19.2=-0.8(t+2)(t-12)$
c) 39.2 m d) 12 s e) $-11.2\frac{\text{m}}{\text{s}}$

9. a) $v(t)=(t-1)^2(8t-11)$ b) $[0,1)\cup\left(1,\frac{11}{8}\right)$ c) $\frac{5}{4}$

10. a)



b) f has a minimum at $x=-5$, a point of inflection at $x=-4$, a maximum at $x=-2$, a point of inflection at $x=0$, a minimum at $x=2$, and a point of inflection at $x=4$
c) 8

11. a) $P(x)=-5x^3+7x^2+x+3$ b) $P(x)=-40x^3+6x^2+3x-2$ c) $P(x)=-x^3-3x^2+9x$

12. $y = -7x - 12$ and $y = 7x + 17$
13. a) \$3000 b) \$31 125 when $q = 75$ c) $q = 80$
14. $\frac{1}{70}$
15. $\frac{16}{\pi + 4}$ wide, with radius $\frac{8}{\pi + 4}$ and height $\frac{16}{\pi + 4}$
16. 2
17. $0.3\frac{\text{mi}}{\text{y}}$
18. $\frac{1}{2}\frac{\text{ft}}{\text{min}}$
19. 441786. 467 J
20. 79 200 J
21. $\frac{343}{6}$