Review Problems

1. What is the value of $\lim_{x\to 0} \dfrac{\sin 3x}{x}$?

   A) 0 B) 1 C) $\dfrac{1}{3}$ D) 3 E) None of these

2. Find $\lim_{x\to\infty} \dfrac{x^2-4}{2+x-4x^2}$.

   A) -2 B) $\dfrac{1}{2}$ C) $-\dfrac{1}{4}$ D) 1 E) Does not exist

3. Given that $f(x) = \dfrac{x^2-4}{x+2}$. Which of the following statements is true?

   A) $f(x)$ is continuous everywhere.

   B) $f(x)$ is removably discontinuous at $x=-2$.

   C) $f(x)$ is nonremovably discontinuous at $x=-2$.

   D) $f(x)$ is removably discontinuous at $x=-2$ and removably discontinuous at $x=2$.

   E) $f(x)$ is removably discontinuous at $x=-2$ and nonremovably discontinuous at $x=2$.

4. Given that $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$, which of the following statements is true about $f(x)$?

   A) $\lim_{x\to 1} f(x)$ exists.

   B) $f(1)$ is defined.

   C) f is continuous at $x=1$.

   D) Both A and B are true

   E) All A, B, and C are true

5. Given that $f(x) = 2(\sin x)^3$. Find $f'(x)$.

   A) $2(\cos x)^3$ B) $6(\sin x)^2$ C) $2(\sin x)^2 \cos x$ D) $6(\sin x)^2 \cos x$ E) none of these

6. Let $f(x) = 2\ln(4e^{-3x})$. Find the value of $f'(0)$.

   A) -6 B) 0 C) 1 D) 2 E) none of these

7. Let $f(x) = \arctan(\cos 2x)$. What is the value of $f'(\pi)$?

   A) 0 B) 1 C) -1 D) -2 E) none of these

8. The area of the region in the first quadrant between the the graph of $y = x\sqrt{4-x^2}$ and the $x-$axis is

A) $\frac{2\sqrt{2}}{3}$ B) $\frac{8}{3}$ C) $2\sqrt{2}$ D) $2\sqrt{3}$ E) $\frac{16}{3}$

9. Given that $f(2) = 8$, $f'(2) = 3$, and $g(x) = \frac{1}{\sqrt[3]{(f(x))^2}}$. Find the exact value of $g'(2)$.

A) -2 B) $\frac{1}{2}$ C) $\frac{2}{3\sqrt[3]{2}}$ D) $-\frac{1}{16}$ E) none of these

10. Let $f(x) = \ln\sqrt{x}$. Evaluate the second derivative of f at $x = 2$.

A) $-\frac{1}{8}$ B) 0 C) -1 D) $\frac{3}{4}$ E) none of these

11. An antiderivative of $f(x) = \cos(3x+6)$ is given by

A) $\sin(3x+6)$ C) $\frac{1}{3}\sin(3x+6)$ E) none of these

B) $-3\sin(3x+6)$ D) $\sin\left(\frac{3}{2}x^2+6x\right)$

12. Find $\frac{dy}{dx}$ at the point $(1,1)$ for the equation $x + 2xy = 3$.

A) $-\frac{5}{2}$ B) $-\frac{3}{2}$ C) $\frac{1}{2}$ D) $\frac{3}{2}$ E) none of these

13. A particle with velocity at any time given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

A) $e^2 - 1$ B) $e - 1$ C) $2e$ D) e^2 E) none of these

14. If f is integrable on the interval $[a,b]$, then the **average value** of f on the interval $[a,b]$ is defined as $\frac{\int_a^b f(x)\,dx}{b-a}$. Find the average value of $f(x) = \frac{1}{x+1}$ on the interval $[0,3]$.

A) $\frac{1}{2}$ B) $\frac{\ln 2}{3}$ C) $\frac{\ln 2}{2}$ D) $\frac{2}{3}\ln 2$ E) none of these

15. Let f be the function defined by $f(x) = (x-3)e^x$. Find all critical numbers of $f(x)$.

A) $x = 1$ B) $x = 2$ C) $x = 3$ D) $x = -2$ E) none of these

16. Evaluate $\int \frac{x^2+x+1}{x^2+1}dx$

A) $x+\ln(x^2+1)+C$ C) $1+\frac{1}{2}\ln(x^2+1)+C$ E) none of these

B) $x+\frac{1}{2}\ln(x^2+1)+C$ D) $1+\ln(x^2+1)+C$

17. Simplify the expression $\frac{d}{dx}\left[\int_1^x \frac{1}{\sqrt{2t^2-1}}dt\right]$

A) $\frac{2x}{\sqrt{2x^2-1}}$ B) $\frac{1}{\sqrt{2x^2-1}}$ C) $2x\sqrt{2x^2-1}$ D) $\frac{1}{\sqrt{2x^2-1}}-1$ E) none of these

18. Compute the slope of the tangent line drawn to the graph of $y=(\ln x)\,e^x$ at $x=2$.

A) e B) $\frac{1}{2}e^2$ C) $(2\ln 2+1)\,e$ D) $\left(\ln 2+\frac{1}{2}\right)e^2$ E) none of these

19. Compute the area of the region bounded by the graph of $y=xe^{x^2}$, the $x-$axis, the lines $x=0$ and $x=2$.

A) $\frac{1}{2}e^3$ B) $\frac{1}{2}(e^4-e)$ C) $\frac{1}{2}(e^4-1)$ D) $\frac{1}{2}e^4$ E) none of these

20. Simplify $\int \tan^2 x dx$.

A) $2\tan x\sec x+C$ C) $\sin x-\tan x+C$ E) none of these

B) $\tan x-x+C$ D) $\frac{1}{3}\tan^3 x+C$

21. The equation of the tangent line to $f(x)=\sqrt{2x+1}$ drawn to $x=4$ is

A) $y+3=-\frac{2}{3}(x+4)$ C) $y+3=-\frac{1}{3}(x+4)$ E) none of these

B) $y-3=\frac{1}{3}(x-4)$ D) $y-3=\frac{2}{3}(x-4)$

22. Find all values of x for which the function $f(x)=\frac{1}{x^2-1}$ is increasing.

A) $x\in(-\infty,-1)\cup(-1,0)$ C) $x\in(-\infty,-1)\cup(0,1)$ E) $x\in(-\infty,\infty)$

B) $x\in(-\infty,-1)\cup(1,\infty)$ D) $x\in(-1,1)$

23. Given that $f(x)=\arcsin(x^2)$, compute $f'(x)$.

A) $\frac{1}{\sqrt{1-x^2}}$ B) $\frac{1}{\sqrt{1-x^4}}$ C) $\frac{2x}{\sqrt{1-x^2}}$ D) $\frac{2x}{\sqrt{1-x^4}}$ E) none of these

24. Oil is leaking from a container at the rate of $R(t) = 2000e^{-0.2t}$ gallons per hour, where t is time, measured in hours. How much oil has leaked out of the container after 10 hours? Round your answer to the nearest gallon.

A) 54 gallons B) 271 gallons C) 865 gallons D) 8647 gallons E) 14 778 gallons

25. All edges of a cube are expanding at a rate of $3\frac{\text{cm}}{\text{min}}$. How fast is the surface area of the cube changing when each edge is 2 cm long?

A) $12\frac{\text{cm}^2}{\text{min}}$ B) $24\frac{\text{cm}^2}{\text{min}}$ C) $36\frac{\text{cm}^2}{\text{min}}$ D) $72\frac{\text{cm}^2}{\text{min}}$ E) none of these

26. If $y = x + \sin(xy)$, then $\frac{dy}{dx} =$

A) $1 + \cos(xy)$ C) $\frac{1 + y\cos(xy)}{1 - x\cos(xy)}$ E) none of these

B) $1 + y\cos(xy)$ D) $\frac{1}{1 - x\cos(xy)}$

27. A virus is spreading through a population in a manner that can be modeled by the function $g(t) = \frac{A}{1 + Be^{-t}}$ where A is the total population, $g(t)$ is the number infected at time t, and B is a constant. What proportion of the population is infected when the virus is spreading the fastest?

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{3}{4}$ E) none of these

28. In a certain town, the rate of deaths at time t due to a particular disease is modeled by $\frac{10000}{(t+3)^3}$ where $t \geq 0$. What is the total number of deaths from this desease predicted by the model?

A) 243 B) 370 C) 556 D) 1111 E) none of these

29. A company has \$120 000 to spend on the development and promotion of a new product. The company estimates that if x is spent on the development and y is spent on promotion, then approximately $\frac{x^{1/2}y^{3/2}}{400\,000}$ items of new product will be sold. Based on this estimate, what is the maximum number of products that the company can sell?

A) 3897 B) 9000 C) 11 691 D) 30 000 E) none of these

30. A company can sell 20 products if it charges \$40 per product. For each dollar decrease or increase in the price, the company can sell one more or one less product, respectively. The total cost of producing q products is $C(q) = 32q + 100$. What is the maximum profit that the company can achieve from manufacturing and selling this product?

A) \$96 B) \$196 C) \$400 D) \$736 E) none of these

Answers

1.	D	11.	C	21.	B
2.	C	12.	B	22.	A
3.	B	13.	A	23.	D
4.	E	14.	D	24.	D
5.	D	15.	B	25.	D
6.	A	16.	B	26.	C
7.	A	17.	B	27.	B
8.	B	18.	D	28.	C
9.	D	19.	C	29.	C
10.	A	20.	B	30.	A