On quiz this Thursday:

1. Differentiating using the definition
2. relative extrema
3. tangent lines

⇒ there is a handout for all three!

Feb 18-6:02 PM

$$\lim_{x\to\infty} \frac{\log_2 24x^2}{\log_2 6x^3}$$

$$\frac{\log_2 24 + \log_2 x^2}{\log_2 6 + \log_2 x^3}$$

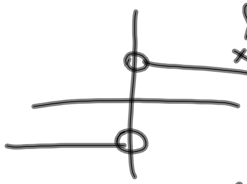
$$\frac{\log_2 24 + 2\cdot\log_2 x}{\log_2 6 + 3\cdot\log_2 x}$$

$$\frac{2\cancel{\log_2 x}\left(\frac{\log_2 24}{2\log_2 x}+1\right)}{3\cancel{\log_2 x}\left(\frac{\log_2 6}{3\log_2 x}+1\right)} \to \frac{2}{3}$$

$\left(\frac{\log_2 24}{2\log_2 x}+1\right) \to 1$, $\left(\frac{\log_2 6}{3\log_2 x}+1\right) \to 1$

$\frac{7+2x}{5+3x}$ as $x\to\infty$

Feb 18-6:06 PM



$$\lim_{x\to 0^-} \frac{|x|}{x} = -1$$

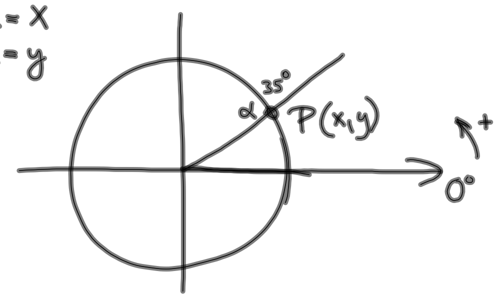
$$\lim_{x\to 0^+} \frac{|x|}{x} = 1$$

$$\lim_{x\to 0} \frac{|x|}{x} = \text{undefined}$$

Feb 18-6:11 PM

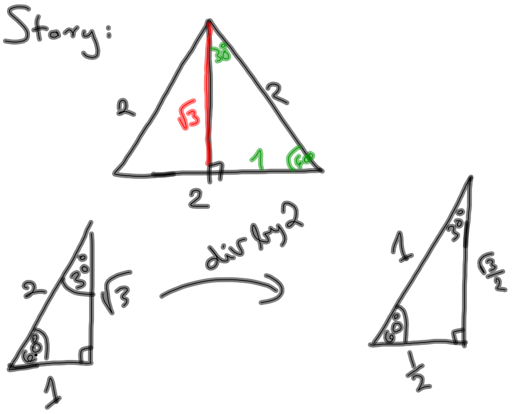
$\cos\alpha = x$

$\sin\alpha = y$

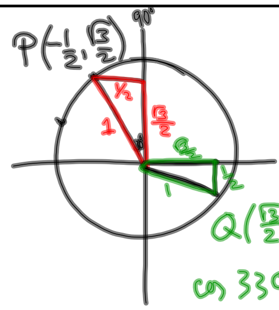


Feb 18-6:18 PM

Story:



Feb 18-6:20 PM



$\cos 120° = -\frac{1}{2}$

$\sin 120° = \frac{\sqrt{3}}{2}$

$\cos 330° = \frac{\sqrt{3}}{2}$

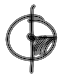
$\sin 330° = -\frac{1}{2}$

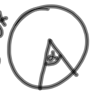
Feb 18-6:23 PM

Area of a circle?

$A = \pi r^2$

$A_{sector} = ?$





$r = 5ft$
$\alpha = 40°$

$A_{circle} = \pi r^2$

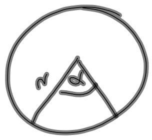
$A_{sector} = \frac{\pi r^2}{360} \cdot 40$

$= \frac{\pi (5ft)^2}{360} \cdot 40$

$= \frac{25\pi}{9} ft^2$ exact value

$\approx 8.726646\ ft^2$ approx.

Feb 18-6:27 PM



$A_{sector} = \frac{\pi r^2}{360} \cdot \alpha$

$A_{sector} = \frac{\pi r^2}{2\pi} \alpha_{rad}$

$A_{sector} = \frac{1}{2} r^2 \alpha_{rad}$

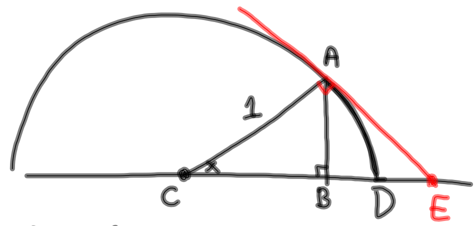
Feb 18-6:31 PM

Theorem: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

We will prove: $\lim_{x \to 0^+} \frac{\sin x}{x} = 1$

x is an angle, measured in radians, very small and positive

Feb 18-6:34 PM

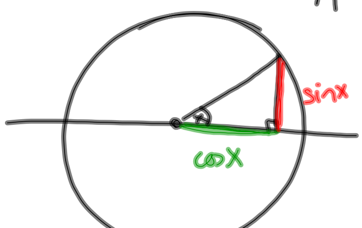


Compare: Area of triangle ABC
Area of sector ACD
Area of triangle ACE

Feb 18-6:39 PM

$A_{triangle}$ ABC

$A = \frac{1}{2} \sin x \cos x$

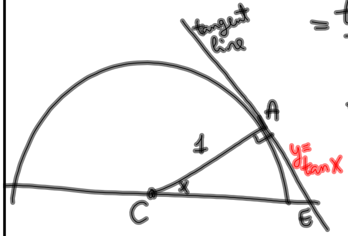


Feb 18-6:42 PM

$A_{sector}\ CAD = \frac{1}{2}(1)^2 \cdot x = \frac{x}{2}$

$A_{triangle}\ ACE = \frac{1}{2} \cdot (1) \tan x$

$= \frac{\tan x}{2}$



tanx from triangle ACE:

$\tan x = \frac{y}{1} \Rightarrow y = \tan x$

Feb 18-6:44 PM

$$\tfrac{1}{2}\sin x\cdot\cos x \le \frac{x}{2} \le \frac{\tan x}{2}$$

$$\sin x\cos x \le x \le \frac{\sin x}{\cos x}$$

as $x\to 0^+$

$$\underset{\substack{\downarrow\\ 1}}{\cos x} \le \frac{x}{\sin x} \le \underset{\substack{\downarrow\\ 1}}{\frac{1}{\cos x}} \qquad \div \sin x$$

By the Sandwhich rule, $\frac{x}{\sin x} \to 1$ as $x \to 0$

Then $\frac{\sin x}{x}$ also approaches 1.

$\lim\limits_{x\to 0^-} \frac{\sin x}{x}$ goes similarly

Feb 18-6:49 PM

Claim 2 $\lim\limits_{x\to 0} \frac{\cos x - 1}{x} = 0$

Pf.: $\lim\limits_{x\to 0} \frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$

$$\lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$

$$\underbrace{\lim_{x\to 0}\frac{\sin x}{x}}_{1}\cdot\underbrace{\lim_{x\to 0}\frac{\sin x}{1+\cos x}}_{0} = 0$$

Feb 18-6:54 PM

Differentiating sinx

$$f'(x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\cos x\cdot\frac{\sin h}{h} + \lim_{h\to 0}\frac{\sin x\cos h - \sin x}{h}$$

$$= \underbrace{\lim_{h\to 0}\cos x}_{\cos x}\cdot\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1} + \underbrace{\lim_{h\to 0}\sin x}_{\sin x}\cdot\underbrace{\lim_{h\to 0}\frac{\cos h - 1}{h}}_{0}$$

$$= \cos x$$

Feb 18-7:06 PM

If $f(x)=\sin x$, then $f'(x)=\cos x$

Feb 18-7:13 PM

Differentiating $\cos x$. If $f(x)=\cos x$, then

$$f'(x) = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\cos x\cdot\cos h - \cos x}{h} - \lim_{h\to 0}\frac{\sin x\cdot\sin h}{h}$$

$$= \underbrace{\lim_{h\to 0}\cos x}_{\cos x}\cdot\underbrace{\lim_{h\to 0}\frac{\cos h - 1}{h}}_{0} - \underbrace{\lim_{h\to 0}\sin x}_{\sin x}\cdot\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1}$$

Feb 18-7:18 PM

If $f(x)=\sin x$, then $f'(x)=\cos x$

If $f(x)=\cos x$, then $f'(x)=-\sin x$

$$\begin{tikzcd} & \sin x \arrow[dr, "()'"] & \\ -\cos x \arrow[ur, "()'"] & & \cos x \arrow[dl, "()'"] \\ & -\sin x \arrow[ul, "()'"] & \end{tikzcd}$$

Feb 18-7:21 PM

The 2 things we are not allowed to forget:

① The derivative measures the slope of the tangent line

② f and f' can always be thought of as location and velocity

Feb 18-7:31 PM

Activity

① C
② H
③ J
④ F
⑤ D
⑥ A
⑦ K
⑧ E
⑨ I
⑩ G
⑪ B

Feb 18-7:54 PM

Graphing polynomials

$f(x) = -2(x+5)(x+3)^2 x^5 (x-2)^4 (x-7)$

If we multiplied out,

$-2x^{13} + \ldots\ldots$

Feb 18-8:00 PM

Step 1. Leading term

$\Rightarrow$ end behavior

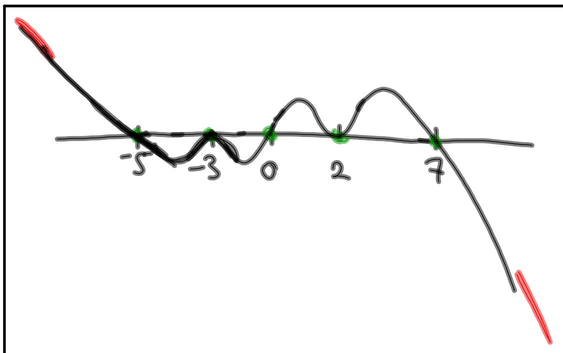
$-2x^{13}$

Step 2. Zeroes

$-5, -3, 0, 2, 7$

and no others !

Feb 18-8:07 PM



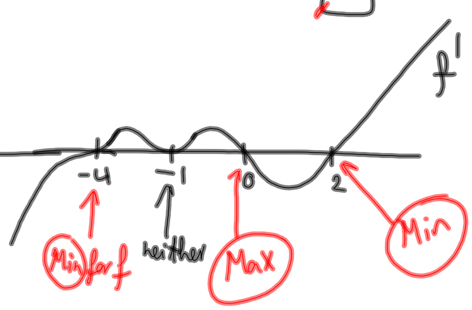
Feb 18-8:10 PM

$f'(x) = 3(x+4)^3 (x+1)^8 x (x-2)^5$

Graph f' and classify its zeroes as max or min or neither for f

Feb 18-8:13 PM

Leading term: $3x^{17}$



Feb 18-8:16 PM