$(3-x) = -(x-3)$

$(3-x)^2 = (x-3)^2$

because $(3-x)^2 = [-(x-3)]^2 = (-1)^2(x-3)^2 = (x-3)^2$

$(3-x)^3 = -(x-3)^3$

$(3-x)^4 = (x-3)^4$

$\vdots$

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$\frac{d}{dx}\left[(x-2)^5(4-x)^7\right]$

$\frac{d}{dx}\left[-(x-2)^5(x-4)^7\right]$

$-\frac{d}{dx}\left[(x-2)^5(x-4)^7\right]$

$-\left[5(x-2)^4(x-4)^7 + (x-2)^5 \cdot 7(x-4)^6\right]$

$-(x-2)^4(x-4)^6\left[5(x-4)+7(x-2)\right]$

$-(x-2)^4(x-4)^6\left[12x-34\right]$

$-12(x-2)^4(x-4)^6\left(x-\frac{17}{6}\right)$ $\quad \frac{34}{12} = \frac{17}{6}$

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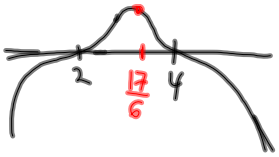
Graph $f(x) = (x-2)^5(4-x)^7$

$= -(x-2)^5(x-4)^7$



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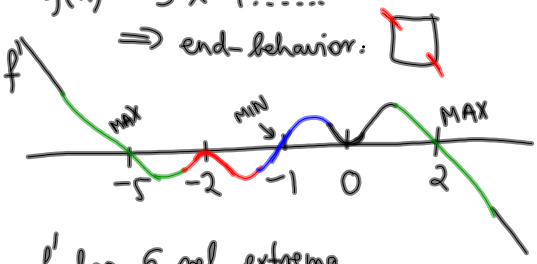
Graphing polynomials

$f(x) = -3(x+5)(x+2)^4(x+1)^3x^2(x-2)^5$

$f(x) = -3x^{15} + \ldots\ldots$

$\Rightarrow$ end-behavior:



f' has 6 rel. extrema

$\Rightarrow$ 6 pts of inflection

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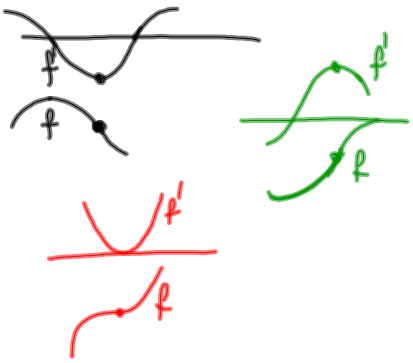

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Ex. $f'(x) = 6(x+4)^3(x+2)^2x(x-2)(x-6)^2$

a) Graph f'

b) Sort out zeroes of f' as max, min or pt of inf. for f

c) All together, how many pts of inflection?

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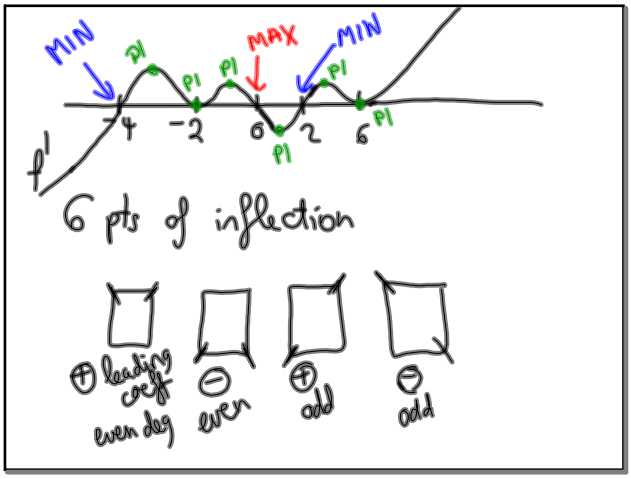

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Implicit differentiation

$(x+2)^2+(y-1)^2=25$ implicit form

$(y-1)^2=25-(x+2)^2$

$y-1=\pm\sqrt{25-(x+2)^2}$

$y=1\pm\sqrt{25-(x+2)^2}$ explicit form

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Implicit differentiation: why?

1. Sometimes it produces better results
2. Sometimes we cannot solve for y.

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$$\frac{d}{dx}\left[(f(x))^6\right]=6\cdot(f(x))^5\cdot f'(x)$$

By the chain rule

$x^3+y^3-2x+5y=x^2+y^2$

y is short for f(x)

$3x^2+3y^2\cdot y'-2+5y'=2x+2yy'$

$3y^2\cdot y'+5y'-2yy'=-3x^2+2+2x$

$y'(3y^2+5-2y)=-3x^2+2x+2$

$$y'=\frac{-3x^2+2x+2}{3y^2+5-2y}$$

Mar 20-6:50 PM

Unit circle

$x^2+y^2=1$ implicit form

explicit form: $y=\pm\sqrt{1-x^2}$

$$\frac{d}{dx}\left(\sqrt{1-x^2}\right)=\frac{1}{2\sqrt{1-x^2}}\cdot(-2x)$$

$$=\frac{-x}{\sqrt{1-x^2}}$$

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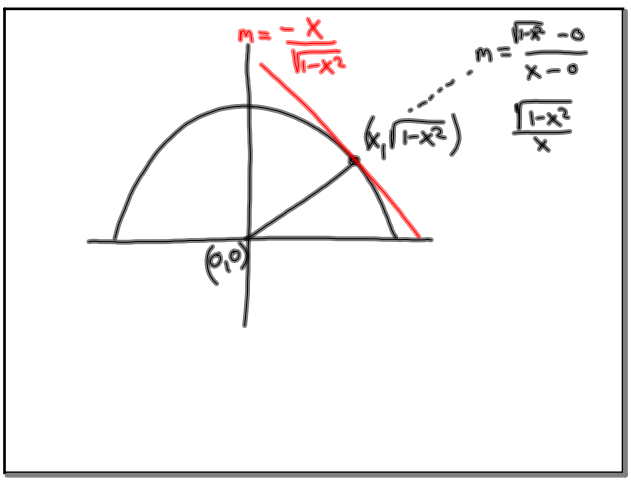

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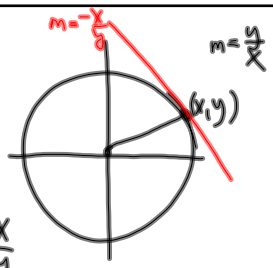
$x^2+y^2=1$

$2x+2yy'=0$

$2yy'=-2x$

$y'=\frac{-2x}{2y}=-\frac{x}{y}$



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$x^3+y^3-2x^2y=x-y^4$

$3x^2+3y^2y'-2\left(2xy+x^2y'\right)=1-4y^3y'$

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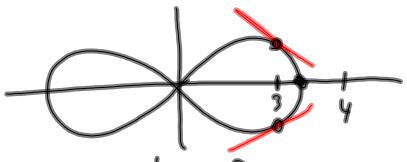

Bernoulli's Lemniscate

$2\left(x^2+y^2\right)^2=25\left(x^2-y^2\right)$

Find all tangent lines drawn to the graph at $x=3$

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$2\left(x^2+y^2\right)^2=25\left(x^2-y^2\right)$ $x=3,\ y=?$

$2\left(9+y^2\right)^2=25\left(9-y^2\right)$

$2\left(y^4+18y^2+81\right)=225-25y^2$

$2y^4+36y^2+162-225+25y^2=0$

$2y^4+61y^2-63=0$

$2\left(y^2\right)^2+61y^2-63=0$ Let $y^2=a$

$2a^2+61a-63=0$

$a_{1,2}=\frac{-61\pm\sqrt{61^2-4(2)(63)}}{4}=\frac{-61\pm\sqrt{65^2}}{4}$

$=\frac{-61\pm65}{4}=\begin{cases}\frac{4}{4}=1\\ \frac{-126}{4}=-\frac{63}{2}\end{cases}$

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Recall $a=y^2$

Either $y^2=1$ or $y^2=-\frac{63}{2}$

$\Downarrow$

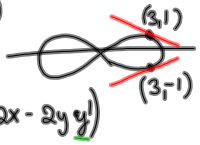
$y=\pm1$

No sol. here

Mar 20-7:20 PM

$2\left(x^2+y^2\right)^2=25\left(x^2-y^2\right)$

$4\left(x^2+y^2\right)\left[2x+2yy'\right]=25\left(2x-2yy'\right)$



$4\left(x^2+y^2\right)2x+4\left(x^2+y^2\right)2yy'=50x-50yy'$

$8\left(x^2+y^2\right)yy'+50yy'=50x-8x\left(x^2+y^2\right)$

$y'\left[4y\left(x^2+y^2\right)+25y\right]=25x-4x\left(x^2+y^2\right)$

$y'=\frac{25x-4x\left(x^2+y^2\right)}{25y+4y\left(x^2+y^2\right)}$ $x=3$ $y=1$

$y_1'=\frac{25(3)-12(10)}{25+4(10)}=\frac{-45}{65}=\frac{-9}{13}$

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