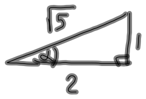
$$\cos\big(\underbrace{\tan^{-1}(\tfrac{1}{2})}_{\alpha}\big)$$

① $\tan\alpha = \frac{1}{2}$

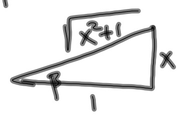


$\cos\alpha = ?$ $\frac{2}{\sqrt{5}}$

$$\sin\big(\underbrace{\tan^{-1}(x)}_{\beta}\big)$$

$\tan\beta = x = \frac{x}{1}$

$\sin\beta = ?$



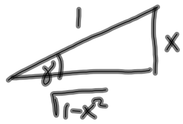
$\sin\beta = \frac{x}{\sqrt{x^2+1}}$

Apr 1-6:00 PM

$\sin(\sin^{-1}(x)) = x$

$$\cos\big(\underbrace{\sin^{-1}x}_{\gamma}\big) = \pm\sqrt{1-x^2}$$

$\sin\gamma = x$



$\cos\gamma = \frac{\sqrt{1-x^2}}{1}$

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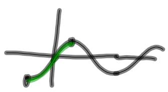
$\frac{d}{dx}(\sin^{-1}x) = ?$

$\sin(\sin^{-1}(x)) = x$

$\cos(\sin^{-1}x) \cdot \frac{d}{dx}(\sin^{-1}(x)) = 1$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\cos(\sin^{-1}x)} = \frac{1}{\pm\sqrt{1-x^2}}$

For the sign, look at graph



sin x restricted increasing

$\Rightarrow$ $\sin^{-1}x$ also increases

$\frac{d}{dx}(\sin^{-1}x)$ is ⊕

Answer is $\frac{1}{\sqrt{1-x^2}}$

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$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$

or $\arcsin x + C$

4e. $f(x) = \log_x(x^2+1)$

$= \frac{\ln(x^2+1)}{\ln x}$

quotient rule

$f'(x) = \frac{\frac{1}{x^2+1}(2x)\cdot\ln x - \ln(x^2+1)\cdot\frac{1}{x}}{(\ln x)^2}$

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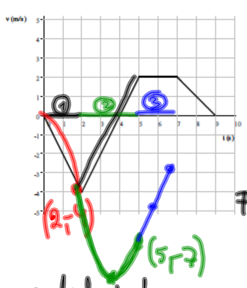

① Velocity function

$y = -2x$

Location function: antiderivative

$F(x) = \int -2x\,dx = -x^2 + C$

$F(0) = 0 \Rightarrow C = 0$

antiderivative is $F(x) = -x^2$

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$(2,-4)$ starting point

velocity function: $y = 2x - 8$

location: antiderivative

$F(x) = \int 2x - 8\,dx = x^2 - 8x + C$ $\quad x^2-8x+16$

$F(2) = -4 = 2^2 - 8(2) + C$ $\quad (x-4)^2$

$-4 = 4 - 16 + C$ $\quad x^2 - 8x + 8$

$-4 = -12 + C$ $\quad x^2-8x+16-16+8$

$8 = C$ $\quad (x-4)^2 - 8$

Apr 1-6:23 PM

Find a polynomial P of degree 3 so that $P(0)=5$ $P'(0)=2$ $P''(0)=4$ $P'''(0)=-24$

$P(x)=ax^3+bx^2+cx+d$ $P(0)=5$ $d=5$

$P'(x)=3ax^2+2bx+c$ $P'(0)=2$ $c=2$

$P''(x)=6ax+2b$ $P''(0)=4=2b$ $b=2$

$P'''(x)=6a$ $P'''(0)=6a=-24$ $a=-4$

$P(x)=-4x^3+2x^2+2x+5$

Apr 1-6:32 PM

$f(x)=\sin x$

$x-\frac{x^3}{6}$

$f(0)=P(0)$

$f'(0)=P'(0)$

$f''(0)=P''(0)$

$f'''(0)=P'''(0)$

Apr 1-6:43 PM

$f(x)=e^x$ $f(0)=1$

$f'(x)=e^x$ $f'(0)=1$

$f''(x)=e^x$ $f''(0)=1$

$\vdots$ $\vdots$

$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots$

Apr 1-6:56 PM

$f(x)=e^{-\frac{x}{2}}$

Find a degree 4 polynomial P with

$f(0)=P(0)$

$f'(0)=P'(0)$

$f''(0)=P''(0)$

$f'''(0)=P'''(0)$

$f^{(4)}(0)=P^{(4)}(0)$

Apr 1-7:00 PM

$f(x)=e^{-\frac{x}{2}}$ $f(0)=1$

$f'(x)=-\frac{1}{2}e^{-\frac{x}{2}}$ $f'(0)=-\frac{1}{2}$

$f''(x)=\frac{1}{4}e^{-\frac{x}{2}}$ $f''(0)=\frac{1}{4}$

$f'''(x)=\frac{-1}{8}e^{-\frac{x}{2}}$ $f'''(0)=\frac{-1}{8}$

$f^{(4)}(x)=\frac{1}{16}e^{-\frac{x}{2}}$ $f^{(4)}(0)=\frac{1}{16}$

Apr 1-7:01 PM

$P(x)=a_4x^4+a_3x^3+a_2x^2+a_1x+a_0$ $P(0)=1$

$P'(x)=4a_4x^3+3a_3x^2+2a_2x+a_1$ $P'(0)=-\frac{1}{2}$

$P''(x)=12a_4x^2+6a_3x+2a_2$ $P''(0)=\frac{1}{4}$

$P'''(x)=24a_4x+6a_3$ $P^{(3)}(0)=-\frac{1}{8}$

$P^{(4)}(x)=24a_4$ $P^{(4)}(0)=\frac{1}{16}$

$a_0=1$ $a_3=\frac{-1}{48}$

$a_1=\frac{-1}{2}$ $a_4=\frac{1}{384}$

$a_2=\frac{1}{8}$

$P(x)=1-\frac{1}{2}x+\frac{1}{8}x^2-\frac{1}{48}x^3+\frac{1}{384}x^4$

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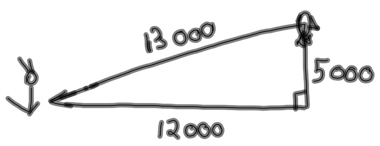

$s' = 480$

$y' = ?$

$x = 12000$

$y = 5000$

$$(12000)^2 + y^2 = s^2$$

$$2yy' = 2ss'$$

$$y' = \frac{ss'}{y} = \frac{13000 \cdot 480}{5000} = 1248$$

Apr 1-7:13 PM

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 10A = \cos^2 5A - \sin^2 5A$$

Apr 1-8:02 PM

$$\left[4 \sin 5x \cdot \cos 5x\right]' =$$

$$4\left[\underbrace{\cos 5x \cdot 5}_{f'} \cdot \underbrace{\cos 5x}_{g} + \underbrace{\sin 5x}_{f} \cdot \underbrace{(-\sin 5x)}_{g'} \cdot 5\right]$$

$$4\left[5\cos^2 5x - 5\sin^2 5x\right] =$$

$$20\left(\cos^2 5x - \sin^2 5x\right) = \boxed{20 \cos 10x}$$

Apr 1-8:10 PM

$$4 \sin 5x \cos 5x$$

$$2 \cdot \left(2 \sin 5x \cos 5x\right) = 2 \sin 10x$$

$$\frac{d}{dx}\left(2 \sin 10x\right) = 2\cos 10x \cdot 10$$

$$= \boxed{20 \cos 10x}$$

Apr 1-8:12 PM

$$\int \sin 3x \cos 3x \, dx$$

$$\int \frac{1}{2} \cdot \left(2 \sin 3x \cos 3x\right) dx$$

$$\int \frac{1}{2} \cdot \sin 6x \, dx = \frac{-\cos 6x}{2 \cdot 6} + C$$

$$= \boxed{\frac{-1}{12} \cos 6x + C}$$

$$(\;)' = \frac{-1}{12} \cdot -\sin 6x \cdot 6 = \frac{1}{2} \sin 6x = \frac{2 \sin 3x \cos 3x}{2}$$

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