Extra Credit Assignment:
due Next Tuesday Apr. 29
Beginning of class

Quiz on Thursday:
- Riemann Sums
- Differentiating inverse trig functions (Proofs too!)
- Antiderivatives after chain rule
- Implicit differentiation
- Optimization

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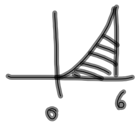
$\int_0^6 x^2\,dx$ definite integral



Signed area under the graph

$\int x^2\,dx$ indefinite integral
Find all antiderivatives for x^2

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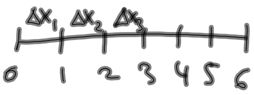
To compute a definite integral:

$\int_0^6 x^2\,dx$



Step 1. State a partition
Step 2. Compute a Riemann Sum
Step 3. Refine partition
$\Rightarrow$ approximations can only get better

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Some approximations overestimate the area. When we refine partitions, those overestimations decrease.

Other approximations underestimate the area. When we refine a partition, these underestimations increase.

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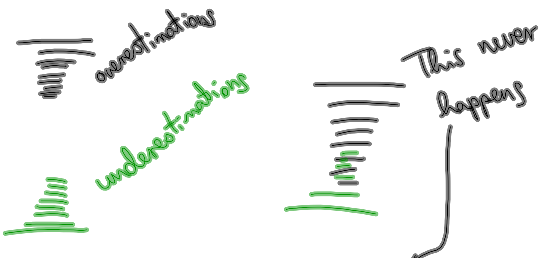

Any underestimate is less than any overestimate.

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Overestimations are bounded from below (by any underestimation) and underestimations are bounded from above. (by any overestimation)

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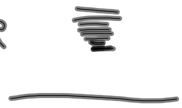
Let's look at overestimations.
As partitions are refined, they decrease. They are also bounded from below.
$\Rightarrow$ they will have a limit



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Same with underestimates.

Case 1.

Case 2.

If the limits are the same, we define that limit to be the area and say that f is integrable

If the limits are different, there is no area, and f is not integrable.

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If f is integrable, then all partitions and all type of Riemann Sums give us the same result.

If f is continuous, then f is integrable.

If f is increasing, then it is integrable.

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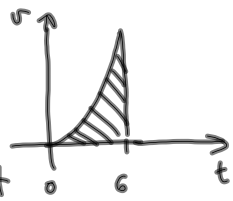
$$\int_0^6 x^2\,dx$$



If we "pretend" that x^2 is the velocity function, then the area under the graph is the distance traveled.

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If velocity function is x^2, then the location function is ~~the~~ an antiderivative.

$F(x) = \frac{x^3}{3}$

$F(0) = 0$
$F(6) = 72$

distance traveled: $72 - 0 = 72$

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Or: $F(x) = \frac{x^3}{3} + 5$

$F(0) = 5$
$F(6) = 77$

$F(6) - F(0) = 72$

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Fundamental Theorem of calculus (part 2) Evaluation theorem:

If f is continuous on $[a,b]$ and $F'=f$ on $[a,b]$, then

$$\int_a^b f(x)dx = F(b)-F(a)$$

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Another way of looking at this:

$$\int_a^b v(t)dt = L(b)-L(a)$$

area under velocity function

distance traveled

change in location

displacement

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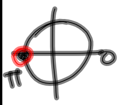
Notation:

$$\int_0^6 x^2 dx = \frac{x^3}{3}\Big|_0^6 = \frac{6^3}{3}-\frac{0^3}{3} = 72$$

$$\int_0^\pi \sin x\, dx = -\cos x\Big|_0^\pi = -\cos\pi-(-\cos 0)$$

$$=-(-1)+1=2$$



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$$\int_1^2 \frac{1}{x^2}dx = -\frac{1}{x}\Big|_1^2 = -\frac{1}{2}-\left(\frac{-1}{1}\right)$$

$$= -\frac{1}{2}+1 = \frac{1}{2}$$

$$\int_{-2}^5 \frac{1}{x^2}dx = \text{undefined}$$

we can NOT apply the fundamental theorem because $f(x)=\frac{1}{x^2}$ is not continuous on $[-2,5]$.

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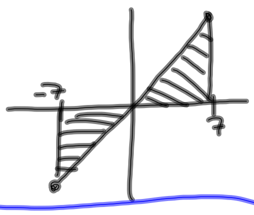
④



If f is an odd function

$$\int_{-a}^a f(x)dx = 0$$

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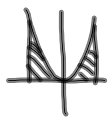
If f is even, then



$$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$$

$$\int_{-6}^6 x^2dx \quad \text{easier: } 2\int_0^6 x^2dx$$

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⑧ $$\int_{-4}^{-1} \frac{1}{x^2}\,dx = \left.\frac{-1}{x}\right|_{-4}^{-1}$$

$$= \left(\frac{-1}{-1}\right) - \left(\frac{-1}{-4}\right) = 1 - \frac{1}{4} = \frac{3}{4}$$

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