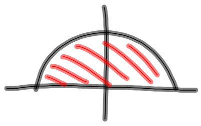
$$2\int_{-1}^{1}\sqrt{1-x^2}\,dx = 2\cdot\frac{\pi}{2} = \pi$$

$$f(x)=\sqrt{1-x^2}$$
$$y=\sqrt{1-x^2}$$
$$y^2=1-x^2$$
$$x^2+y^2=1$$



$$A=\frac{\pi r^2}{2}=\frac{\pi\cdot 1^2}{2}=\frac{\pi}{2}$$

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Summation Notation
Fundamental Theorem
Integration by Substitution

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Summation notation

$$\sum_{n=1}^{7}(n^2-2) = (1^2-2)+(2^2-2)+(3^2-2)+(4^2-2)+(5^2-2)+(6^2-2)+(7^2-2)$$
$$=\frac{7\cdot8\cdot15}{6}-2\cdot7 = 126$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

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sum

$$\sum_{n=1}^{100} n^2+1 = 1^2+1+2^2+1+3^2+1\ldots\ldots+100^2+1$$
$$=\frac{100\cdot101\cdot201}{6}+1\cdot100 = 338\,450$$

product

$$\prod_{k=1}^{12}\frac{k}{k+1} = \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdots\cdots\frac{11}{12}\cdot\frac{12}{13}$$
$$=\frac{1}{13}$$

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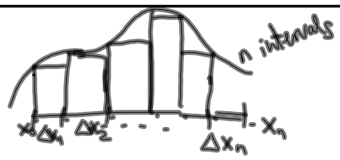
Riemann sums:



left sum: $f(x_0)\Delta x_1+f(x_1)\Delta x_2+\ldots+f(x_n)\Delta x_n$

right

$$f(x_1)\cdot\Delta x_1+f(x_2)\Delta x_2+\ldots+f(x_n)\cdot\Delta x_n$$

$$L=\sum_{j=0}^{n-1} f(x_j)\Delta x_{j+1}$$

$$R=\sum_{i=1}^{n} f(x_i)\Delta x_i$$

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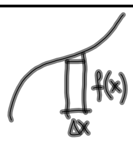
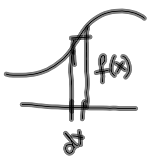
Riemann Sum:



$$\sum f(x_i)\Delta x_i$$



$$\int f(x)\,dx$$

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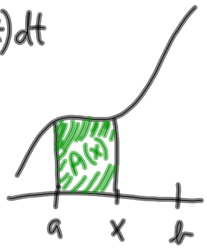
Fundamental theorem of calculus (Part 1)

Suppose f is integrable on $[a,b]$

Define $A(x) = \int_a^x f(t)\,dt$

Then A is differentiable and

$A' = f$



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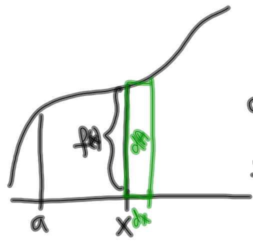

$dA = f(x) \cdot dx$

$\frac{dA}{dx} = f(x)$

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$$\frac{d}{dx}\left(\int_2^x \sin t\,dt\right) = \sin x$$

$$\frac{d}{dx}\left(\int_5^x \ln t\,dt\right) = \ln x$$

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Integration by Substitution

Reverses the Chain rule
(does other things as well.)

$f(x) = (3x^4 - x^2 + 5)^{100}$

$f'(x) = 100\,(3x^4 - x^2 + 5)^{99}\,(12x^3 - 2x)$

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$$\int 200\,\underbrace{(3x^4 - x^2 + 5)^{99}}_{f'(g(x))} \cdot \underbrace{(6x^3 - x)}_{g'(x)}\,dx$$

Let $u = 3x^4 - x^2 + 5$

$\frac{du}{dx} = 12x^3 - 2x$

$du = (12x^3 - 2x)\,dx$ Solve for dx

$\frac{du}{12x^3 - 2x} = dx$

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$$\int 200\,(3x^4 - x^2 + 5)^{99}\,(6x^3 - x)\,dx =$$

$$\int 200\,u^{99}\,\cancel{(6x^3 - x)}\,\frac{du}{\cancel{12x^3 - 2x}_{\,2}}$$

$$\int 200\,u^{99} \cdot \frac{du}{2} = \int 100\,u^{99}\,du$$

$$= \cancel{100}\,\frac{u^{100}}{\cancel{100}} + C = u^{100} + C = (3x^4 - x^2 + 5)^{100} + C$$

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After substitution of (at least) one expression in the integral and dx, we should have:

1) completely eliminated x (if not, the substitution did not work.)
2) the new integral should be something we can easily integrate

if either one of these fails, the substitution did not work.

Do not expect the first substitution for x to be the one that works. Keep trying.

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$$\int_0^2 (-3x+4)e^{-3x^2+8x}\,dx =$$

If $x=0$, then $u=-3\cdot 0^2+8\cdot 0 = 0$

If $x=2$, then $u=-3\cdot 2^2+8\cdot 2 = 4$

$\boxed{\begin{array}{l} \text{Let } u=-3x^2+8x \\ du=(-6x+8)dx \\ \frac{du}{-6x+8} = dx \end{array}}$

$$= \int_{?}^{?} (-3x+4)e^u \frac{du}{-6x+8} = \int_0^4 e^u \cdot \frac{du}{2} = \left.\frac{e^u}{2}\right|_0^4 = \frac{e^4 - e^0}{2} = \frac{e^4-1}{4}$$

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$$\int \frac{1}{x^2+1}dx = \tan^{-1}x + C$$

$$\int \frac{x}{x^2+1}dx =$$

Let $u=x^2+1$
$du=2x\,dx$
$\frac{du}{2x}=dx$

$$\int \frac{x}{u}\cdot\frac{du}{2x} = \int \frac{1}{2u}du = \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|x^2+1| + C = \frac{1}{2}\ln(x^2+1) + C$$

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$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$$

Let $u=\sqrt{x}$
$du=\frac{1}{2\sqrt{x}}dx$
$2\sqrt{x}\,du=dx$
$2u\,du=dx$

$$\int \frac{\cos u}{\cancel{u}}\cdot 2\cancel{u}\,du$$

$$2\int \cos u\,du = 2\sin u + C = 2\sin\sqrt{x} + C$$

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$$\int_0^3 \sqrt{5x+1}\,dx$$

Let $u=5x+1$
$du=5dx$
$\frac{du}{5}=dx$

$$\int_1^{16}\sqrt{u}\,\frac{du}{5} = \left.\frac{2}{15}u^{3/2}\right|_1^{16}$$

$$= \frac{2}{15}\left(16^{3/2} - 1^{3/2}\right) = \frac{2}{15}(64-1)$$

$$= \frac{2}{3\cdot 5}\cdot 63 = \frac{42}{5}$$

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$$\int_{-1}^{1}\sqrt{1-x^2}\,dx$$

idea: want: $x=\sin\theta$

Let $\theta = \sin^{-1}x$
$\Downarrow$
$\sin\theta = x$
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$x=\sin\theta$
$dx=\cos\theta\,d\theta$

$$\int_{-\pi/2}^{\pi/2}\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$$\int_{-\pi/2}^{\pi/2}\cos\theta\cdot\cos\theta\,d\theta$$

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$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} + \frac{\cos 2\theta}{2} \, d\theta$$

$$\boxed{\begin{gathered}\cos 2\theta = 2\cos^2\theta - 1 \\ \frac{1+\cos 2\theta}{2} = \cos^2\theta\end{gathered}}$$

$$\left.\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\left(\frac{\pi}{4} + \underset{=0}{\frac{\sin \pi}{4}}\right) - \left(\frac{-\pi}{4} + \underset{=0}{\frac{\sin(-\pi)}{4}}\right) = \frac{\pi}{2}$$

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